WEAKER FORM OF NANO OPEN SETS IN R$^+$

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Abstract: In 1963 Norman Levine initiated the concept of semi open sets and its continuity. O.Njastad introduced α -open set in classical topology in 1965. Lellis Thivagar et al introduced nano topological space with respect to the subset X of a finite universe which is defined in terms of lower and upper approximations. The new topological space is called nano because it contains atmost five elements. The elements of the nano topological space are called nano open sets. He also continued to expose certain weak forms of nano open sets namely nano semi open sets and nano α -open sets. This definition is extended to the infinite universe, the real line. The nano topological space is defined with respect to an interval of R^+ which is defined interms of lower and upper approximation of an interval. The objective of this work is to define nano semi open sets and nano α -open sets in R^+ and characterize the properties of these sets. An attempt is made to compare the weaker form of nano open sets in R^+ .
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Introduction: Norman Levine and O.Njastad introduced respectively semi open sets and α -open sets.Lellis Thivagar et al [5] introduced nano topological space with respect to a subset X of an finite universe which is defined interms of lower and upper approximations of X. He also introduced the nano topology in the infinite universe, positive real line and the equivalence relation is induced by a sequence. Here we introduce nano semi open sets, nano pre-open sets and nano α -open sets in the infinite universe positive real line.The variation in the behaviour of the nano weaker open sets in the finite universe and the infinite universe are studied. The characterisation of these nano weaker open sets and the comaparision between them are also considered.

Preliminaries:

Definition 2.1 *[5]:* Let U be a non-empty finite set of objects called the universe and R be an equivalance relation on U named as the indiscernibility relation.Elements belonging to the same equivalance class are said to indiscernible with one another.The pair (U,R) is said to be the approximation space.

- The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is $L_R(X)$ = $\cup\{R(x) : R(x) \subseteq X\}$ where R(x)denotes the equivalence class determined by x.
- The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is $U_R(X)$ = $\cup\{R(x) : R(x) \cap X \neq \emptyset\}$

- The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$.That is $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2 *[5]:* Let U be the universe,R be an equivalance relation on U and $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where X $\subseteq$ U. Then $\tau_R(X)$ satisfies the following axioms

- U and $\emptyset \in \tau_R(X)$.
- The union of the elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.
- The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.
- That is $\tau_R(X)$ is a topology on U called the nanotopology on U with repect to X. We call (U, $\tau_R(X)$) as the nanotopological space. The elements of $\tau_R(x)$ are called as nano open sets.

Definition 2.3 *[5]:* If (U, $\tau_R(X)$) is a nano topological space with respect to X where X $\subseteq$ U and if A $\subseteq$ U,then the nano interior of A is defined as the union of all nano-open subsets of A and it is denoted by Nint(A).That is Nint(A)is the largest nano-open subset of A.The nano closure of A is defined as the intersection of all nanoclosed sets containing A and it is denoted by Ncl(A).That is Ncl(A)is the smallest nano closed set containing A.

Definition 2.4 *[7]:* Let U be a nonempty finite universe and X $\subseteq$ U

- If $L_R(X) = \emptyset$ and $U_R(X) \neq U$ then $\tau_R(X) = \{U, \emptyset, U_R(X)\}$
- If $L_R(X) = U_R(X)$ = X then the nano topology $\tau_R(X) = \{U, \emptyset, L_R(X)\}$
- If $L_R(X) \neq \emptyset$ and $U_R(X) = U$ then $\tau_R(X) = \{U, \emptyset, L_R(X), B_R(X)\}$
- If $L_R(X) \neq U_R(X)$ where $L_R(X) \neq \emptyset$ and $U_R(X) \neq U$ then $\tau_R(X)$ = $\{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$ is the discrete nano topology on U
- If $L_R(X) = \emptyset$ and $U_R(X) = U$ then $\tau_R(X) = \{U, \emptyset\}$ the indiscrete nano topology on U

Definition 2.5 *:* Let R^+ the set of all nonnegative real numbers.Let Seq $\subseteq R^+$ be a sequence of real numbers defined by $x_1, x_2, x_3, \ldots x_n, \ldots$ such that $x_1 < x_2 < x_3 < \ldots < x_n < \ldots$. The Seq defines the partition $\pi(Seq)$ of R^+ by $\pi(Seq) = \{0, (0, x_1), x_1, (x_1, x_2), \ldots.(x_i, x_{i+1}), \ldots\}$ where (x_i, x_{i+1}) denote open intervals R^+. The ordered pair $(R^+, \pi^*(Seq))$ is an approximation space where $\pi^*(Seq)$ is the equivalence relation associated with $\pi(Seq)$.

