A BASIC STRUCTURE OF NEARLY OPEN SETS IN NEUTROSOPHIC CRISP INFRA TOPOLOGICAL SPACE

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Abstract: As a new branch of philosophy, the neutrosophy was presented by Smarandache in 1980. It was presented as the study of origin, nature, and scope of neutralities as well as their interactions with different ideational spectra. In this paper we introduce the neutrosophic crisp infra set and the notion of neutrosophic crisp infra topological space. Also we construct the basic concepts of the neutrosophic crisp infra topology. The basic structure of nearly open sets in neutrosophic crisp infra topological space are also investigated. In addition to these, we introduce the definition of neutrosophic crisp infra continuous function and neutrosophic crisp infra relation. Furthermore, some properties of these concepts are investigated.

Keywords: Neutrosophic Crisp Infra Set, Neutrosophic Crisp Infra Topology, Neutrosophic Crisp Infra Open Sets And Neutrosophic Crisp Infra Relation.

Introduction: The idea of "Neutrosophic set" was first given by Smarandache [7]. The fuzzy sets was introduced by zadeh[10] in 1965. The intutionistic fuzzy sets was introduced by Atanassov[5,6] in the year 1983. Salama et al [1,2,3] investigated the neutrosophic operations in the year 2012. And he introduced the new concept of neutrosophic crisp topological space by generalizing the crisp topological space to the notion of neutrosophic crisp set.
In this paper we introduce the structure of some classes of neutrosophic crisp infra sets(NCIS). Some basic operations and properties of NCIS are discussed. The concept of neutrosophic crisp infra topology also introduced. We establish some properties of neutrosophic crisp infra topological space with supporting proofs.

Preliminaries: We recollect some relevant basic preliminaries and in particular, the work of Smarandache in [7] and Salama et al. [8]. Smarandache introduced the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where $]^{-}0\,,1^{+}[$ is non-standard unit interval. Salama et al. [3] considered some possible definitions for basic concepts of the neutrosophic crisp set and its operations.

Definition 2.1[3]: Let X be a non-empty fixed set. A neutrosophic crisp set (NCS) A is an object having the form A= $< A_1, A_2, A_3>$ where A_1, A_2 and A_3 are subsets of X satisfying $A_1 \cap A_2 = \phi$, $A_1 \cap A_3 = \phi$, $A_2 \cap A_3 = \phi$.

Definition 2.2[3]: A neutrosophic crisp topology(NCT) on a non-empty sets X is a family of neutrosophic crisp subsets in Γ satisfying the following axioms.
(i)$\emptyset_N, X_N \in \Gamma$
(ii) $A_1 \cap A_2 \in \Gamma$ for any A_1 and $A_2 \in \Gamma$.
(iii) $\cup A_j \in \Gamma$ for all $\{A_j : j \in J\} \subseteq \Gamma$.

In this case the pair (X, Γ) is called a neutrosophic crisp topological space(NCTS) in X. The elements in Γ are called neutrosophic crisp open sets(NCOS) in X. A neutrosophic crisp set F is closed if and only if its complement F^c is an open neutrosophic crisp set.

Definition 2.3[4]: Let X be any arbitrary set. An Infra –topological space on X is a collection τ_{iX} subsets of X such that the following axioms are satisfying:
Ax-1: ϕ , X $\epsilon\tau_{iX}$.
Ax-2:The intersection of the elements of any sub collection of τ_{iX} in X
i.e) If $O_i\epsilon\tau_{iX}$,$1\leq i \leq n \rightarrow \cap O_i \epsilon\tau_{iX}$.
Terminology, the order pair (X,τ_{iX}) is called infra-topological space. We simply say X is an infra space.

Definition 2.4[4]: Let (X,τ_{iX}) be an infra-topological space and A$\subset X$. A is called an infra open set (IOS) if A τ_{iX} .

Definition 2.5[4]: Let (X,τ_{iX}) be an infra topological space. A subset C $\subset X$ is called infra-closed set (ICS) in X if X \C is infra-open set in X.
(i.e) C is infra-closed set (ICS) iff
X \C $\epsilon\tau_{iX}$.

