LOWER LEVEL SUBHEMIRINGS OF AN INTERVAL VALUED ANTI-FUZZY SOFT SUBHEMIRING OF A HEMIRING

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Abstract: Today is a universe of uncertainty with its related issues, which can be all around took care of by soft set theory. In this paper, we propose lower level subhemirings of an interval valued anti-fuzzy soft subhemiring of a hemiring, its writes with cases and some new operators based on weights. We additionally consider their properties.

Keywords: Interval Valued Fuzzyset, Interval Valued Anti-Fuzzy Soft Subhemiring, Lower Level Anti-Fuzzy Soft Suhemirings of A Hemiring.

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1. Introduction: All things considered, situation, we confront such a large number of uncertainties, in all kinds of different backgrounds. Zadeh'sclassical idea of fuzzy set[20] is solid to manage such kind of issues. Since the start of fuzzy set hypothesis, there are recommendations for higher request fuzzy sets for various applications in numerous fields. Among higher fuzzy sets intuitionistic fuzzy set presented by Atanassov have been observed to be extremely valuable and material. Soft set hypothesis has gotten much consideration since its presentation by Molodtsov [8].The idea and essential properties of soft set hypothesis are introduced in [11,6]. Later on Maji et al. [7] have proposed the hypothesis of fuzzy soft set. Majumdar et al. have additionally summed up the idea of fuzzy soft sets. Maji et al. [6] stretched out fuzzy soft sets to intuitionistic fuzzy soft sets which depends on the mix of the intuitionistic fuzzy set and soft set. Yang et al. displayed the idea of the interim esteemed fuzzy soft sets by consolidating the interim esteemed fuzzy sets and soft set. They have likewise given a calculation to settle interim esteemed fuzzy soft set based basic leadership issues. In this paper we introduce the notion of interval valued fuzzy soft suhemirings of a hemiring and in the second section we also introduce the notion of interval valued anti-fuzzy soft subhemrings. Mainly we generalize the results of lower level interval valued anti-fuzzy soft subhemiring of a hemiring.

2. Preliminaries:

2.1 Definition: Let R be a hemiring. An intuitionistic fuzzy soft subset (F, A) of R is said to be an intuitionistic fuzzy soft subhemiring (IFSSHR) of R in case it satisfies the going with conditions:
(i) $\mu_{(F,A)}(x_{(F,A)} + y_{(F,A)}) \geq \min\{\mu_{(F,A)}(x_{(F,A)}), \mu_{(F,A)}(y_{(F,A)})\}$,(ii) $\mu_{(F,A)}(x_{(F,A)}\, y_{(F,A)}) \geq \min\{\mu_{(F,A)}(x_{(F,A)}), \mu_{(F,A)}(y_{(F,A)})\}$,(iii) $\nu_{(F,A)}(x_{(F,A)} + y_{(F,A)}) \leq \max\{\nu_{(F,A)}(x_{(F,A)}), \nu_{(F,A)}(y_{(F,A)})\}$,(iv) $\nu_{(F,A)}(x_{(F,A)}\, y_{(F,A)}) \leq \max\{\nu_{(F,A)}(x_{(F,A)}), \nu_{(F,A)}(y_{(F,A)})\}$, for all $x_{(F,A)}$ and $y_{(F,A)}$ in R.

2.2 Definition: Let R be a hemiring. An intuitionistic fuzzy soft subhemiring A of R is said to be an intuitionistic fuzzy soft normal subhemiring (IFNSHR) of R if it satisfies the going with conditions: (i) $\mu_{(F,A)}(x_{(F,A)}\, y_{(F,A)}) = \mu_{(F,A)}(y_{(F,A)}\, x_{(F,A)})$, (ii) $\nu_{(F,A)}(x_{(F,A)}\, y_{(F,A)}) = \nu_{(F,A)}(y_{(F,A)}\, x_{(F,A)})$, for all $x_{(F,A)}$ and $y_{(F,A)}$ in R.

2.3 Definition: Let (F, A) and (G, B) be intuitionistic fuzzy soft subsets of sets H and J, exclusively. The product of (F,A) and (G,B), implied by (F,A) x(G,B), is described as (F,A) x(G,B) $=\{ \langle (x_{(F,A)}, y_{(G,B)}), \mu_{(F,A)\, x(G,B)}(x_{(F,A)},y_{(G,B)}), \nu_{(F,A)\, x(G,B)}(x_{(F,A)},y_{(G,B)}) \rangle$ / for all $x_{(F,A)}$ in H and $y_{(G,B)}$ in J }, where $\mu_{(F,A)\, x(G,B)}(x_{(F,A)}, y_{(G,B)}) = \min\{ \mu_{(F,A)}(x_{(F,A)}), \mu_{(G,B)}(y_{(G,B)}) \}$ and $\nu_{(F,A)\, x(G,B)}(x_{(F,A)}, y_{(G,B)}) = \max\{ \nu_{(F,A)}(x_{(F,A)}), \nu_{(G,B)}(y_{(G,B)}) \}$.

