QUALITATIVE BEHAVIOR OF A DISCRETE PREY-PREDATOR MODEL WITH HOLLINGTYPE II RESPONSE

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Abstract: Investigation of the discrete time predator- prey model involving Holling type II functional response in the non-negative quadrant R_+^2 is carried out. For different parameter values time plots and phase portraits are acquired. Bifurcation diagrams are used for the discussion of abrupt change in the qualitative behavior of the system for the interior fixed point in R_+^2. The bifurcation of the system for Growth parameter 'r', interaction parameter 'a' and Holling type parameter 'b' are illustrated with 2-D and 3-D bifurcation diagrams. The simulations are performed numerically not only to support the theoretical analysis but to study the rich and complex behavior exhibited by the discrete system.

Keywords: Discrete System, Prey-Predator, Holling Type II, Stability, Fixed Point.

Predator-Prey Model: The building blocks of the ecosystems are for sure the predator-prey system as the biomasses are grown out of their resources. For the very existence of species, they compete and evolve for the purpose of finding the food resources. In order to model inter relationships between the species, various environmental factors are to be considered. More realistic food web chain models have been considered by the introduction of functional responses. The amount of time available for food searching is the simple functional response. Of the three Holling type functional responses, second functional response also known as prey dependent response function plays a crucial role and attracted many authors to study the dynamics of different prey predator systems using Holling type II response [4].

The population regulation is a density dependent process since the population density regulates the population growth rate. The amount of prey consumed by predators is assumed to have no upper limit in a basic predator-prey model. As the prey density increases, time needed for hunting is limited and thus there is an upper bound for the consumption rate. Complex problem of the two interacting species can be studied with discrete models. Discrete-time models described by difference equations are suitable to study the complexity of the dynamical behaviors in the interactions. In fact discrete time models are more reasonable when populations have non-overlapping generations [1, 2, 3, 6, 7].

Discussion of Model and Equilibrium Points: The purpose of this paper is to analyze the dynamical complexity of a discrete-time predator prey model represented by a system of Difference equations with Holling Type II functional response as follows

$$\begin{aligned} x(t+1) &= rx(t)[1-x(t)] - \frac{ax(t)}{b+x(t)}y(t) \\ y(t+1) &= (1-c)y(t) + \frac{ax(t)}{b+x(t)}dy(t) \end{aligned} \qquad (1)$$

where x(t) and y(t) are the prey and predator densities respectively. To be meaningful biologically, the parameters assume positive values. The logistic growth of the prey is considered with the growth parameter r. Here the introduction of the Holling type II functional response denoted by $\frac{ax(t)}{b+x(t)}$ explains the rate of prey consumption by a predator with change in the food density.

Parameter	Description
a	Maximum consumption rate
b	Number of prey at which predation rate is maximal
c	Natural death rate of predator
d	Reduction rate of predator

The equilibrium points of the system (1) are

1. $E_0 = (0,0)$
2. $E_1 = \left(\frac{r-1}{r}, 0\right)$
3. $E_2 = \left(\frac{bc}{ad-c}, bd\left[\frac{r-1}{ad-c} - \frac{rcb}{(ad-c)^2}\right]\right)$

The Equilibrium point E_0 is a trivial point with no prey and predator which is not an interesting phenomenon in an ecosystem. E_1 is a predator free equilibrium and the coexistence E_2 is a interior fixed point of the system (1).

Jacobian matrix of the transformation for the system (1) at (x^*, y^*)

$$J(x^*, y^*) = \begin{bmatrix} r - 2rx^* - \frac{ab}{(b+x^*)^2}y^* & -\frac{ax^*}{b+x^*} \\ \frac{abd}{b+x^*}y^* & (1-c) + \frac{adx^*}{b+x^*} \end{bmatrix}. \qquad (2)$$

is required for the stability analysis [5].

Stability Analysis: In this section, we will discuss the stability of the system (1) at interior equilibrium point $\boldsymbol{E_2}$.

Theorem 1 : The Equilibrium point E_2 is a sink if

$$b < min\left\{\frac{(r-1)(ad-c)}{rc}, \frac{c(1-r)(ad-c) - 2 - 2\left(\frac{rc+ad-c}{ad}\right)}{\frac{2rc}{ad} - \frac{4rc}{ad-c} - \frac{rc^2}{ad}}\right\}$$

and source if

$$b > \frac{(r-1)(ad-c)}{rc}$$

At E_2, the co-efficient matrix obtained from (2) is

$$J(E_2) = \begin{bmatrix} W & -\frac{c}{d} \\ d\left(r - \frac{2bcr}{ad-c} - W\right) & 1 \end{bmatrix}. \qquad (3)$$

and the eigenvalues obtained from $|J(E_2) - \lambda I| = 0$ are

$$\lambda_{1,2} = \frac{(W+1) \pm \sqrt{(W+1)^2 - 4\left[c\left(r - \frac{2rbc}{ad-c} - W\right)\right]}}{2}$$

where $W = r - \frac{2bcr}{ad-c} - \frac{1}{ad}[(r-1)(ad-c) - rcb]$. The following corollary is a consequence of Theorem (1)