Definition 2.6[9]: Let (X,τ_{iX}) be an infra topological space. A set 'A' is called infra semi-open if A $\subseteq icp\ (iip\ (A))$and infra semi- closed set if $iip\ (icp\ (A)) \subseteq$ A.

Definition 2.7[9]: Let (X,τ_{iX}) be an infra topological space. A set 'A' is called infra pre-open if A $\subseteq iip\ (icp\ (A))$ and infra pre-closed set if $icp\ (iip\ (A)) \subseteq A$.

Definition 2.8[9]: Let (X,τ_{iX}) be an infra topological space. A set 'A' is called infra α-open if A $\subseteq iip\ (icp(iip(A)))$and infra α - closed set if $icp\ (iip\ (icp(A))) \subseteq A$.

Definition 2.9[9]: Let (X,τ_{iX}) be an infra topological space. A set 'A' is called infra β -open if A $\subseteq icp\ (iip\ (icp(A)))\ and$ infra β - closed set if $iip(icp(iip(A))) \subseteq A$.

Neutrosophic Crisp Infra Sets and its Operations in Neutrosophic Crisp Infra Topological Space:
We introduce and study the concepts of neutrosophic crisp infra sets and its operations.

Definition 3.1: For a neutrosophic crisp set A= < A_1, A_2, A_3> of the non-empty fixed set X, the neutrosophic crisp infra set A^i is defined to be the following triple structure: A^i= < A_1^i , A_2^i , A_3^i> , where
$A_1^i = A_1 \cap (A_2 \cap A_3)^c$,
$A_2^i = A_2 \cap (A_1 \cap A_3)^c$,
$A_3^i = A_3 \cap (A_1 \cap A_2)^c$.
And $A_j^i \subseteq A_j\ \forall$ j= 1, 2, 3.

Lemma 3.2: Let X be a non-empty fixed sample space. A neutrosophic crisp set A is an object having the form A= < A_1, A_2, A_3>. Then A^i= < $A_1 \cap (A_2 \cap A_3)^c, A_2 \cap (A_1 \cap A_3)^c , A_3 \cap (A_1 \cap A_2)^c$ > is also neutrosophic crisp set.

Definition 3.3: Let A^i= < A_1^i , A_2^i , A_3^i> be a NCIS on X, then the complement of the set A, may be defined in three different ways:
$(C_1)\ A^{i^c} =<\ A_1^{i^c} , A_2^{i^c} , A_3^{i^c} >$
$(C_2)\ A^{i^c} =< A_3^{i^c} , A_2^{i^c} , A_1^{i^c} >$
$(C_1)\ A^{i^c} =< A_3^{i^c} , A_2^{i^c} , A_1^{i^c} >$
The following are the several relations and operations between neutrosophic crisp infra sets.

Definition 3.4: Let X is a non-empty set, and the NCISs A^i and B^i in the form

A^i= < A_1^i , A_2^i , A_3^i> and B^i= < B_1^i , B_2^i , B_3^i> .Then the infra intersection and infra union of any two NCISs are defined as follows:
1)The neutrosophic crisp infra intersection of A^i, B^i defined in two ways:
i) $A^i \cap B^i = < A_1^i \cap B_1^i , A_2^i \cap B_2^i , A_3^i \cup B_3^i >$
ii)$A^i \cap B^i = < A_1^i \cap B_1^i , A_2^i \cup B_2^i , A_3^i \cup B_3^i >$
2)The neutrosophic crisp infra union of A^i, B^i defined in two ways: i) $A^i \cup B^i =$
$$< A_1^i \cup B_1^i , A_2^i \cap B_2^i , A_3^i \cap B_3^i >$$
ii) $A^i \cup B^i =$
$$< A_1^i \cup B_1^i , A_2^i \cup B_2^i , A_3^i \cap B_3^i >$$

Definition 3.5: Let X is a non-empty set, and the NCISs A^i and B^i in the form
A^i= < A_1^i , A_2^i , A_3^i> and B^i= < B_1^i , B_2^i , B_3^i>, then we may consider two possible definitions for subsets ($A^i \subseteq B^i$).
i) $A^i \subseteq B^i \Leftrightarrow A_1^i \subseteq B_1^i , A_2^i \subseteq B_2^i , A_3^i \supseteq B_3^i$ or
ii) $A^i \subseteq B^i \Leftrightarrow A_1^i \subseteq B_1^i , A_2^i \supseteq B_2^i , A_3^i \supseteq B_3^i$

