MORPHOLOGICAL OPERATORS ON THE COMPLETE LATTICE OF FUZZY HYPERGRAPHS

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Abstract: Inthis work we study some basic morphological operators on the lattice of fuzzy hypergraph of a crisp hypergraph. Basic morphological operators defined in this paper can be used as an effective tool in image processing.

Keywords: Complete Lattice, Dilation, Erosion, Fuzzy Hypergraph, Mathematical Morphology.

1. Introduction: Mathematical morphology is a set theoretic based approach. Originally, it is developed for binary images by Georges Matheron and Jean Serra. It was extended to gray-level images in the late 1970's.This technique is used to extract characteristic features of the images. Considering digital objects carrying structural information, mathematical morphology has been developed on graphs[1],[2],[7], simplicial complexes[8], and hypergraphs[3],[4],[9],[13], [17].

Graph theoretic methods have found tremendous applications in image processing. Vincent [1] proposed morphological operators on a graph. Cousty, Najman and Serra[2] defined morphological operators on various lattices formed by the graph. Rosenfeld [12] proposed that gray scale images may be represented as fuzzy graphs. Considering the complex aspect of an image, Bloch and Bretto[3] have proposed a hypergraph model to represent an image. The theory of hypergraphs have now become an active area of research in the field of analysis of images. Bloch and Bretto [13] introduced mathematical morphology on hypergraphs by forming various lattices on hypergraphs. Bino, Unnikrishnan, Kannan and Ramkumar [4] developed morphological operators on hypergraphs by defining a vertex-hyperedge correspondence.

In this paper, we introduce basic morphological operators acting on the complete lattice of fuzzy hypergraphs that may be termed as dilation and erosion. Following this introduction, section 2 presents some basic definitions and concepts. Section 3 involves the formation of complete lattice structure. In section 4 we have defined the morphological operators. Finally, section 5 deals with the conclusion.

2. Preliminaries:

2.1 Morphological Operators and Complete Lattice: Complete lattice theory is accepted as an appropriate theoretical framework for Mathematical morphology [14]. Thus we can define morphological operators for any type of image data, as long as complete lattice structure can be introduced on the pixel intensity range. A complete lattice is a partially ordered set such that for any family of elements, we can always find a supremum and an infimum.

Let δ be any operator on lattice (L, ≤), then δ is

(a) increasing if X ≤ Y implies $\delta(X) \leq \delta(Y)$

(b) extensive if X ≤ $\delta(X)$ for every X∈ L

(c) anti-extensive if $\delta(X) \leq X$ for every X ∈ L

Given two lattices L_1 and L_2, an operator $\delta: L_1 \rightarrow L_2$ is called a dilation when it preserves the supremum: that is, for every $X \subseteq L_1$, $\delta(V_1X) = V_2\{\delta(x)|x \in X\}$, where V_1 is the supremum in L_1 and V_2 the supremum in L_2. Similarly, an operator which preserves the infimum is called an erosion. Two operators $\varepsilon: L_1 \rightarrow L_2$and

δ: $L_2 \to L_1$ form an adjunction (ε,δ) when for any x in L_2 and any y in L_1, we have $\delta(x) \leq_1 y \Leftrightarrow x \leq_2 \varepsilon(y)$, where $\leq_1$ and $\leq_2$ denote the order relations on L_1 and L_2 respectively. If the pair (ε,δ) is an adjunction, then ε is an erosion and δ is a dilation. If L_1, L_2, L_3 are three lattices and if δ: $L_1 \to L_2$, δ^1: $L_2 \to L_3$, ε: $L_2 \to L_1$ and ε^1: $L_3 \to L_2$ are four operators such that (ε, δ) and (ε^1, δ^1) are adjunctions, then their composition ($\varepsilon_0\varepsilon^1$, $\delta_0\delta^1$) is also adjunction.

2.2 Hypergraphs and Fuzzy Hypergraphs: Hypergraph theory was introduced in 1960's as a generalization of graph theory. A (crisp) hypergraph on a set X is a pair H = (X, E) where X is a finite set and E is a finite family of nonempty sets of X which satisfy the condition: Every member of X is contained in some member of E. X is called the vertex set and E is called the hyperedge set of H. Before defining a fuzzy hypergraph let us recall some definitions of fuzzy sets.
A fuzzy set A in a universe of discourse U is characterized by a membership function μ_A which takes the values in the unit interval [0, 1]:

that is, μ_A: U→ [0, 1].