Corollary 1: The Equilibrium point is a Saddle if

$$\frac{(r-1)(ad-c)}{rc} < b < min\left\{\frac{(r-1)(ad-c)}{rc}, \frac{c(1-r)(ad-c) - 2 - 2\left(\frac{rc+ad-c}{ad}\right)}{\frac{2rc}{ad} - \frac{4rc}{ad-c} - \frac{rc^2}{ad}}\right\}$$

Example 1: For the parameter values $r = 1.5, a = 0.3, b = 0.27, c = 0.02, d = 0.35$ with initial condition (0.2,0.4) the eigenvalues are $\lambda_1 = 1.978480893$ and $\lambda_2 = 0.0033118247$. Since $|\lambda| > 1$ and $|\lambda_2| < 1$, the stability criteria is satisfied. Therefore, the equilibrium point is stable and both the prey and predator converge to the equilibrium. The phase portrait in Figure-(1) represents the sink as the spiral moves inwards to the equilibrium point $E_2 = (0.06352941176, 0.4499377163)$.

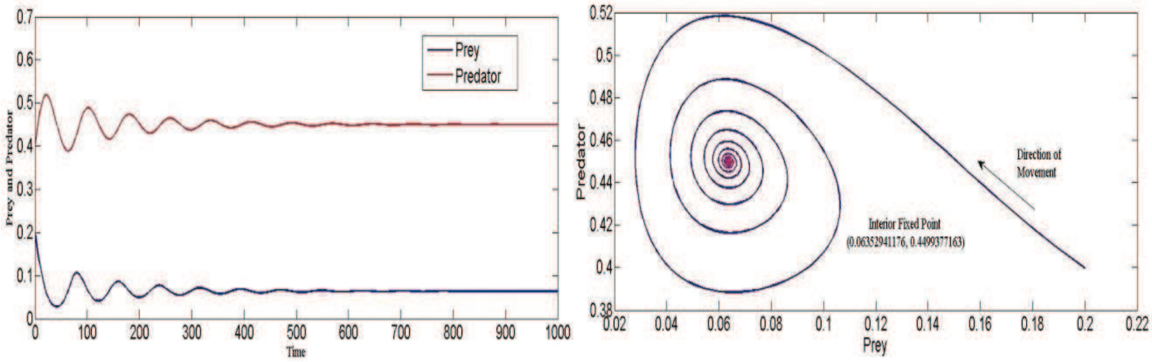

Figure 1: Stable solution of E_2

Example 2: The fixed point of the system (1) is unstable for the parameter values $r = 1.75, a = 0.3, b = 0.27, c = 0.02, d = 0.35$ with initial condition (0.2,0.4).The existence of limit cycle is illustrated by phase portrait in Figure-(2).

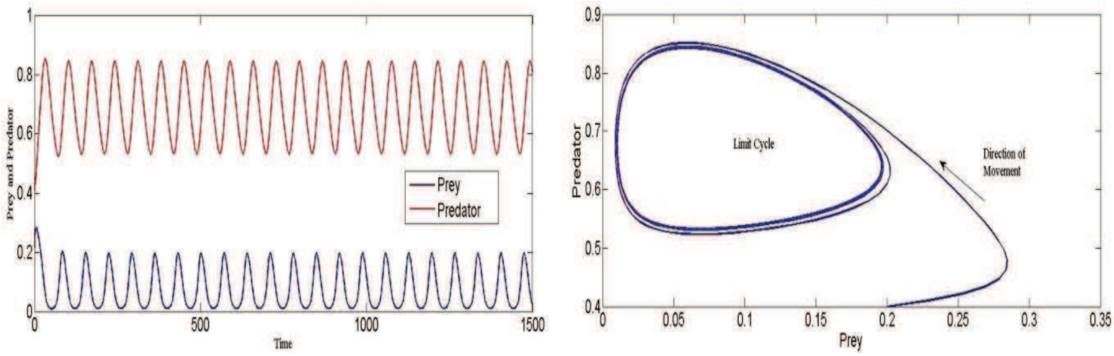

Figure 2: Periodic Solution of E_2

Qualitative Behavior of the System*:* In this section, we will discuss the change in qualitative behavior of the system with small change in growth parameter r and Holling type parameters a and b. Here we take bifurcation parameter along the horizontal axis and set of values of logistic function visited asymptotically for all initial conditions along vertical axis.

Bifurcation forr: Considering r as the bifurcation parameter ranging between $3.0 - 4.0$ with values of other parameters fixed. The values are assigned as $a = 0.6, b = 0.33, c = 0.2, d = 0.5$. The bifurcation graph for r, x and r, x and y is represented in Figure-(3).

