# A NEW NOTION OF SOFT MULTISET

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**Abstract:** Due to the absence of adequacy of parameterization concepts we slightly move from classical mathematics to modern mathematics for solving uncertainty problems occurs in engineering, environmental science, medical science, physics, chemistry and so many fields. At present Fuzzy sets, rough sets, soft sets and combination of these three sets are acting as the emerging mathematical tools. Many researchers in mathematics initiated so many hybrid versions of the above sets. Bipolar soft set was constructed by bipartition soft set, was initiated by Faruk et al.[9]. Softmultiset was introduced by Babitha et.al[6]. In this paper let us see the concept of bipolar soft multiset and its operations.

**Keywords:** Bipolar Soft Multiset, Multiset, Soft Multiset, Soft Set.

**Introduction:**The modern mathematical tools like fuzzy, soft and rough sets helps to overcome an intricacy problems occur in many fields. The concept of soft set was initiated by Molodtsov [4 ]. After his work, so many researchers in Mathematics and computer science gave their own supports to develop softset theory through new ideas based on soft set. At present the applications of soft theory is extended to many fields [ 5-8]. Soft multiset theory was originated by Babitha et.al [6]. Bipolar soft set was acquainted by Faruketal [9]. Bipartition on the attribute set is the basic concept of bipolar soft set [9]. This bipolar softset, partition on the attribute set is the bricks of bijective soft set[10]. This bipolar is differing from bipolar fuzzy soft set. In this paper we introduce bipolar soft multiset and some of it properties.

The following chapters contain the following allocation. Section 2 helps to recollect the basic definitions, section3 introduces bipolar soft multiset, section 4 concludes this paper.

**Preliminaries:**Throughout this paper, we will denote initial universe set of parameters and power set of U, $E$ and P(U)respectively.

In this section we recollect the definitions and rotations as mentioned by Molodstov [ 4 ] and Majiet. al [ 11 ]. We also recall definition of soft multiset[6 ].

**Definition 2.1.** Let∪ be an initial universe set and E be a set of parameters. LetP(U) denotes the power set of U and A CE. A pair (F, A) is called a soft set over ∪, where F is a mapping given by F : A→P (∪)

**Definition 2.2**. Let A = $\{e_{1,}e_2 \ldots \ldots e_n\}$ be a set of parameters. The NOT set of A denoted by ˥A is defined by ˥A = $\{˥e_{1,}˥e_{2,}.....˥e_{n,}\}$ where ˥$e_{i,}$= not $e_{i,}$∀ I = 1, 2,......n.

**Definition 2.3** Let∪ be universal mset and E be set of parameters. Then an ordered pair (F, E) is called a soft multiset where F is a mapping given by F: A→PW (∪),PW (∪) is defined as the set of all whole submset of M.

**Definition 2.4.** Let M be a multiset. Then the relative compliment of a whole submset$M_1$ of M is given by $M_1^r$= $^{m_i}/_{x_i}$ where $C_M^{(xi)}$= 0 Foreveryx$_i$ in $M_1$ and $m_i$ is the count of $x_i$ in M.

**Bipolar Soft Multiset:**In this section, we will introduce bipolar soft multiset and also investigate some of its properties.

**Definition 3.1** Let Ebe a parameters set and $E_1, E_2$ be two nonempty subsets of E such that $E_1 \cup E_2$ = E and $E_1 \cap E_2 = \Phi$ If F :$E_1$ →PW (∪) and G: $E_2$ →PW (∪) are two mappings such that $F(e) \cap G(f(e))= \Phi$ then triple (F,G,E) is called bipolar soft multiset where F: $E_1 \rightarrow E_2$ is a bijective function. Set of all bipolar soft multisets over ∪ is denoted by BMS. We can represent a bipolar soft multiset (F, G, E) as follows.

(F, G,E) = $\left\{ \begin{matrix} < \left(e, F(e), \left(f(e), G1f1(e)\right)\right) > \\ e \in E_1 \ and \ F(e) \cap G(f(e)) = \Phi \end{matrix} \right\}$

**Remark 3.2.** Tabular representation of bipolar soft multiset is as follows.

Let ∪ $=\left\{ {k_1}/{u_1}, {k_2}/{u_2} \dots \dots .. {k_m}/{u_m} \right\}$ and

E= $\{e_1, e_2, \dots\dots e_n\}$ Then.

| (F,G,E) | $(F(e_1), G(f(e_1)))$ | $(F(e_2), G(f(e_2)))$ | … | $(F(e_n), G(f(e_n)))$ |
|---|---|---|---|---|
| $u_1$ | $(P_{11,}q_{11,})$ | $(P_{12,}q_{12,})$ | | $(P_{1n,}q_{1n,})$ |
| $u_2$ | $(P_{21,}q_{21,})$ | $(P_{22,}q_{22,})$ | | $(P_{2n,}q_{2n,})$ |
| . . . | | | | |
| $u_n$ | $(P_{m1,}q_{m1,})$ | $(P_{m2,}q_{m2,})$ | | $(P_{mn,}q_{mn,})$ |
| | | | | |