2.4 Definition: Let (F,A) be an intuitionistic fuzzy soft subset in a set S, the most grounded intuitionistic fuzzy Soft relation on S, that is an intuitionistic fuzzy soft relation on ((F,A) is (G,V) given by $\mu_{(G,V)}(x_{(G,V)}, y_{(G,V)}) = \min\{ \mu_{(F,A)}(x_{(F,A)}), \mu_{(F,A)}(y_{(F,A)}) \}$ and $\nu_{(G,V)}(x_{(G,V)}, y_{(G,V)}) = \max\{ \nu_{(F,A)}(x_{(F,A)}), \nu_{(F,A)}(y_{(F,A)}) \}$, for all $x_{(F,A)}$ and $y_{(F,A)}$ in S.

2.5 Definition: An intuitionistic fuzzy soft subhemiring A of a hemiring R is presently as an intuitionistic fuzzy soft characteristic $\mu_{(F,A)}(x_{(F,A)}) = \mu_{(F,A)}(f(x_{(F,A)}))$ and $\nu_{(F,A)}(x_{(F,A)}) = \nu_{(F,A)}(f(x_{(F,A)}))$, for all $x_{(F,A)}$ in R and f in Aut (R)

2.6 Definition: (R, +, ·) and (R^{I}, +, ·) be any two Hemirings. Let $f : R \rightarrow R^{I}$ be any function and (F,A) be an intuitionistic fuzzy soft subhemiring in R, (G,V) be an intuitionistic fuzzy soft subhemiring in f(R) = R^{I}, portrayed by $\mu_{(G,V)}(y_{(G,V)}) = \sup_{x\in f^{-1}(y)} \mu_{(F,A)}(x_{(F,A)})$ and $\nu_{(G,V)}(y_{(G,V)}) = \inf_{x\in f^{-1}(y)} \nu_{(F,A)}(x_{(F,A)})$, for all $x_{(F,A)}$ in R and $y_{(G,V)}$ in R^{I}. At the point (F, A) is presently as a preimage of (G, V) under f and is indicated by $f^{-1}((G, V))$.

2.7 Definition: Let (F, A) be an intuitionistic fuzzy soft subhemiring of a hemiring (R, +, ·) and an in R. By at the point the pseudo intuitionistic fuzzy soft coset $(a(F,A))^{p}$ is described by $((a\mu_{(F,A)})^{p})(x_{(F,A)}) = p(a)\, \mu_{(F,A)}(x_{(F,A)})$ and $((a\nu_{(F,A)})^{p})(x_{(F,A)}) = p(a)\, \nu_{(F,A)}(x_{(F,A)})$, for every $x_{(F,A)}$ in R and for some p in P.

2.8 Definition: Let (F, A) be an intuitionistic fuzzy soft subset of X. For α, β in [0, 1], the level soft subset of (F,A) is the set , $(F,A)_{(\alpha,\beta)} = \{ x_{(F,A)}\in X : \mu_{(F,A)}(x_{(F,A)}) \geq \alpha, \nu_{(F,A)}(x_{(F,A)}) \leq \beta \}$. This is called an intuitionistic fuzzy soft level subset of A.

2.9 Definition: Let (F, A) be a fuzzy soft subset of X. For α in [0, 1], the lower level soft subset of (F, A) is the set $A_{\alpha} = \{ x_{(F,A)}\in X : \mu_{(F,A)}(x_{(F,A)}) \leq \alpha\}$.

3. Lower Level Subhemirings Of An Interval Valued Anti-Fuzzy Soft Subhemiring Of A Hemiring

3.1. Theorem: Let [F,A] be an interval valued anti-fuzzy soft subhemiring of a hemiring R. Then for α in [0,1], $[F,A]_{\alpha}$is a lower level soft subhemiring of R.

Proof: For all $x_{[F,A]}$ and $y_{[F,A]}$ in $[F,A]_{\alpha}$,we have, $\mu_{[F,A]}(x_{[F,A]}) \leq \alpha$ and $\mu_{[F,A]}(y_{[F,A]}) \leq \alpha$.Now, $\mu_{[F,A]}(x_{[F,A]} + y_{[F,A]}) \leq \max \{ \mu_{[F,A]}(x_{[F,A]}), \mu_{[F,A]}(y_{[F,A]}) \} \leq \max \{ \alpha, \alpha \} = \alpha$,which implies that $\mu_{[F,A]}(x_{[F,A]}+ y_{[F,A]}) \leq \alpha$.And, $\mu_{[F,A]}(x_{[F,A]}y_{[F,A]}) \leq \max\{ \mu_{[F,A]}(x_{[F,A]}), \mu_{[F,A]}(y_{[F,A]}) \} \leq \max\{ \alpha, \alpha \}= \alpha$,which implies that $\mu_{[F,A]}(x_{[F,A]}y_{[F,A]}) \leq \alpha$.Therefore, $\mu_{[F,A]}(x_{[F,A]}+ y_{[F,A]}) \leq \alpha$ and $\mu_{[F,A]}(x_{[F,A]}y_{[F,A]}) \leq \alpha$.Therefore , $x_{[F,A]} + y_{[F,A]}$ and $x_{[F,A]}y_{[F,A]}$ in $[F,A]_{\alpha}$.Hence $[F,A]_{\alpha}$ is a lower level subhemiring of a hemiring R.