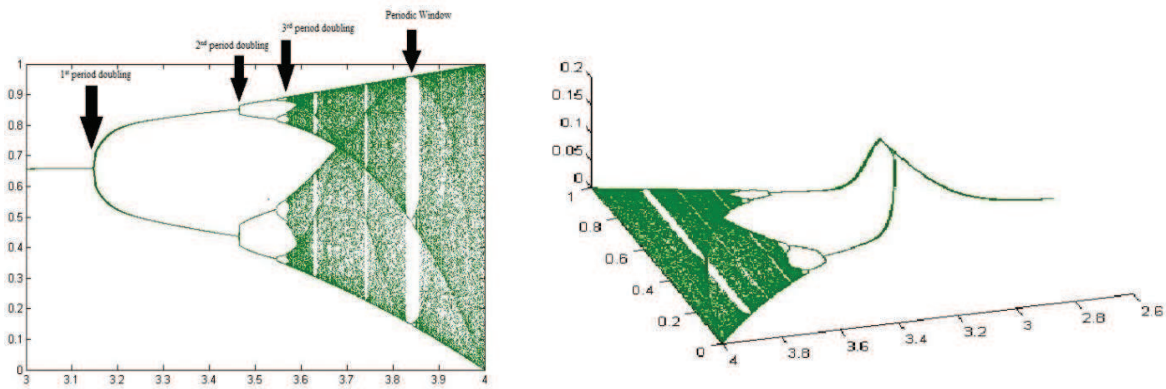

Figure 3: Bifurcation Diagram for r

Bifurcation for a: Considering $\mathbf{0.2-0.99}$ range for the bifurcation parameter **a** and the values of parameters $\mathbf{b=0.22, r=2.99, c=0.27, d=0.6}$. Figure-(4) explains the bifurcation graph for **a** and Prey. The graph of bifurcation for **a** , Prey and Predator is also given.

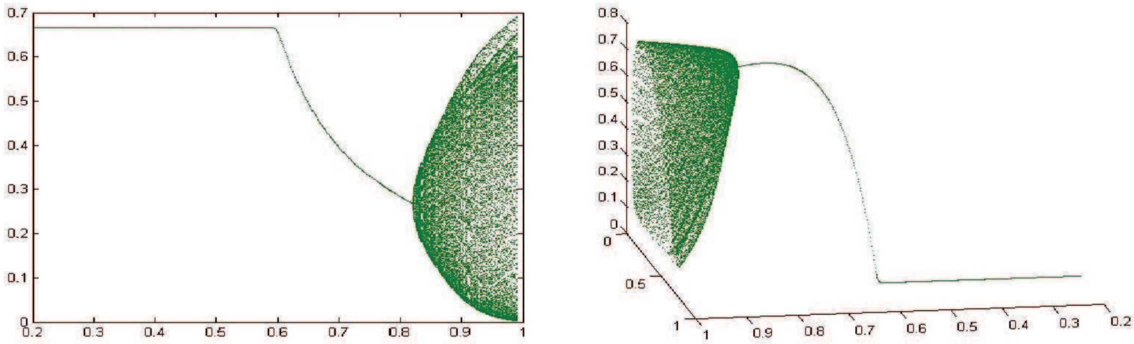

Figure 4: Bifurcation diagram for a

Bifurcation for b: With $\mathbf{0.2-1.4}$ as range for the bifurcation parameter **a** and the values of parameters $\mathbf{b=0.22, r=2.99, c=0.27}$ and $\mathbf{d=0.6}$, Figure-(5) explains the bifurcation graph for **b** and Prey, also for **b** , Prey and Predator.

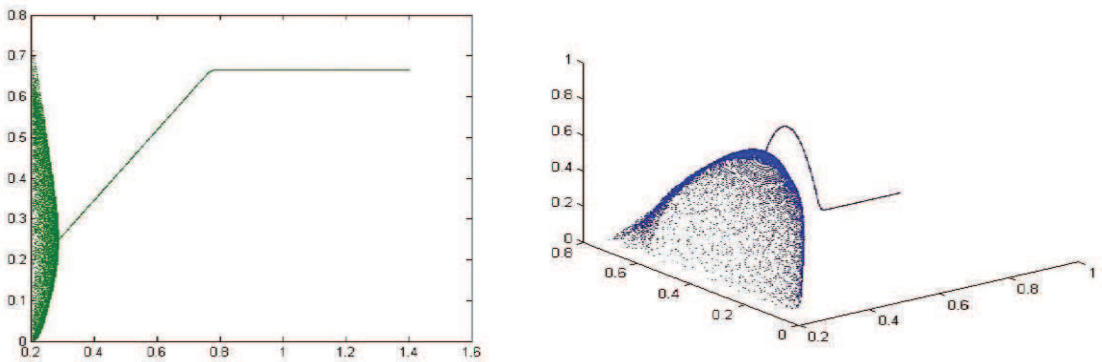

Figure 5: Bifurcation Diagram for b

Conclusion: The Dynamical properties of the discrete predator prey model with Holling type II response are discussed with suitable examples. Numerical simulations are performed to verify the theoretical analysis and bifurcation graphs are provided explaining the qualitative behavior of the predator prey model. Thus, rich dynamics of the model is visible through the diagrams provided.

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