A STUDY OF THE LEVEL OF SET FUNCTION USED TO DESCRIBE IMMERSED BODIES ON CARTESIAN GRID

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Abstract: In this paper we discuss wall boundary conditions for kinetic model are presented highlighting the need of devising a new boundary condition to deal with the asymptotic preserving properties. Here we introduced a new boundary condition is proposed is devoted to its enforcement at the desired order of accuracy on a Cartesian grid .we illustrates with numerical result the need of such a boundary condition and the accuracy of the proposed method. Here we also tested the sensibility the solution with respect to the value of tol in the case of BGK model. We compare the solution given by BGK model with the with the new Euler –AP boundary condition .The solution is polluted by the spurious fluxes at the boundary.

Keywords: Regime, Body –Fitted Grid, Cartesian Grid, Bilinear Interpolation And Embedded Boundary Method.

1. Introduction: A common feature of complex flows is the presence of bith rare fied and hydro dynamic regimes in the same field .This is the particularly true in the industry when dealing with vacuum pumps and hypersonic atmospheric re –entry vehicles for instance .For such ; this crucial effect makes difficult to reproduce experiments in real condition.This difficulty justify the use of numerical simulations.

The objective of this work is the simulation of complex rarefied flows with a particular care on the asymptotic behavior towards the hydro dynamic regime and on the viability of the numerical modeling for realistic test cases .The adaptability of the schemes too high performance computing is an essential aspect taken into account in order to reduce the computational time requirments for the simulation of complex cases .In particular ,we will devote our investigation to two relevant industrial problems:the particle dynamics in a nozzle plume of a satellite thuruster in a rarefied environment and a re-entry capsule. The case of nozzle plume has been investigated by 39 and shows that after firing , particle dust pollutes a collar even infront of the nozzle seefig1

2. Basic Methods: To compute the fictitious state at first order, the Maxwellian distribution function at the wall is built as presented with the same tangential velocity and the opposite normal velocity;

$$\S_{refl} = \S - 2\big((\S - U_W).n_w\big)n_w$$

Where $\S_{refl}$ the particle velocity after reflection , § the velocity before reflection U_W, the wall velocity and $n_w$the normal to the wall.This hold for each particle such that $(\S - U_W)\,.\,n_w > 0$. For $(\S - U_W).n_w < 0$,the distribution function on the boundary is already known and equal to the one in the fluid cell .The distribution function for the boundary is already known and equal to the one in the fluid cell. The distribution function for the boundary condition has to be computed only for $(\S - U_W)\,.\,n_w > 0$ .The entire distribution function f_s enforcing the boundary condition is then

$$f_s = \{f_w \text{ for } (\S - U_W).n_w < 0$$
$$f_w\left(\S_{refl}\right) \text{ for } (\S - U_W)\,.\,n_w > 0$$

In the reduced model, the same procedure is applied to ∅ and ¥ .T guarantees zero mass and energy fluxes (now U_W is set to zerofor simplicity:

$$F_{mass} = \int_{\S n_w < 0.} \S n_w f_w\,(\S) d\S + \int_{\S n_w > 0.} \S n_w f_w\,(\,\S_{refl}) d\S$$

$$= \int_{\S . n_w < 0.} \S . n_w f_w\,(\S) d\S + \int_{\S . n_w < 0.} - \S . n_w f_w\,(\,\S\,) d\S$$

= 0

$$F_{energy} \;=\; \int_{\S . n_w < 0.} |\,\S|^2\, \S . n_w f_w\,(\S) d\S + \int_{\S . n_w > 0.} |\,\S|^2\, \S . n_w f_w(\,\S_{refl}) d\S\,(\S) d\S$$

= $\int_{\S . n_w < 0.} |\,\S|^2\, \S . n_w f_w\,(\S) d\S + \int_{\S . n_w < 0.} |\,\S|^2 (-\,\S . n_w f_w(\,\S\,) d\S\,(\S)) d\S$

= 0

AP Impermeability Condition: A cure is proposed here to remove this spurious effect. It will be describe in the case of the BGK model
Let us assume that the distribution fuction is a Maxwellian (k_n number close to 0)

Then imposing impermeability condition at the wallcorrespond to impose a Maxwellian distribution function at the wall exactly as described.However, in this case the velocity must have the same tangential component of the fluid next to the boundary with zero wall normal component and temperature must be the same of the fluid . Therefore the contrast with what is done tangential velocity and temperature are extrapolated from fluid to the wall . If f_M such a boundary condition we have