3.2. Theorem: Let [F,A] be an interval valued anti-fuzzy soft subhemiring of a hemiring R. Then two lower level soft subhemiring $[F,A]_{\alpha_1}$, $[F,A]_{\alpha_2}$ and α_1, α_2 are in [0,1] with $\alpha_1<\alpha_2$ of [F,A] are equal if and only if there is no x in R such that $\alpha_2> [\mu_{[F,A]}](x_{[F,A]}) >\alpha_1$.

Proof: Assume that $[F,A]_{\alpha_1} = [F,A]_{\alpha_2}$.Suppose there exists $x_{[F,A]}$ in R such that $\alpha_2> [\mu_{[F,A]}](x_{[F,A]}) >\alpha_1$.Then $[F,A]_{\alpha_1} \subseteq [F,A]_{\alpha_2}$ implies $x_{[F,A]}$ belongs to $[F,A]_{\alpha_2}$, but not in $[F,A]_{\alpha_1}$.This is contradiction to $[F,A]_{\alpha_1} =$

$[F,A]_{\alpha_2}$.Therefore there is no $x \in R$ such that $\alpha_2 > [\mu_{[F,A]}](x_{[F,A]}) > \alpha_1$.Conversely if there is no $x_{[F,A]} \in R$ such that $\alpha_2 > [\mu_{[F,A]}](x_{[F,A]}) > \alpha_1$.Then $[F,A]_{\alpha_1} = [F,A]_{\alpha_2}$.

3.3 Theorem: Let R be a hemiring and [F,A] be a fuzzy soft subset of R such that $[F,A]_\alpha$ be a subhemiring of R. If α in [0,1], then [F,A] is an interval valued anti-fuzzy soft subhemiring of R.

Proof: Let R be a hemiring and x and y in R.Let $[\mu_{[F,A]}](x_{[F,A]}) = \alpha_1$ and $[\mu_{[F,A]}](y_{[F,A]}) = \alpha_2$.

Case (i): If $\alpha_1 < \alpha_2$, then $x_{[F,A]}, y_{[F,A]} \in [F,A]_{\alpha_2}$.As $[F,A]_{\alpha_2}$ is a subhemiring of R, $x_{[F,A]} + y_{[F,A]}$ and $x_{[F,A]}y_{[F,A]}$ in $[F,A]_{\alpha_2}$.Now, $\mu_{[F,A]}(x_{[F,A]} + y_{[F,A]}) \leq \alpha_2 = \max\{\alpha_1, \alpha_2\} = \max\{\mu_{[F,A]}(x_{[F,A]}), \mu_{[F,A]}(y_{[F,A]})\}$, which implies that $\mu_{[F,A]}(x_{[F,A]} + y_{[F,A]}) \leq \max\{\mu_{[F,A]}(x_{[F,A]}), \mu_{[F,A]}(y_{[F,A]})\}$, for all $x_{[F,A]}$ and $y_{[F,A]}$ in R.Also, $\mu_{[F,A]}(x_{[F,A]}y_{[F,A]}) \leq \alpha_2 = \max\{\alpha_1, \alpha_2\} = \max\{\mu_{[F,A]}(x_{[F,A]}), \mu_{[F,A]}(y_{[F,A]})\}$, Which implies that $\mu_{[F,A]}(x_{[F,A]}y_{[F,A]}) \leq \max\{\mu_{[F,A]}(x_{[F,A]}), \mu_{[F,A]}(y_{[F,A]})\}$, for all $x_{[F,A]}$ and $y_{[F,A]}$ in R.

Case (ii): If $\alpha_1 > \alpha_2$, then $x_{[F,A]}$ and $y_{[F,A]}$ in $[F,A]_{\alpha_1}$.As $[F,A]_{\alpha_1}$ is a subhemiring of R, $x_{[F,A]} + y_{[F,A]}$ and $x_{[F,A]}y_{[F,A]}$ in $[F,A]_{\alpha_1}$.Now, $\mu_{[F,A]}(x_{[F,A]} + y_{[F,A]}) \leq \alpha_1 = \max\{\alpha_2, \alpha_1\} = \max\{\mu_{[F,A]}(y_{[F,A]}), \mu_{[F,A]}(x_{[F,A]})\}$,which implies that $\mu_{[F,A]}(x_{[F,A]} + y_{[F,A]}) \leq \max\{\mu_{[F,A]}(x_{[F,A]}), \mu_{[F,A]}(y_{[F,A]})\}$, for all $x_{[F,A]}$ and $y_{[F,A]}$ in R.Also, $[\mu_{[F,A]}](x_{[F,A]}y_{[F,A]}) \leq \alpha_2 = \max\{\alpha_2, \alpha_1\} = \max\{\mu_{[F,A]}(y_{[F,A]}), \mu_{[F,A]}(x_{[F,A]})\}$, which implies that $\mu_{[F,A]}(x_{[F,A]}y_{[F,A]}) \leq \max\{\mu_{[F,A]}(x_{[F,A]}), \mu_{[F,A]}(y_{[F,A]})\}$, for all $x_{[F,A]}$ and $y_{[F,A]}$ in R.