Proposition 3.6: For any neutrosophic crisp infra set A^i, and the suitable choice of ϕ_N^i and X_N^i , the following are hold:
i) $\phi_N^i \subseteq A^i$, $\phi_N^i \subseteq \phi_N^i$.
ii)$A^i \subseteq X_N^i$, $X_N^i \subseteq X_N^i$.

Proposition 3.7: For any neutrosophic crisp infra sets A^i and B^i on X , then the following are true:
i) $(A^i \cap B^i)^c = A^{i^c} \cup B^{i^c}$
ii) $(A^i \cup B^i)^c = A^{i^c} \cap B^{i^c}$
The generalization of the operations of intersection and union given in definition 3.4 to arbitrary family of neutrosophic crisp infra subsets are as follows:

Proposition 3.8: Let $\{A_j : j \in J\}$ be arbitrary family of neutrosophic crisp infra subsets in X , then
1)$\bigcap_j A_j$ may be defined as the following types:
i) $\bigcap_j A_j = < \cap A_{j1}^i , \cap A_{j2}^i , \cup A_{j3}^i >$ or
ii) $\bigcap_j A_j = < \cap A_{j1}^i , \cup A_{j2}^i , \cup A_{j3}^i >$
2)$\bigcup_j A_j$ may be defined as the following types:
i) $\bigcup_j A_j = < \cup A_{j1}^i , \cup A_{j2}^i , \cap A_{j3}^i >$ or
ii) $\bigcup_j A_j = < \cup A_{j1}^i , \cap A_{j2}^i , \cap A_{j3}^i >$

Definition 3.9: A neutrosophic crisp infra topology(NCIT) on a non-empty sets X is a family of neutrosophic crisp infra subsets in Γ_X^i satisfying the following axioms:
(i)$\emptyset_N, X_N \in \Gamma_X^i$
(ii) $A_1^i \cap A_2^i \in \Gamma$ for any A_1^i and $A_2^i \in \Gamma_X^i$.
In this case the pair (X, Γ_X^i) is called a neutrosophic crisp infra topological space(NCITS) in X. The elements in Γ_X^i are called neutrosophic crisp infra open sets(NCIOS) in X. A neutrosophic crisp set F is infra closed if and only if its complement F^c is an infra open neutrosophic crisp set.

Definition 3.10: Let (X, Γ_X^i) be NCITS and A^i= < A_1^i , A_2^i , A_3^i> be a NCIS in X. Then the neutrosophic crisp infra closure of A (NCicp(A^i)) and neutrosophic crisp infra interior of A (NCiip(A^i)) of A are definied by
NCicp(A^i)= $\cap$ $\{C_i : C_i \textit{ is an NCICS in X and } A^i \subseteq C_i$
NCiip(A^i)= $\cup$ $\{O_i : O_i \textit{ is an NCIOS in X and } O_i \subseteq A^i$
Where NCICS is a neutrosophic crisp infra closed set and NCIOS is a neutrosophic crisp infra open set.
It can be also shown that NCicp(A^i) is a NCICS and NCiip(A^i) is a NCIOS in X.
a) A^i is in X if and only if NCicp(A^i) $\supseteq A^i$.
b) A^i is a NCICS in X if and only if NCiip(A^i) =A^i.

Proposition 3.11: For any neutrosophic crisp infra set A^i in $(X,\ \Gamma_X^i)$ we have

a) NNicp(A^i) = $(NCiip(A^i))^c$

b) NNiip(A^i) = $(NCicp(A^i))^c$

Proof: a)Let A^i= < A_1^i , A_2^i , A_3^i> and suppose that the family of neutrosophic crisp infra subsets contained in A^i are indexed by the family if NCISs contained by the family A^i={<A_{j1}^i , A_{j2}^i , A_{j3}^i >: i ∈ J}. Then We see that we have two types of NCiip(A^i)= {<∪ A_{j1}^i ,∪ A_{j2}^i ,∩ A_{j3}^i>} or NCiip(A^i)= {<∪ A_{j1}^i ,∩ A_{j2}^i ,∩ A_{j3}^i>}.