Definition 2.6 *:* Let R^+ the set of all nonnegative real numbers be the universe. Let $\pi^*(Seq)$ be the equivalance relation associated with a partition $\pi(Seq)$ of R^+. The ordered pair ($R^+, \pi^*(Seq)$)is an approximation space. Let Q(x)=[o,x] where x $\in$ R^+ is a closed subset of R^+. The sequence lower,sequence upper and sequence boundary approximation of Q(X) with respect to $\pi^*(Seq)$ are defined as follows

• The Seq-lower approximation of $Q(x)$ with respect to $\pi^*(Seq)$ is denoted by $L_{\pi^*(Seq)}Q(x)$ and it is defined as

$L_{\pi^*(Seq)}Q(x) = \{y \in R^+ : Seq(y) \subseteq Q(x)\}$.

• The Seq-upper approximation of $Q(x)$ with respect to $\pi^*(Seq)$ is denoted by $U_{\pi^*(Seq)}Q(x)$ and it is defined as $U_{\pi^*(Seq)}Q(x) = \{y \in R^+ : Seq(y) \cap Q(x) \neq \varnothing\}$

• The Seq-boundary approximation of $Q(x)$ with respect to $\pi^*(Seq)$ is denoted by $B_{\pi^*(Seq)}Q(x)$ and it is defined as

$B_{\pi^*(Seq)}Q(x) = U_{\pi^*(Seq)}Q(x) - L_{\pi^*(Seq)}Q(x)$

Then $\tau_{\pi^*(Seq)}[Q(x)] = \{R^+, \varnothing, L_{\pi^*(Seq)}Q(x), U_{\pi^*(Seq)}Q(x), B_{\pi^*(Seq)}Q(x)\}$ forms a nano topology on R^+ .This nano topology depends on the sequence Seq defined on R^+ (i.e) this topology is induced by sequence. ($R^+, \tau_{\pi^*(Seq)}[Q(x)]$,)is nano topological space.

Definition 2.7 *:* If ($R^+, \tau_{\pi^*(Seq)}Q(x)$) is a nano topological space with respect to Q(x) where Q(x) $\subseteq R^+$ and if A $\subseteq R^+$,then the nano interior of A is defined as the union of all nano-open subsets of A and it is denoted by $\mathrm{N}int(A)$ .That is $\mathrm{N}int(A)$ is the largest nano open subset of A. The nano closure of A is defined as the intersection of all nanoclosed sets containing A and it is denoted by $\mathrm{N}cl(A)$. That is $\mathrm{N}cl(A)$ is the smallest nano closed set containing A.

Nano Semi open Sets in R $^+$

Definition 3.1 *:* If ($R^+, \tau_{\pi^*(Seq)}Q(x)$) is a nano topological space with respect to $Q(x)$ where $Q(x)$ $\subseteq R^+$. Then A $\subseteq R^+$ is said to be nano semi open if A $\subseteq$ $\mathrm{N}cl(\mathrm{N}int(A))$.

Example 3.2 *:* Let R^+ be the set of all non negative real numbers.Let N = $\{1,2,3,...\}$ be a sequence in R^+. The partition is defined as

$\pi(N)$ $= \{0,(0,1),1,(1,2),2,(2,3),......n(n,n+1),...\}$. $\pi^*(N)$ is the equivalence relation on R^+. Let Q(x)= [0,2] then $\tau_{\pi^*(N)}Q(x)$ = $\{R^+, \varnothing, [0,2]\}$ A=[0,3] is a nano semi open set but B=[0,1] is not a nano semi open set.

Proposition 3.3 *:* In a nano topological space $(R^+, \tau_{\pi^*(Seq)}Q(x))$ if $\tau_{\pi^*(Seq)}Q(x)$ = $\{R^+, \varnothing, [0,x]\}$ then R^+, $\varnothing$ and all the subsets of the form [0,n], n $\geq$ x are the only nano semi open sets.

Proof : Let $\tau_{\pi^*(Seq)}Q(x)$ = $\{R^+, \varnothing, [0,x]\}$. Let A be a subset of R^+ of the form [0,n] where n $\leq$ x then $\mathrm{N}int(A)$ is empty. Therefore A does not contained in $\mathrm{N}cl(\mathrm{N}int(A))$. Hence A is not nano semi open in R^+. If A be a subset of R^+ of the form [0,n] where n $\geq$ x then $\mathrm{N}int(A)$ is [0,x] and $\mathrm{N}cl(\mathrm{N}int(A))$ is R^+. Therefore A is a subset of R^+. Hence A is a nano semi open set.Obviously $R^+, \varnothing$ are nano semi open sets. In a nano topological space $(R^+, \tau_{\pi^*(Seq)}Q(x)$ if

$\tau_{\pi^*(Seq)}Q(x)$ = $\{R^+, \varnothing, L_{\pi^*(Seq)}Q(x), U_{\pi^*(Seq)}Q(x), B_{\pi^*(Seq)}Q(x)\}$ then R^+, $\varnothing$,all the subsets of the form [0,n], n $\geq$ integral part of x, [0,n),n $\geq$ an integer which is greater than x and (a,b), which contains $B_{\pi^*(Seq)}Q(x)$ and a $<$ b are the only nano semi open sets.