Case (iii): If $\alpha_1 = \alpha_2$.

It is trivial. In all the cases, [F,A] is an interval valued anti-fuzzy soft subhemiring of a hemiring R.

3.4. Theorem: Let [F,A] be an interval valued anti-fuzzy soft subhemiring of a hemiring R. If any two lower level soft subhemirings of [F,A] belongs to R, then their intersection is also lower level soft subhemiring of A in R.

Proof: Let $\alpha_1, \alpha_2 \in [0,1]$.

Case (i): If $\alpha_1 < \mu_{[F,A]}(x_{[F,A]}) < \alpha_2$, then $[F,A]_{\alpha_1} \subseteq [F,A]_{\alpha_2}$.Therefore, $[F,A]_{\alpha_1} \cap [F,A]_{\alpha_2} = [F,A]_{\alpha_1}$, but $[F,A]_{\alpha_1}$ is a lower level soft subhemiring of [F,A].

Case (ii): If $\alpha_1 > [\mu_{[F,A]}](x_{[F,A]}) > \alpha_2$, then $[F,A]_{\alpha_2} \subseteq [F,A]_{\alpha_1}$.Therefore, $[F,A]_{\alpha_1} \cap [F,A]_{\alpha_2} = [F,A]_{\alpha_2}$, but A_{α_2} is a lower level soft subhemiring of [F,A].

Case (iii): If $\alpha_1 = \alpha_2$, then $[F,A]_{\alpha_1} = [F,A]_{\alpha_2}$.In all cases, intersection of any two lower level soft subhemirings is a lower level soft subhemiring of [F,A].

3.5. Theorem: Let [F,A] be an interval valued anti-fuzzy softsubhemiring of a hemiring R. If any two lower level soft subhemirings of [F,A] belongs to R, then their union is also a lower level soft subhemiring of [F,A] in R.

Proof: Let $\alpha_1, \alpha_2 \in [0,1]$.

Case (i): If $\alpha_1 < \mu_{[F,A]}(x_{[F,A]}) < \alpha_2$, then $[F,A]_{\alpha_1} \subseteq [F,A]_{\alpha_2}$.Therefore,$[F,A]_{\alpha_1} \cup [F,A]_{\alpha_2} = [F,A]_{\alpha_2}$, but $[F,A]_{\alpha_2}$ is a lower level soft subhemiring of A.

Case (ii): If $\alpha_1 > [\mu_{[F,A]}](x_{[F,A]}) > \alpha_2$, then $[F,A]_{\alpha_2} \subseteq [F,A]_{\alpha_1}$.Therefore , $[F,A]_{\alpha_1} \cup [F,A]_{\alpha_2} = [F,A]_{\alpha_1}$, but $[F,A]_{\alpha_1}$ is a lower level soft subhemiring of [F,A].

Case (iii): If $\alpha_1 = \alpha_2$, then $[F,A]_{\alpha_1} = [F,A]_{\alpha_2}$.In all cases, union of any two lower level soft subhemiring is also a lower level soft subhemiring of [F,A].

3.6. Theorem: The homomorphic image of a lower level soft subhemiring of an interval valued anti-fuzzy soft subhemiring of a hemiring R is a lower level soft subhemiring of an interval valued anti-fuzzy soft subhemiring of a hemiring R^I.