The value $\mu_A(u)$, $u \in U$ represents the grade of membership of u in A and is a point in [0, 1]
Let A and B be two fuzzy sets on a set X. We say that A is contained in B or A is a subset of B, if $A(x) \leq B(x)$ for every $x \in X$.The standard union of A and B is a fuzzy set, A∪B:X→[0, 1], given by (A∪B)(x) = max{A(x),B(x)} for every x ∈ X. Similarly the intersection of A and B is a fuzzy set, A∩B: X→ [0, 1], given by (A∩B)(x) = min {A(x), B(x)}. Let A be a fuzzy set on X, then complement of A, denoted by A^c is a fuzzy set, A^c: X→ [0, 1], given by $A^c(x)$ = 1-A(x). The support of a fuzzy set A is the set of all those elements in the universal set that have a non-zero membership grade. It is denoted by supp A.
Now, we are in a position to define a fuzzy hypergraph. Let X be a finite set and σ be a fuzzy subset of X. Let $\mathcal{E}$ be a collection of fuzzy subsets of X such that for each $\mu \in \mathcal{E}$ and $x \in X$, $\mu(x) \leq \sigma(x)$. Also X $= \cup_{\mu \in \mathcal{E}}$ supp μ. Then H= (X,E,σ,$\mathcal{E}$) is called a fuzzy hypergraph on the fuzzy set σ.The fuzzy hypergraph H=(X,E,σ,$\mathcal{E}$) is also called a fuzzy hypergraph on X= supp σ.The fuzzy set σ defines a condition for membership in the edge set. For instance, suppose that e∈E, such that e is composed of the vertices x_1, x_2, x_3,……,x_k. Then the membership grade $\mu \in \mathcal{E}$ of e will satisfy $\mu(e) \leq \sigma(x_1) \wedge \sigma(x_2) \wedge \ldots\ldots \sigma(x_k)$[15].

3. Formation of Complete Lattice: In this section we shall define a partial ordering on fuzzy hypergraph that will lead to the formation of a complete lattice. The fuzzy morphological operators will be defined on this lattice. Consider the hypergraph H= (X,E). Let Ħ be the set of all fuzzy hypergraphs H= (X,E,σ_i ,$\mathcal{E}_i$) defined on H= (X,E).Let 0 be a fuzzy hypergraph in Ħ with all vertices and edges of membership degree 0 and 1 be a fuzzy hypergraph in Ħ with all vertices and edges of membership degree 1.

Definition3.1: Let H_1=(X,E,σ_1,$\mathcal{E}_1$) and H_2=(X,E,σ_2,$\mathcal{E}_2$) ∈ Ħ. The relation ⊆ onĦ is defined as follows: $H_1 \subseteq H_2$ if and only if $\sigma_1 \subseteq \sigma_2$ and $\mathcal{E}_1 \subseteq \mathcal{E}_2$

Result 3.2: (Ħ, ⊆) is a poset.
Proof: Clearly this relation is reflexive
If $H_1 \subseteq H_2$ and $H_2 \subseteq H_1$, then $H_1 = H_2$. Therefore, the relation is anti-symmetric.
If $H_1 \subseteq H_2$ and $H_2 \subseteq H_3$, then evidently $\mathcal{E}_1 \subseteq \mathcal{E}_2$ and $\sigma_1 \subseteq \sigma_2$. Also$\mathcal{E}_2 \subseteq \mathcal{E}_3$ and $\sigma_2 \subseteq \sigma_3$.
From this we get,$\mathcal{E}_1 \subseteq \mathcal{E}_3$ and $\sigma_1 \subseteq \sigma_3$. Thus, $H_1 \subseteq H_3$. That is, the relation is transitive. Hence, (Ħ, ⊆) is a poset.