Theorem 3.5 *:* Union of two nano semi open sets in R^+ is nano semi open in R^+

Proof : From the definition of nano semi open sets, the proof is obvious. Intersection of two nano semi open sets in R^+ is not nano semi open in R^+. Let R^+ be the set of all non negative real numbers.
Let N = $\{1,2,3,...\}$ be a sequence in R^+. The partition is defined as
$\pi(N) = \{0,(0,1),1,(1,2),2,(2,3),......n(n,n+1),...\}$. $\pi^*(N)$ is the equivalence relation on R^+. Let Q(x)= [0,2]then $\tau_{\pi^*(N)}Q(x) = \{R^+,\varnothing,[0,2]\}$ The intervals A=[0,3] and B=[0,4] are nano semi open sets. A $\cap$ B =(3,4] is not nano semi open.
Remark 3.7 *:* In $\mathsf{N}T_1$ type, that is $\tau_R(X) = \{\mathsf{U},\varnothing,U_R(X)\}$, intersection of two nano semi open sets is is nano semi open. But in R^+ intersection of two nano semi open sets is not nano semi open.

Theorem 3.8 *:* Every nano open set in R^+ is a nano semi open set in R^+.
Remark 3.9 *:* Converse of the above theorem is not true.Let N = $\{1,2,3,...\}$ be a sequence in R^+. The partition is defined as
$\pi(N) = \{0,(0,1),1,(1,2),2,(2,3),......n(n,n+1),...\}$. $\pi^*(N)$ is the equivalence relation on R^+. Let Q(x)= [0,2]then $\tau_{\pi^*(N)}Q(x) = \{R^+,\varnothing,[0,2]\}$ A=[0,4] is a nano semi open set but not a nano open set.

Theorem 3.10 *:* In a nano topological space $(R^+,\tau_{\pi^*(Seq)}Q(x)$ if $\tau_{\pi^*(Seq)}Q(x) = \{R^+,\varnothing,[0,x]\}$ then the set of all nano semi open sets of R^+ does not form a topology.
Proof : Intersection of two nano semi open sets in R^+ is not nano semi open in R^+.Therefore the set of all nano semi open sets in R^+ does not form a topology. If A and B are nano semi closed in R^+, then A $\cap$ B is nano semi closed in R^+

Theorem 3.11 *:If A and B are nano semi closed in R+, then A\B is nano semi closed in R+*
Proof : Since A and B are nano semi closed in R^+, A^C and B^C are nano semi open in R^+. Since union of two nano semi open sets is nano semi open and $A^C \cup B^C = (A \cap B)^C$. Theerefore A $\cap$ B is nano semi closed in R^+.

4 Nano α-open Sets and Nano pre-open sets in R+ :
Definition 4.1 *:* If ($R^+,\tau_{\pi^*(Seq)}Q(x)$) is a nano topological space with respect to $Q(x)$ where $Q(x)$ $\subseteq R^+$.Then A $\subseteq R^+$ is said to be nano α -open if A $\subseteq$ $\mathsf{N}int(\mathsf{N}cl(\mathsf{N}int(A)))$.
Example 4.2 *:* Let R^+ be the set of all non negative real numbers.Let N = $\{1,2,3,...\}$ be a sequence in R^+. The partition is defined as $\pi(N) = \{0,(0,1),1,(1,2),2,(2,3),......n(n,n+1),...\}$. $\pi^*(N)$ is the equivalence relation on R^+. Let Q(x)= [0,2]then $\tau_{\pi^*(N)}Q(x) = \{R^+,\varnothing,[0,2]\}$. A= [0,3] is a nano α -open set.