Proof: Let $(R, +, .)$ and $(R^{I}, +, .)$ be any two hemirings and $f : R \rightarrow R^{I}$ be a homomorphism.That is, $f(x + y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$, for all x and y in R.Let $[G,V] = f([F,A])$, where $[F,A]$ is an interval valued anti-fuzzy soft subhemiring of a hemiring R.Clearly $[G,V]$ is an interval valued anti- fuzzy soft subhemiring of a hemiring R^{I}.Let $x_{[F,A]}$ and $y_{[F,A]}$ in R, implies $f(x_{[G,V]})$ and $f(y_{[G,V]})$ in R^{I}.Let $[F,A]_{\alpha}$is a lower level subhemiring of $[F,A]$.That is, $\mu_{[F,A]}(x_{[F,A]}) \leq \alpha$ and $\mu_{[F,A]}(y_{[F,A]}) \leq \alpha$; $\mu_{[F,A]}(x_{[F,A]}+ y_{[F,A]}) \leq \alpha$, $\mu_{[F,A]}(x_{[F,A]}y_{[F,A]}) \leq \alpha$.We have to prove that $f([F,A]_{\alpha})$ is a lower level subhemiring of (G,V).Now, $\mu_{[G,V]}(f(x_{[G,V]})) \leq \mu_{[F,A]}(x_{[F,A]}) \leq \alpha$, which implies that $\mu_{[G,V]}(f(x_{[G,V]})) \leq \alpha$;and $\mu_{[G,V]}(f(y_{[G,V]})) \leq \mu_{[F,A]}(y_{[F,A]}) \leq \alpha$, which implies that $\mu_{[G,V]}(f(y_{[G,V]})) \leq \alpha$ and$\mu_{[G,V]}(f(x_{[G,V]}) + f(y_{[G,V]})) = \mu_{[G,V]}(f(x_{[G,V]} + y_{[G,V]}))$, as f is a homomorphism$\leq \mu_{[F,A]}(x_{[F,A]}+ y_{[F,A]}) \leq \alpha$,which implies that $\mu_{[G,V]}(f(x_{[G,V]}) + f(y_{[G,V]})) \leq \alpha$.Also, $\mu_{[G,V]}(f(x_{[G,V]})f(y_{[G,V]})) = \mu_{[G,V]}(f(x_{[G,V]}y_{[G,V]}))$, as f is a homomorphism$\leq \mu_{[F,A]}(x_{[F,A]}y_{[F,A]}) \leq \alpha$,which implies that $\mu_{[G,V]}(f(x_{[G,V]})f(y_{[G,V]})) \leq \alpha$.Therefore, $\mu_{[G,V]}(f(x_{[G,V]}) + f(y_{[G,V]})) \leq \alpha$, $\mu_{[G,V]}(f(x_{[G,V]})f(y_{[G,V]})) \leq \alpha$.Hence $f([F,A]_{\alpha})$ is a lower level subhemiring of an interval valued anti-fuzzy soft subhemiring $[G,V]$ of a hemiring R^{I}.

3.7. Theorem: The homomorphic pre-image of a lower level soft subhemiring of an interval valued anti-fuzzy soft subhemiring of a hemiring R^{I} is a lower level soft subhemiring of an interval valued anti-fuzzy soft subhemiring of a hemiring R.

Proof: Let $(R, +, .)$ and $(R^{I}, +, .)$ be any two hemirings and $f : R \rightarrow R^{I}$ be a homomorphism.That is, $f(x + y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$ for all x and y in R.Let $[G,V] = f([F,A])$, where $[G,V]$ is an interval valued anti-fuzzy soft subhemiring of a hemiring R^{I}.Clearly $[F,A]$ is an interval valued anti-fuzzy soft subhemiring of a hemiring R.Let $f(x_{[G,V]})$ and $f(y_{[G,V]})$ in R^{I}, implies x and y in R.Let $f([F,A]_{\alpha})$ is a lower level subhemiring of $[G,V]$.That is, $\mu_{[G,V]}(f(x_{[G,V]})) \leq \alpha$ and $\mu_{[G,V]}(f(y_{[G,V]})) \leq \alpha$; $\mu_{[G,V]}(f(x_{[G,V]}) + f(y_{[G,V]})) \leq \alpha$, $\mu_{[G,V]}(f(x_{[G,V]})f(y_{[G,V]})) \leq \alpha$.We have to prove that $[F,A]_{\alpha}$ is a lower level soft subhemiring of $[F,A]$.Now, $\mu_{[F,A]}(x_{[F,A]}) = \mu_{[G,V]}(f(x_{[G,V]})) \leq \alpha$, implies that $\mu_{[F,A]}(x_{[F,A]}) \leq \alpha$;$\mu_{[F,A]}(y_{[F,A]}) = \mu_{[G,V]}(f(y_{[G,V]})) \leq \alpha$, implies that $\mu_{[F,A]}(y_{[F,A]}) \leq \alpha$ and $\mu_{[F,A]}(x_{[F,A]} + y_{[F,A]}) = \mu_{[G,V]}(f(x_{[G,V]}+ y_{[G,V]})) = \mu_{[G,V]}(f(x_{[G,V]}) + f(y_{[G,V]}))$, as f is a homomorphism $\leq \alpha$,which implies that $\mu_{[F,A]}(x_{[F,A]} + y_{[F,A]}) \leq \alpha$.Also, $\mu_{[F,A]}(x_{[F,A]}y_{[F,A]}) = \mu_{[G,V]}(f(x_{[G,V]}y_{[G,V]})) = \mu_{[G,V]}(f(x_{[G,V]})f(y_{[G,V]}))$, as f is a homomorphism $\leq \alpha$.which implies that $[\mu_{[F,A]}](x_{[F,A]}y_{[F,A]}) \leq \alpha$.Therefore, $\mu_{[G,V]}(f(x_{[G,V]}) + f(y_{[G,V]})) \leq \alpha$, $\mu_{[G,V]}(f(x_{[G,V]})f(y_{[G,V]})) \leq \alpha$.Hence, $[F,A]_{\alpha}$is a lower level soft subhemiring of an interval valued anti-fuzzy intervalvaluedsoft subhemiring $[F,A]$ of R.

3.8. Theorem: The anti-homomorphic image of a lower level soft subhemiring of an interval valued anti-fuzzy soft subhemiring of a hemiring R is a lower level soft subhemiring of an interval valued anti- fuzzy soft subhemiring of a hemiring R^{I}.