Definition 3.3: Let H_1=(X,E,σ_1,$\mathcal{E}_1$) and H_2=(X,E,σ_2 ,$\mathcal{E}_2$) ∈ Ħ. We define union (∪) and intersection (∩) of H_1 and H_2 as

$$H_1 \cup H_2 = (X,E, \sigma_1 \cup \sigma_2, \mathcal{E}_1 \cup \mathcal{E}_2)$$
$$H_1 \cap H_2 = (X,E, \sigma_1 \cap \sigma_2, \mathcal{E}_1 \cap \mathcal{E}_2)$$

The above definition can be generalized to any finite number of fuzzy hypergraphs.

Result 3.4: Suppose $\boldsymbol{H} = \{H_i\}_{i \in I}$ (I being an index set) be a family of elements of Ħ.
Then Sup $\boldsymbol{H} = \cup_{i \in I} H_i$ and Inf $\boldsymbol{H} = \cap_{i \in I} H_i$
Proof: Let H_i = (X,E, σ_i, $\mathcal{E}_i$)

Therefore, $\cup_{i\in I} H_i = (X, E, \cup_{i\in I}\sigma_i, \cup_{i\in I}\mathcal{E}_i)$
By definition, $H_i \subseteq \cup_{i\in I} H_i$ for every i=1, 2, 3… k
Now let $H_i \subseteq Y$ for every i=1, 2, 3… k, where $Y=(X, E, \sigma_Y, \mathcal{E}_Y)$
Then $\sigma_i \subseteq \sigma_Y$ and $\mathcal{E}_i \subseteq \mathcal{E}_Y$ for every i∈I
By the property of fuzzy sets,
$\cup_{i\in I}\sigma_i \subseteq \sigma_Y$ and $\cup_{i\in I}\mathcal{E}_i \subseteq \mathcal{E}_Y$
This implies, $\cup_{i\in I} H_i \subseteq Y$
Since Y is an arbitrary fuzzy hypergraph, the above inclusion is true for all such fuzzy hypergraphs Y
Thus Sup $\boldsymbol{H} = \cup_{i\in I} H_i$
In a similar manner, it can be shown that Inf $\boldsymbol{H} = \cap_{i\in I} H_i$

Definition 3.5: The complement of a fuzzy hypergraph H= $(X, E, \sigma, \mathcal{E})$ is defined as
$H^c = (X^c, E^c, \sigma^c, \mathcal{E}^c)$

Remark 3.6: From results 3.2 and 3.4, it is evident that the set of all fuzzy hypergraphs Ħ of a hypergraph H=(X, E) forms a complete lattice. But the complement of a fuzzy hypergraph need not be a fuzzy hypergraph. Hence, Ħ is not a Boolean algebra. Thus, we have now laid a foundation on which fuzzy operators could be defined.

4. Fuzzy Hypergraph Morphology: In this section, we define fuzzy operations on the fuzzy vertex set and fuzzy hyperedge set of a fuzzy hypergraph H= $(X,E,\sigma,\mathcal{E})$ defined on the (crisp) hypergraph H= (X, E). We investigate such operators and we study their properties. Then, based on these operators, we propose dilations and erosions acting on the lattice. Recall that dilation is extensive, increasing and distributes over supremum whereas erosion is anti-extensive, increasing and distributes over infimum.

Definition 4.1: We define two fuzzy sets on X and two fuzzy sets on E as follows:

1. The fuzzy set $\delta_\sigma : X \to [0,1]$ is defined by $\delta_\sigma(x) = V_{x\in v(e_i)}\{\sigma(v(e_i)), i\in I \mid e_i \in E\}$ for every x∈X
2. The fuzzy set $\varepsilon_\mu : E \to [0,1]$ is defined by $\varepsilon_\mu(e_i) = min_{i\in I}\{\wedge_{x\in v(e_i)}\mu(e_i) \mid e_i \in E, \mu \in \mathcal{E}\}$
3. The fuzzy set $\delta_\mu : E \to [0,1]$ is defined by $\delta_\mu(e_i) = max_{i\in I}\{V_{x\in v(e_i)}\mu(e_i) \mid e_i \in E, \mu \in \mathcal{E}\}$
4. The fuzzy set $\varepsilon_\sigma : X \to [0,1]$ is defined by $\varepsilon_\sigma(x) = \wedge_{x\in v(e_i)}\{\sigma(v(e_i)), i\in I \mid e_i \in E\}$ for every $x\in X$