Hence (NCiip(A^i))^c={<∩ A_{j1}^i, ∩ A_{j2}^i, ∪ A_{j3}^i>} or (NCiip(A^i))^c={<∩ A_{j1}^i, ∪ A_{j2}^i, ∪ A_{j3}^i>}.

Hence NCicp(A^{i^c})= (NCiip(A^i))^c , which is analogous to (a).

Proposition 3.12:

Let (X, Γ_X^i) be a NCITS and A^i, B^i be two neutrosophic crisp infra sets in X. Then the following properties hold:

(a) NCiip(A^i) ⊆ A^i,
(b) A^i ⊆ NCicp(A^i),
(c) A^i ⊆ B^i ,NCiip(A^i) ⊆ NCiip(B^i),
(d) A^i ⊆ B^i,NCicp(A^i) ⊆ NCicp(A^i),
(e) NCiip(A^i ∩ B^i) =NCiip(A^i) ∩ NCiip(B^i),
(f) NCicp(A^i ∪ B^i) = NCicp(A^i) ∪ NCicp(B^i),
(g) NCiip($\phi_N^i{}_N^i$) = X_N^i,
(h) NCicp(ϕ_N^i) = ϕ_N^i

Proof: (a), (b) and (c) are obvious; (c) follows from (a) and from definitions.

Example 3.13: Let X={a, b, c, d, e, f}, A_1^i=<{a, b, c, d}>,{e},{f}>, B_1^i=<{a, b, c,},{d},{e}> are NCIS. Then the complement may be equal as:

1. A^{i^c}=<{e, f},{a, b, c, d, f},{a, b, c, d}>
   A^{i^c}=<{f}, {e}, {a, b, c, d}>
   A^{i^c}=<{f}, {a, b, c, d, f}, {a, b, c, d}>
2. B^{i^c}=<{e, f},{a, b, c, d, f},{a, b, c, d}>
   B^{i^c}=<{f}, {e}, {a, b, c, d}>
   B^{i^c}=<{f}, {a, b, c, d, f}, {a, b, c, d}>
3. A^i ∩ B^i may be equals the following forms:
   A^i ∩ B^i =< {a, b, c,}, {e, d}, {f, e}>
   A^i ∩ B^i =< {a, b, c,}, ϕ, {f, e}>
4. A^i ∪ B^i may be equal the following forms:
   $A^i \cup\ B^i =<\{\mathrm{a,b,c,d}\},\{\mathrm{e}\},\phi>$
   $A^i \cup\ B^i =<\{\mathrm{a,b,c,d}\},\phi,\{\mathrm{f}\}>$

Here we give the definition relation on neutrosophic crisp infra sets and study of its properties.
Let X, Y, Z be three ordinary non-empty sets.

Definition 3.14: The Cartesian productof two neutrosophic crisp infra sets A^i and B^i is a neutrosophic crisp infra set $A^i \times B^i$ is given by $A^i \times B^i = \langle A_1^i \times\ B_1^i, A_2^i \times B_2^i, A_3^i\ \times B_3^i\rangle$.
We will call a neutrosophic crisp infra relation $R^i \subseteq\ A^i \times B^i$ on $X\ \times Y$ is denoted as NCIR($X\ \times Y$).

Definition 3.15: Let R^i be a neutrosophic crisp infra relation on $X\ \times Y$, then the inverse of R^i is dented by ${R^i}^{-\mathbf{1}} \subseteq\ B^i \times A^i$ on $Y\ \times X$.