Proof: Let $(R, +, .)$ and $(R^{I}, +, .)$ be any two hemirings and $f: R \rightarrow R^{I}$ be an anti-homomorphism.That is, $f(x + y) = f(y) + f(x)$ and $f(xy) = f(y)f(x)$, for all x and y in R.

Let $[G,V] = f([F,A])$, where $[F,A]$ is an interval valued anti-fuzzy soft subhemiring of R.Clearly $[G,V]$ is an interval valued anti-fuzzy soft subhemiring of R^{I}.Let $x_{[F,A]}$ and $y_{[F,A]}$ in R, implies $f(x_{[G,V]})$ and $f(y_{[G,V]})$ in R^{I}.Let $[F,A]_{\alpha}$is a lower level soft subhemiring of $[F,A]$.That is, $\mu_{[F,A]}(x_{[F,A]}) \leq \alpha$ and $\mu_{[F,A]}(y_{[F,A]}) \leq \alpha$.$\mu_{[F,A]}(y_{[F,A]} + x_{[F,A]}) \leq \alpha$, $\mu_{[F,A]}(y_{[F,A]}x_{[F,A]}) \leq \alpha$.We have to prove that $f([F,A]_{\alpha})$ is a lower level soft subhemiring of $[G,V]$.Now, $\mu_{[G,V]}(f(x_{[G,V]})) \leq \mu_{[F,A]}(x_{[F,A]}) \leq \alpha$, which implies that $\mu_{[G,V]}(f(x_{[G,V]})) \leq \alpha$;$\mu_{[G,V]}(f(y_{[G,V]})) \leq \mu_{[F,A]}(y_{[F,A]}) \leq \alpha$, which implies that $\mu_{[G,V]}(f(y_{[G,V]})) \leq \alpha$.Now, $\mu_{[G,V]}(f(x_{[G,V]}) + f(y_{[G,V]})) = \mu_{[G,V]}(f(y_{[G,V]} + x_{[G,V]}))$, as f is an anti-homomorphism $\leq \mu_{[F,A]}(y_{[F,A]} + x_{[F,A]}) \leq \alpha$, which implies that, $\mu_{[G,V]}(f(x_{[G,V]}) + f(y_{[G,V]})) \leq \alpha$.Also, $\mu_{[G,V]}(f(x_{[G,V]})f(y_{[G,V]})) = \mu_{[G,V]}(f(y_{[G,V]}x_{[G,V]}))$, as f is an anti-homomorphism $\leq \mu_{[F,A]}(y_{[F,A]}x_{[F,A]}) \leq \alpha$,which implies that $\mu_{[G,V]}(f(x_{[G,V]})f(y_{[G,V]})) \leq \alpha$.Therefore, $\mu_{[G,V]}(f(x_{[G,V]}) + f(y_{[G,V]})) \leq \alpha$and $\mu_{[G,V]}(f(x_{[G,V]})f(y_{[G,V]})) \leq \alpha$.Hence $f([F,A]_{\alpha})$ is a lower level soft subhemiring of an interval valued anti-fuzzy soft subhemiring $[G,V]$ of R^{I}.

3.9. Theorem: The anti-homomorphic pre-image of a lower level soft subhemiring of an interval valued anti-fuzzy soft subhemiring of a hemiring R^{I} is a lower level soft subhemiring of an interval valued anti-fuzzy soft subhemiring of a hemiring R.

Proof: Let $(R, +, .)$ and $(R^{I}, +, .)$ be any two hemirings and $f : R \rightarrow R^{I}$ be an anti-homomorphism.That is, $f(x + y) = f(y) + f(x)$ and $f(xy) = f(y)f(x)$, for all x and y in R.Let $[G,V] = f([F,A])$, where $[G,V]$ is an interval valued anti-fuzzy soft subhemiring of a hemiring R^{I}.Clearly $[F,A]$ is an interval valued anti-fuzzy soft subhemiring of a hemiring R.Let $f(x_{[G,V]})$ and $f(y_{[G,V]})$ in R^{I}, implies $x_{[F,A]}$ and $y_{[F,A]}$ in