Using these fuzzy sets, we define two fuzzy hypergraphs are follows:
Definition 4.2: Define δ:Ħ→Ħ and ε: Ħ→Ħ by

$$\delta(H) = (X, E, \delta_\sigma, \delta_\mu);\ X = \cup_{\delta_\mu} \mathrm{supp}\,\delta_\mu$$
$$\varepsilon(H) = (X, E, \varepsilon_\sigma, \varepsilon_\mu);\ X = \cup_{\varepsilon_\mu} \mathrm{supp}\,\varepsilon_\mu$$

The fuzzy sets and fuzzy hypergraphs defined in the above definitions satisfy several properties of dilations and erosions which make them the fuzzy analogues of dilations and erosion of (crisp) hypergraph.

Remark 4.3: From the very definitions stated in 4.1 and 4.2, it is evident that
(a) $\delta(H)$ and ε(H) are fuzzy hypergraphs belonging to Ħ
(b) $\sigma \subseteq \delta_\sigma$ and $\mu \subseteq \delta_\mu$
(c) $\varepsilon_\sigma \subseteq \sigma$ and $\varepsilon_\mu \subseteq \mu$
(d) $H \subseteq \delta(H)$ and $\varepsilon(H) \subseteq H$ for every H∈Ħ (The proof of which follows from (b) and (c) of this remark)
Hence, we can say that δ_σ, δ_μ and δ are extensive and ε_σ, ε_μ and ε are anti- extensive, which is one of the properties satisfied by the operators dilation and erosion respectively.

Theorem 4.4: Let $H_1=(X,E,\sigma_1,\mathcal{E}_1)$ and $H_2=(X,E,\sigma_2,\mathcal{E}_2)$ be two fuzzy hypergraphs in Ħ. Then,
(a) $H_1 \subseteq H_2$ implies $\delta(H_1) \subseteq \delta(H_2)$
(b) $H_1 \subseteq H_2$ implies $\varepsilon(H_1) \subseteq \varepsilon(H_2)$
Proof: (a) Let $H_1 \subseteq H_2$. Then clearly, $\sigma_1 \subseteq \sigma_2$ and $\mathcal{E}_1 \subseteq \mathcal{E}_2$ for every $x\in v(e_i)$
Now, $\sigma_1 \subseteq \sigma_2$ imply $\sigma_1(v(e_i)) \le \sigma_2(v(e_i))$ for every $x\in v(e_i)$

By the property of fuzzy sets, $V_{x\in v(e_i)}\{\sigma_1(v(e_i))\} \leq V_{x\in v(e_i)}\{\sigma_2(v(e_i))\}$

Therefore, $\delta_{\sigma_1}(x) \leq \delta_{\sigma_2}(x)$ for every $x\in v(e_i)$

Thus $\delta_{\sigma_1}\subseteq\delta_{\sigma_2}$

Next, $\mu_1\subseteq\mu_2$ implies $\mu_1(e_i) \leq\mu_2(e_i)$ for every $e_i \in E$

By the property of fuzzy sets, $V_{x\in v(e_i)}\mu_1(e_i) \leq V_{x\in v(e_i)}\mu_2(e_i)$ for every $e_i \in E$

Therefore, max $\{V_{x\in v(e_i)}\mu_1(e_i)\} \leq$ max $\{V_{x\in v(e_i)}\mu_2(e_i)\}$ for every $e_i \in E$

i.e.; $\delta_{\mu_1}(e_i) \leq \delta_{\mu_2}(e_i)$.

Thus we get, $\delta_{\mu_1}\subseteq\delta_{\mu_2}$

Hence, we have shown that $\delta_{\sigma_1}\subseteq\delta_{\sigma_2}$ and $\delta_{\mu_1}\subseteq\delta_{\mu_2}$. This together imply $\delta(H_1) \subseteq \delta(H_2)$.

(b) The proof of this part can be obtained in a similar way by replacing V by ∧ and max by min in the proof of part (a).