Example 3.16: Let X={a, b, c, d, e, f}, A_1^i=<{a, b, c, d}>,{e},{f}>, B_1^i=<{a, b, c,},{d},{e}> are NCIS. Then the product of neutrosophic crisp infra sets given by

1) $A^i \times B^i$ =
$\langle\{(\mathrm{a,a}),(\mathrm{a,b}),(\mathrm{a,c}),(\mathrm{b,a}),(\mathrm{b,b}),(\mathrm{b,c}),(\mathrm{c,a}),(\mathrm{c,b}),(\mathrm{c,c}),(\mathrm{d,a}),(\mathrm{d,b}),(\mathrm{d,c}),(\mathrm{d,d})\},\ \{(\mathrm{e,d})\},\{(\mathrm{f,e})\}\rangle$

2) $B^i \times A^i$ = ⟨{(a, a), (a, b), (a, c), (a, d), (b, a), (b, b), (b, c), (b, d), (c, a), (c, b), (c, c), (c, d)}, {(d, e)}, {(e, f)}⟩
And R_1^i = ⟨{(a, a)}, {(c, c)}, {(d, d)}⟩ .
Here $R_1^i \subseteq A^i \times B^i$ on $X \times Y$.

A New Forms Of Nearly Open Sets In Neutrosophic Crisp Infra Topological Space: In this section, we introduce the concept of neutrosophic crisp infra open sets such as neutrosophic crisp infra semi-open set, neutrosophic crisp infra pre- open set, neutrosophic crisp infra α-open set, neutrosophic crisp infra β –open in neutrosophic crisp infra topological space and study some of its properties.

Definition 4.1: Let (X, Γ_X^i) be a neutrosophic crisp infra topological space. A set $A^i = < A_1^i , A_2^i , A_3^i >$ be a NCIS in X, then 'A^i' is called
i)Neutrosophic crisp infra semi-open iff $A^i \subseteq NCicp\ (NCiip\ (A^i))$.
ii)Neutrosophic crisp infra pre-open iff $A^i \subseteq NCiip\ (NCicp\ (A^i))$
iii)Neutrosophic crisp infra α-open iff $A^i \subseteq NCiip\ (NCicp(NCiip(A^i)))$
iv)Neutrosophic crisp infra β -open iff $A^i \subseteq NCicp\ (NCiip\ (NCicp(A^i)))$.
We shall denote the class of all Neutrosophic crisp infra α-open sets NCIT$^{\alpha}$, the class of all Neutrosophic crisp infra β –open sets NCIT$^{\beta}$, the class of all Neutrosophic crisp infra pre-open NCIT^{P} and the class of all Neutrosophic crisp infra semi-open NCIT^{S} .

Remark 4.2: A class consisting of exactly all a neutrosophic crisp infra α – structure (resp. NCIβ -structure). Evidently NCIT ⊆ NCIT$^{\alpha}$ ⊆ NCIT$^{\beta}$. We notice that every non-empty neutrosophic crisp infra β- open has NCI α -nonempty interior. If all neutrosophic crisp infra set the following $\{B_j^i\}_{j\in I}$ are NCI β −Open sets, then $\{\cup_j B_j^i\}_{j\in I} \subset NCicp\ (NCiip\ (NCicp(A^i)))$ that is a NCIβ- structure is a neutrosophic infra closed with respect to arbitrary neutrosophic crisp infra unions. We shall now characterize NCIT$^{\alpha}$ in terms NCIT$^{\beta}$.

Theorem 4.3: Let (X, Γ_X^i)be a NCTS. A NCT$^{\alpha}$ consists of exactly those neutrosophic crisp set A^i for which $A^i \cap B^i \in$ NCT$^{\alpha}$ for $B^i \in$ NCT$^{\beta}$

Theorem 4.4: Every neutrosophic crisp infra NCIα- structure is a neutrosophic crisp infra topology.
Proof: NCIT$^{\beta}$ contains the neutrosophic crisp infra empty set and is an infra closed with respect to arbitrary unions. A standard result gives the class of those neutrosophic crisp infra sets A for which $A^i \cap B^i \in$ NCT$^{\alpha}$ for all $B^i \in$ NCT$^{\beta}$ constitutes a neutrosophic crisp infra topology, hence the theorem . Hence forth we shall also use the term NCI$^{\alpha}$- topology for NCI α - structure two neutrosophic crisp infra topology deterring the same NCI α - structure shall be called NCI$^{\alpha}$- equivalent, and the equivalence classes shall be called NCI$^{\alpha}$- classes.

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