R.Let f([F,A] $_\alpha$) is a lower level soft subhemiring of [G,V].That is, $\mu_{[G,V]}$ (f($x_{[G,V]}$)) ≤ α and $\mu_{[G,V]}$(f($y_{[G,V]}$)) ≤ α; $\mu_{[G,V]}$(f($y_{[G,V]}$) + f($x_{[G,V]}$)) ≤ α, $\mu_{[G,V]}$(f($y_{[G,V]}$) f($x_{[G,V]}$)) ≤ α.We have to prove that A_α is a lower level subhemiring of A.Now, $\mu_{[F,A]}$ ($x_{[F,A]}$) = $\mu_{[G,V]}$ (f($x_{[G,V]}$)) ≤ α, which implies that $\mu_{[F,A]}$ ($x_{[F,A]}$) ≤ α; $\mu_{[F,A]}$ ($y_{[F,A]}$) = $\mu_{[G,V]}$ (f($y_{[G,V]}$)) ≤ α, which implies that $\mu_{[F,A]}$ ($y_{[F,A]}$) ≤ α.Now, $\mu_{[F,A]}$ ($x_{[F,A]}$ + $y_{[F,A]}$) = $\mu_{[G,V]}$ (f($x_{[G,V]}$ + $y_{[G,V]}$)) = $\mu_{[G,V]}$ (f($y_{[G,V]}$) + f($x_{[G,V]}$)), as f is an anti-homomorphism≤ α, which implies that $\mu_{[F,A]}$ ($x_{[F,A]}$ + $y_{[F,A]}$) ≤ α.Also,$\mu_{[F,A]}$($x_{[F,A]}y_{[F,A]}$) = $\mu_{[G,V]}$ (f($x_{[G,V]}y_{[G,V]}$)) = $\mu_{[G,V]}$(f($y_{[G,V]}$)f($x_{[G,V]}$)), as f is an anti-homomorphism ≤ α,which implies that $\mu_{[F,A]}$ ($x_{[F,A]}y_{[F,A]}$) ≤ α.Therefore, $\mu_{[G,V]}$ (f($x_{[G,V]}$) + f($y_{[G,V]}$)) ≤ α and $\mu_{[G,V]}$(f($x_{[G,V]}$) f($y_{[G,V]}$)) ≤ α.Hence $[F,A]_\alpha$ is a lower level soft subhemiring of an interval valued anti-fuzzy soft subhemiring [F,A] of R.

3.10. Theorem: Let (R, +, .) be a hemiring and [F,A] be a non empty subset of R. Then [F,A] is a subhemiring of R if and only if [H,B] = < $\overline{\chi_{[F,A]}}$ > is an interval valued anti-fuzzy soft subhemiring of R, where $\overline{\chi_{[F,A]}}$ is the characteristic function.

Proof: Let (R, +, .) be a hemiring and [F,A] be a non empty subset of R. First let [F,A] be a subhemiring of R. Take x and y in R.

Case (i): If $x_{[F,A]}$ and $y_{[F,A]}$ in [F,A], then $x_{[F,A]}$ + $y_{[F,A]}$, $x_{[F,A]}y_{[F,A]}$ in [F,A], since [F,A] is a subhemiring of R, $\overline{\chi_{[F,A]}}$ ($x_{[F,A]}$) = $\overline{\chi_{[F,A]}}$ ($y_{[F,A]}$) = $\overline{\chi_{[F,A]}}$ ($x_{[F,A]}$ + $y_{[F,A]}$) = $\overline{\chi_{[F,A]}}$ ($x_{[F,A]}y_{[F,A]}$) =0. So, $\overline{\chi_{[F,A]}}$ ($x_{[F,A]}$+ $y_{[F,A]}$) ≤ max { $\overline{\chi_{[F,A]}}$ ($x_{[F,A]}$), $\overline{\chi_{[F,A]}}$ ($y_{[F,A]}$) }, for all $x_{[F,A]}$ and $y_{[F,A]}$ in R, $\overline{\chi_{[F,A]}}$ ($x_{[F,A]}y_{[F,A]}$) ≤ max { $\overline{\chi_{[F,A]}}$ ($x_{[F,A]}$), $\overline{\chi_{[F,A]}}$ ($y_{[F,A]}$) }, for all $x_{[F,A]}$ and $y_{[F,A]}$ in R.

Case (ii): If $x_{[F,A]}$ in [F,A], $y_{[F,A]}$ not in[F,A] (or $x_{[F,A]}$ not in [F,A], $y_{[F,A]}$ in [F,A]), then $x_{[F,A]}$ + $y_{[F,A]}$, $x_{[F,A]}y_{[F,A]}$ may or may not be in [F,A], $\overline{\chi_{[F,A]}}$ ($x_{[F,A]}$) =0, $\overline{\chi_{[F,A]}}$ (y) =1 (or $\overline{\chi_{[F,A]}}$ ($x_{[F,A]}$) =1, $\overline{\chi_{[F,A]}}$ ($y_{[F,A]}$) =0), $\overline{\chi_{[F,A]}}$ ($x_{[F,A]}$+$y_{[F,A]}$) = $\overline{\chi_{[F,A]}}$ ($x_{[F,A]}y_{[F,A]}$) =0 (or 1).
Clearly $\overline{\chi_{[F,A]}}$ ($x_{[F,A]}$ + $y_{[F,A]}$) ≤ max { $\overline{\chi_{[F,A]}}$ ($x_{[F,A]}$), $\overline{\chi_{[F,A]}}$ ($y_{[F,A]}$) }, for all $x_{[F,A]}$ and $y_{[F,A]}$ in R, $\overline{\chi_{[F,A]}}$ ($x_{[F,A]}y_{[F,A]}$) ≤ max { $\overline{\chi_{[F,A]}}$ ($x_{[F,A]}$), $\overline{\chi_{[F,A]}}$ ($y_{[F,A]}$) }, for all $x_{[F,A]}$ and $y_{[F,A]}$ in R.