Thus we have shown that δ_σ , δ_μ, δ, ε_σ ,ε_μ and ε are all increasing, which is yet another property possessed by morphological operators: dilation and erosion.

Theorem 4.5: Let $H_1=(X,E,\sigma_1,\mathcal{E}_1)$ and $H_2 =(X,E,\sigma_2,\mathcal{E}_2)$ be two fuzzy hypergraphs in Ӈ. Then,

(a) $\delta(H_1\cup H_2) = \delta(H_1) \cup \delta(H_2)$

(b) $\varepsilon(H_1\cap H_2) = \varepsilon(H_1) \cap \varepsilon(H_2)$

Proof: Clearly, $H_1\cup H_2 = (X,E,\sigma_1\cup \sigma_2, \mathcal{E}_1\cup\mathcal{E}_2)$ and, $H_1\cap H_2 = (X,E,\sigma_1\cap \sigma_2, \mathcal{E}_1\cap\mathcal{E}_2)$

(a) $\delta_{\sigma_1\cup\sigma_2}(x) =V_{x\in V(e_i)}\{ \sigma_1\cup \sigma_2(v(e_i))\}$

$=V_{x\in V(e_i)}[\max\{\sigma_1(v(e_i)), \sigma_2(v(e_i)) \}]$

$=\max \{V_{x\in V(e_i)}(\sigma_1(v(e_i)), V_{x\in V(e_i)} (\sigma_2(v(e_i))\}$

$= \max \{\delta_{\sigma_1}(x), \delta_{\sigma_2}(x)\}$ for every $x\in v(e_i)$, $i\in I$

$= \delta_{\sigma_1}(x)\cup \delta_{\sigma_2}(x)$ for every $x\in v(e_i)$, $i\in I$

$= (\delta_{\sigma_1} \cup \delta_{\sigma_2})(x)$ for every $x\in v(e_i)$, $i\in I$

Thus, $\delta_{\sigma_1\cup\sigma_2} = \delta_{\sigma_1} \cup \delta_{\sigma_2}$... (1)

Now, consider, $\delta_{\mu_1\cup\mu_2}(e_i) = \max \{V_{x\in v(e_i)}\mu_1 \cup \mu_2(e_i)\}$ for every $e_i\in E$, $x\in v(e_i)$, $i\in I$

$= \max \{V_{x\in v(e_i)}(\max (\mu_1(e_i), \mu_2(e_i)))\}$

$= \max \max \{V_{x\in v(e_i)}\mu_1(e_i), V_{x\in v(e_i)}\mu_2(e_i)\}$

$= \max \{\max (V_{x\in v(e_i)}\mu_1(e_i)), \max (V_{x\in v(e_i)}\mu_2(e_i))\}$

$=\max \{\delta_{\mu_1}(e_i), \delta_{\mu_2}(e_i)\}$ for every $e_i\in E$, $x\in v(e_i)$, $i\in I$

$= \delta_{\mu_1} \cup \delta_{\mu_2}(e_i)$ for every $e_i\in E$, $x\in v(e_i)$, $i\in I$

Thus, $\delta_{\mu_1\cup\mu_2} = \delta_{\mu_1} \cup \delta_{\mu_2}$.. (2)

From (1) and (2), $\delta(H_1\cup H_2) = \delta(H_1) \cup \delta(H_2)$

(b) The proof of this part can be obtained in a similar way by replacing V by ∧ and max by min in the proof of part (a).

From the above theorem it is evident that the fuzzy operations that we defined distributes over supremum and infimum.

Remark 4.6: By the limelight of the above theorems, we can call δ_σ, δ_μ, ε_σ and ε_μ to be the dilation of fuzzy vertex set, dilation of fuzzy hyperedge set, erosion of fuzzy vertex set and erosion of fuzzy hyperedge set respectively.

Also, we can call $\delta(H)= (X,E,\delta_\sigma, \delta_\mu)$; $X= \cup_{\delta_\mu}$ supp δ_μ as the dilation of the fuzzy hypergraph $H =(X,E, \sigma,\mathcal{E})$ and $\varepsilon(H)= (X, E, \varepsilon_\sigma, \varepsilon_\mu)$; $X= \cup_{\varepsilon_\mu}$ supp ε_μ as the erosion of the fuzzy hypergraph $H =(X, E,\sigma,\mathcal{E})$.