Proposition 4.3 *:* In a nano topology space $(R^+,\tau_{\pi^*(Seq)}Q(x)$ if $\tau_{\pi^*(Seq)}Q(x) = \{R^+,\varnothing,[0,x]\}$ then R^+, $\varnothing$ and all the subsets of the form [0,n], n $\geq$ x are the only nano α -open sets.
Theorem 4.4 *:* Union of two nano α -open sets in R^+ is a nano α -open set in R^+
Proof : From the definition of nano α open sets the proof is obvious. Intersection of two nano α -open sets in R^+ is not a nano α -open set in R^+

Theorem 4.6 *:* Every nano open set in R^+ is nano α -open set in R^+.
Remark 4.7 *:* [(i)]Converse of the above theorem is not true. [(ii)] In Nano topology the set of all nano α -open sets form a topology. But in R^+ the set of all nano α -open sets does not form a topology.
Definition 4.8 *:* If ($R^+, \tau_{\pi^*(Seq)} Q(x)$)is a nano topological space with respect to $Q(x)$ where $Q(x) \subseteq R^+$. Then A $\subseteq R^+$ is said to be nano pre-open if A $\subseteq$ $\mathsf{N}int(\mathsf{N}cl(A))$.
Example 4.9 *:* Let R^+ be the set of all non negative real numbers. Let N = $\{1,2,3,...\}$ be a sequence in R^+. The partition is defined as $\pi(N)$ = $\{0,(0,1),1,(1,2),2,(2,3),......n(n,n+1),...\}$. $\pi^*(N)$ is the equivalence relation on R^+. Let Q(x)= [0,2]then $\tau_{\pi^*(N)} Q(x)$ = $\{ R^+, \varnothing, [0,2]\}$ The intervals A=[0,3] is a nano pre-open set.

Proposition 4.10 *:* In a nano topology space $(R^+, \tau_{\pi^*(Seq)} Q(x)$ if $\tau_{\pi^*(Seq)} Q(x)$ = $\{ R^+, \varnothing, [0,x]\}$ then R^+, $\varnothing$,all the intervals of the form [0,n] and all the intervals contained in [0,x] are nano pre-open sets.
Proof : If $\tau_{\pi^*(Seq)} Q(x)$ = $\{ R^+, \varnothing, [0,x]\}$ then $\mathsf{N}int(\mathsf{N}cl(R^+))$= R^+ and $\mathsf{N}int(\mathsf{N}cl(\varnothing))$ = $\varnothing$. Therefore R^+ and $\varnothing$ are nano pre-open. For all the intervals of the form [0,n] and all the intervals contained in [0,x] the nano closure is R^+ ,hence these set are nano pre-open.Let A be an interval which is neither in the form [0,n] nor contained in [0,x], then Nano closure of A is $(x, \varnothing)$ whose interior is empty set. Therefore A is not pre open. In a nano topology space $(R^+, \tau_{\pi^*(Seq)} Q(x))$ if $\tau_{\pi^*(Seq)} Q(x)$ = $\{ R^+, \varnothing, L_{\pi^*(Seq)} Q(x), U_{\pi^*(Seq)} Q(x), B_{\pi^*(Seq)} Q(x)\}$ then R^+, $\varnothing$ and all intervals whose nano closure is R^+ are the nano-pre open sets.

5 *Nano regular open sets:*

Definition 5.1 *:* If ($R^+, \tau_{\pi^*(Seq)} Q(x)$) is a nano topological space with respect to $Q(x)$ where $Q(x) \subseteq R^+$. Then A $\subseteq R^+$ is said to be nano regular open if $\mathsf{N}int(\mathsf{N}cl(A)) = A$.
Proposition 5.2 *:* In a nano topology space $(R^+, \tau_{\pi^*(Seq)} Q(x)$ if $\tau_{\pi^*(Seq)} Q(x)$ = $\{ R^+, \varnothing, [0,x]\}$ then R^+ and $\varnothing$ are the only nano regular open sets.
Proof : If $\tau_{\pi^*(Seq)} Q(x)$ = $\{ R^+, \varnothing, [0,x]\}$ then the nano closed sets are $R^+, \varnothing, (x,\infty)$. Nano closure of any sub set of R^+ is one among the three nano closed sets. $\mathsf{N}int(R^+)$ = $R^+)$, $\mathsf{N}int(\varnothing)$ = $\varnothing$ and $\mathsf{N}int(x,\infty)$ = $\varnothing$. Therefore R^+ and $\varnothing$ are the only nano regular open sets. In a nano topology space $(R^+, \tau_{\pi^*(Seq)} Q(x))$ if $\tau_{\pi^*(Seq)} Q(x)$ = $\{ R^+, \varnothing, L_{\pi^*(Seq)} Q(x), U_{\pi^*(Seq)} Q(x), B_{\pi^*(Seq)} Q(x)\}$ then R^+ and $\varnothing$ are the only nano regular open sets.
Proof : Proof as similar to the previous theorem.

Conclusion: We define weaker form of nano open sets such as nano semi open sets, nano α -open set, nano pre open sets and nano regular open sets on the infinite universe positive real line and study its properties. The behaviour these sets in the infinite universe positive real line and the finite universe are compared. This can be extended further to study the separation axioms and solve the real life problems.

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