$f_M = \frac{\rho'_w}{(2\pi T_w)^{3/2}} \exp\left(-\frac{|\,(\S - U'_W)|^2}{2T_w}\right)$ ....A

Where ρ'_w is the density verifying a zero mass flux , T_w is the temperature extrapolated from from the fluid and U'_W with a zero normal velocity component

$U'_W = U_W - (u.n_w)n_w.$
Let us now calculate the fluxes through the wall unit normal n_w .In hydrodynamic regime we have at the boundary

$f_w(\S) = \frac{\rho_W}{(2\pi T_W)^{3/2}} \exp\left(-\frac{|\,(\S - U_W)|^2}{2T_W}\right)$ ....B

Where ρ_w is the gas density close to the wall.
Then the fluxes through the wall are:
$F_{mass} = \frac{\rho_W}{(2\pi T_W)^{3/2}} \int_{\S . n_w < 0.} \S . n_w \exp\left(-\frac{|\,(\S - U_W)|^2}{2T_W}\right) d\S \; + \frac{\rho'_W}{(2\pi T_W)^{3/2}} \int_{\S . n_w < 0.} \S . n_w \exp\left(-\frac{|\,(\S - U'_W)|^2}{2T_W}\right) d\S$
= 0
The mass fluxes are identically zero because the density ρ'_w is computed such that it is zero.
$F_{energy} = \frac{\rho_W}{(2\pi T_W)^{3/2}} \int_{\S . n_w < 0.} .\, |\,\S|^2\, \S . n_w \exp\left(-\frac{|\,(\S - U_W)|^2}{2T_W}\right) d\S \; + \frac{\rho'_W}{(2\pi T_W)^{3/2}} \int_{\S . n_w > 0.} \S . n_w \exp\left(-\frac{|\,(\S - U'_W)|^2}{2T_W}\right) d\S$

If the velocity at the boundary U_w has a zero normal velocity component then the energy flux is zero . If not,the energy flux is not zero because the fluxes are calculated with an upwind scheme and then the distribution function for $\S . n_w < 0$ is taken from fluid and not the boundary condition imposes a zero normal velocity .The energy flux goes quickly to zero .At steady state .the mass and energy fluxes are always identically zero.The boundary condition is then expressed as
$f_{AP} = f_w$ for $(\S - U_w)\, n_w < 0$
f_M for $(\S - U_w)\, n_w > 0$

Boundary Condition on Cartesian Grid: In Cartesian grid we use Gaussian distribution function $\boldsymbol{G_f}$, The diffuse and specular reflection schemes are similar .Also it should be noted that this technique can be easily applied to the case of body fitted grids.

In case of solid immersed in the flow ,a fictitious state has to be created in the solid to compute the transport step numerically between a fluid cell and a cell containing the solid .The idea is first to compute the equivalent distribution function at the solid interface satisfying the imposed boundary condition and then create a fictitious state in the neighbour solid cellcalled ghost cell that respect the boundary value given approximiation order .To do so few parameters on the boundary are needed.

First Order AP-Scheme: To compute the fictitious state at first order ,the Maxwellian distribution function at the wall is built with the tangential velocity $\boldsymbol{U_A.\tau_{i,j1}}$ and temperature $\boldsymbol{T_A}$ taken from the fluid cell and a zero relative normal velocity $(\boldsymbol{U_A - U_W}).\boldsymbol{n_{i,j1}}$:

$T_A = T_{i,j1}$

$(U_A - U_W).\tau_{i,j1} = (U_{i,j1} - U_W)\tau_{i,j1}$

$(U_A - U_W).n_{i,j1} = 0$

The density ρ_A is calculated to the distribution function in cell (I,j1) invoking mass conservation through the wall.The Maxwellian built with ρ_A, U_A, T_A is then simply imposed as the state in the first solid cell$(i+1, j1)$. The part of the boundary condition f_s corresponding to the specular reflection and f_d corresponding to the diffuse boundary condition can be also easily computed from $f_{i,j1}$ and imposed velocity and temperature .Thus f_b is fully constructed.

Conclusion: In this paper ,we discuss without a particular treatment, standard methods are not AP in presence of interior boundaries .Because they exhibit spurious energy fluxes through the boundaries when the hydrodynamic limit is approached conditions. These artefact are purely numerical and a possible cure relies on a more accurate treatment of the boundary condition at the price of costly higher order interpolations in velocity space . Here we proposed an efficient boundary condition which ensures that an AP schemes remains up to wall boundary . We illustrated this result by comparing several numerical schemes to model the impermeability condition for the BGK model with emphasis on asymptotic preserving properties in the Euler limit.

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