Now to justify our claims, let us show that $(\varepsilon_\sigma,\delta_\sigma)$ is an adjunction of fuzzy vertex set, $(\varepsilon_\mu,\delta_\mu)$ is an adjunction of fuzzy hyperedge set and (ε,δ) is an adjunction of the fuzzy hypergrph.

Theorem 4.7: Let $H_1=(X, E,\sigma_1,\mathcal{E}_1)$ and $H_2=(X, E, \sigma_2,\mathcal{E}_2)$ be two fuzzy hypergraphs such that $H_1, H_2 \in$Ӈ. Then $\delta(H_1) \subseteq H_2$ if and only if $H_1\subseteq\varepsilon(H_2)$

Proof: The theorem can be proved in two parts. In part (1) we will show that $\delta_{\sigma_1}\subseteq\sigma_2$ if and only if $\sigma_1\subseteq\varepsilon_{\sigma_2}$ and in part (2) we will prove that $\Delta_{\mu_1}\subseteq\mu_2$ if and only if $\mu_1\subseteq\varepsilon_{\mu_2}$.

Proof of part (1): Consider a fixed x∈ X

Let $\delta_{\sigma_1} \subseteq \sigma_2$

Then, $\delta_{\sigma_1}(x) \leq \sigma_2(x)$

$\Leftrightarrow V_{x\in v(e_i)}\{\sigma_1(v(e_i))\} \leq \sigma_2(v(e_i))$ for every x ∈ v(e$_i$), i∈I

$\Leftrightarrow \sigma_1(v(e_i)) \leq \wedge_{x\in v(e_i)}\sigma_2(v(e_i))$ for every x ∈ v(e$_i$), i∈I

$\Leftrightarrow \sigma_1(v(e_i)) \leq \varepsilon_{\sigma_2}(v(e_i))$ for every x ∈ v(e$_i$), i∈I

$\Leftrightarrow \sigma_1(x) \leq \varepsilon_{\sigma_2}(x)$ for every x ∈ v(e$_i$), i∈I

$\Leftrightarrow \sigma_1 \subseteq \varepsilon_{\sigma_2}$

This completes the proof of part (1)

Proof of part (2): For a fixed e ∈ E,

Let $\delta_{\mu_1} \subseteq \mu_2$

Then, $\delta_{\mu_1}(e) \leq \mu_2$ (e)

$\Leftrightarrow$ max $\{V_{x\in v(e)}\mu_1(e)\} \leq \mu_2$ (e)

$\Leftrightarrow \mu_1$ (e) $\leq \mu_2$ (e) for every x ∈ v(e)

$\Leftrightarrow \mu_1$ (e) $\leq \wedge_{x\in v(e)}\mu_2$ (e)

$\Leftrightarrow \mu_1$ (e) ≤ min $\{\wedge_{x\in v(e)}\mu_2$ (e)$\}$

$\Leftrightarrow \mu_1$ (e) $\leq \varepsilon_{\mu_2}$ (e)

This is true for every e ∈ E.

Therefore, $\delta_{\mu_1} \subseteq \mu_2$ if and only if $\mu_1 \subseteq \varepsilon_{\mu_2}$

This completes proof of part (2)

Remark 4.8: The above theorem shows that $(\varepsilon_\sigma, \delta_\sigma)$, $(\varepsilon_\mu, \delta_\mu)$ and (ε, δ) are adjunctions which in turn strongly assets our claim that these operators are fuzzy analogues of the morphological operators: dilation and erosion.

5.Conclusion: The proposed approach in this paper has been established by constructing a fuzzy hypergraph- based learning framework. We introduced a lattice structure using all the fuzzy hypergraph on a (crisp) hypergraph. We defined a set of fuzzy operations and studied their algebraic properties. It has been shown that these operators satisfies all the properties of the morphological operators: dilation and erosion. Since image analysis is a field that involves uncertainty and ambiguity, our approach has adverse application in this stream.

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