Case (iii): If $x_{[F,A]}$ and $y_{[F,A]}$ not in [F,A], then $x_{[F,A]}$ +$y_{[F,A]}$, $x_{[F,A]}y_{[F,A]}$ may or may not be in [F,A], $\overline{\chi_{[F,A]}}$ ($x_{[F,A]}$) = $\overline{\chi_{[F,A]}}$ ($y_{[F,A]}$) = 1, $\overline{\chi_{[F,A]}}$ ($x_{[F,A]}$ + $y_{[F,A]}$) = $\overline{\chi_{[F,A]}}$ ($x_{[F,A]}y_{[F,A]}$) = 0 or 1.Clearly $\overline{\chi_{[F,A]}}$ ($x_{[F,A]}$+ $y_{[F,A]}$) ≤ max { $\overline{\chi_{[F,A]}}$ ($x_{[F,A]}$), $\overline{\chi_{[F,A]}}$ ($y_{[F,A]}$) }, for all $x_{[F,A]}$ and $y_{[F,A]}$ in R. $\overline{\chi_{[F,A]}}$ ($x_{[F,A]}y_{[F,A]}$) ≤ max { $\overline{\chi_{[F,A]}}$ ($x_{[F,A]}$), $\overline{\chi_{[F,A]}}$ ($y_{[F,A]}$}), for all $x_{[F,A]}$ and $y_{[F,A]}$ in R.
So in all the three cases, we have [H,B] is an interval valued anti-fuzzy soft subhemiring of a hemiring R.Conversely, let $x_{[F,A]}$ and $y_{[F,A]}$ in [F,A], since [F,A] is a non empty subset of R, so, $\overline{\chi_{[F,A]}}$ ($x_{[F,A]}$) = $\overline{\chi_{[F,A]}}$ ($y_{[F,A]}$) = 0.Since [H,B]= < $\overline{\chi_{[F,A]}}$ > is an interval valued anti-fuzzy soft sub hemiring of R, we have $\overline{\chi_{[F,A]}}$ ($x_{[F,A]}$ + $y_{[F,A]}$) ≤ max { $\overline{\chi_{[F,A]}}$ ($x_{[F,A]}$), $\overline{\chi_{[F,A]}}$ ($y_{[F,A]}$) }= max {0, 0 }= 0, $\overline{\chi_{[F,A]}}$ ($x_{[F,A]}y_{[F,A]}$) ≤ max { $\overline{\chi_{[F,A]}}$ ($x_{[F,A]}$), $\overline{\chi_{[F,A]}}$ ($y_{[F,A]}$) }= max {0, 0 } = 0.Therefore $\overline{\chi_{[F,A]}}$ ($x_{[F,A]}$ + $y_{[F,A]}$) = $\overline{\chi_{[F,A]}}$ ($x_{[F,A]}y_{[F,A]}$) = 0.
Hence $x_{[F,A]}$ + $y_{[F,A]}$ and $x_{[F,A]}y_{[F,A]}$in [F,A], so [F,A] is an interval valued anti-fuzzy soft subhemiring of R.

References:

1. Ali, M.I., F. Feng, X.Y. Liu, W.K. Min and M. Shabir, 2008. On some new operations in soft set theory. Comput. Math. Appl., pp: 2621-2628.

2. Ali, M.I., M. Shabir and M. Naz, 2011. Algebraic structures of soft sets associated with new operations. Comput. Math. Appl., 61: 2647-2654.
3. Atanassov, K., 1986. fuzzy sets. Fuzzy Sets and Systems, 20: 87- 96.
4. Feng, F., Y.B. Jun and X.Z. Zhao, 2008. Soft semirings, Computers and Math. with Appl., 56: 2621-2628.
5. Gau, W.L. and D.J. Buehrer, 1993. Vague sets. IEEE Trans. System, Man and Cybernet., 23 (2): 610-614.
6. Maji, P.K., R. Biswas and A.R. Roy, 2003. Soft set theory, Computers and Math. with Appl., 45: 555-562.
7. Maji, P.K., R. Biswas and A.R. Roy, 2001. Fuzzy soft sets. The J. Fuzzy Math., 9: 589-602.
8. Molodtsov, D., 1999. Soft set theory first results. Comput. Math. Appl., 37: 19-31
9. Pie, D. and D. Miao, 2005. From soft sets to information systems. Granular computing, IEEE Inter. Conf. 2: 617-621.
10. Roy, A.R. and P.K. Maji, 2007. A fuzzy soft set theoretic approach to decision making problems. J.of Comp. and App. Math., 203: 412-418.
11. Solairaju .A and R.Nagarajan, Charactarization of interval valued Anti fuzzy Left h-ideals over Hemirings, Advances in fuzzyMathematics,Vol.4,No.2, 129-136(2006).
12. Shabir, M. and M. Naz, 2011. On soft topological spaces, Comp. and Math. with App., 61: 1786 -1799.
13. Zadeh, L.A., 1965. Fuzzy sets. Inform. and Control, 8: 338-353.

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