Pij = $\left\{ \begin{matrix} k_i & u_i \in^{ki} & f(ej) \\ 0 & u_i \notin^{ki} & f(ej) \end{matrix} \right\}$&

Qij = $\left\{ \begin{matrix} k_i & u_i \in^{ki} & G(f(ej)) \\ 0 & u_i \notin^{ki} & G(f(ej)) \end{matrix} \right\}$

$E_1 \cap E_2 = \Phi$ & $f$ is bijective from $E_1$ $toE_2$ , Pij and qij ,must not be $k_i$ in same time.

**Example 3.3:** Let ∪ be universal mset indicates some of the cars in carcompany and E = $\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$ be the set of all parameters. Here $e_{i\ (i=1,2,\dots\dots 8)}$indicate the parameters "cheap", "expensive", "topend", "basic", "Expected features", "high features", "high quality engine", "low quality engine", respectively.

Therefore we can chose $E_1$ and $E_2$ set as $E_1 = \{e_1, e_3, e_4, e_5, e_7,\}$and $E_2 = \{e_2, e_4, e_6, e_8\}$. The bijective function F is defined by f($e_i$)= ˥$e_i$

Hence,

$f(e_{1,}) = \urcorner e_{1,} = e_{2,}$

$f(e_{3,}) = \urcorner e_3 = e_4$

$f(e_{5,}) = \urcorner e_5 = e_6$

$f(e_{7,}) = \urcorner e_7 = e_8$

The bipolar soft sets ($F_1$,$G_1$,E) and $(F_2, G_2, E)$ to select a car.

($F_1$,G1,E)={ <(e1,{$k_1/u_1$, $k_4/u_4$, $k_5/u_5$}),
(e2,{$k_2/u_2$, $k_3/u_3$})>,<(e3,{$k_2/u_2$, $k_7/u_7$, $k_8/u_8$}), (e2,{$k_4/u_4$, $k_5/u_5$,$k_6/u_6$})>,< ($e_5$, {$u_1$,$u_5$}),($e_6$, {$u_2$,$u_4$,$u_6$,$u_8$})>,<($e_7$, {$u_8$,$u_3$,$u_6$,$u_7$}), ($e_8$, {$u_2$,$u_5$})>}

($F_2$,$G_2$,E)={<($e_1$, {$u_1$,$u_2$,$u_4$}),
($e_2$, {$u_3$,$u_5$,$u_6$,$u_7$})>, <($e_3$, {$u_2$,$u_8$}),
($e_4$, {$u_1$,$u_4$,$u_5$,$u_6$,$u_7$})>, <($e_5$, {$u_1$,$u_4$,$u_5$}),
($e_6$, {$u_2$,$u_6$,$u_7$,$u_8$,})>, <($e_7$, {$u_8$}),

| $(F_1, G_1, E_1)$ | $(F_1(e_1), G(f(e_1)))$ | $(F_1(e_3), G(f(e_3)))$ | $(F_1(e_5), G(f(e_5)))$ | $(F_1(e_7), G(f(e_7)))$ |
|---|---|---|---|---|
| $u_1$ | $(k_1,0)$ | $(k_1,0)$ | $(k_1,0)$ | $(0,0)$ |
| $u_2$ | $(0,k_2)$ | $(0,0)$ | $(0,k_2)$ | $(0,k_2)$ |
| $u_3$ | $(0,k_3)$ | $(0,0)$ | $(0,0)$ | $(k_3,0)$ |
| $u_4$ | $(k_4,0)$ | $(0,k_4)$ | $(0,k_4)$ | $(0,0)$ |
| $u_5$ | $(k_5,0)$ | $(0,k_5)$ | $(k_5,0)$ | $(0,k_5)$ |
| $u_6$ | $(0,0)$ | $(0,k_6)$ | $(0,k_6)$ | $(k_6,0)$ |
| $u_7$ | $(0,0)$ | $(k_7,0)$ | $(0,0)$ | $(k_7,0)$ |
| $u_8$ | $(0,0)$ | $(k_8,0)$ | $(0,k_8)$ | $(k_8,0)$ |

$(e_8, \{u_1,u_2,u_5\})>\}$.
Tabular representations of bipolar and soft multisets $(F_1,G_1,E)$ and $(F_2,G_2,E)$ are in table..........

| $(F_2, G_2, E_2)$ | $(F_2(e_1), G(f(e_1)))$ | $(F_1(e_3), G(f(e_3)))$ | $(F_1(e_5), G(f(e_5)))$ | $(F_1(e_7), G(f(e_7)))$ |
|---|---|---|---|---|
| $u_1$ | $(k_1,0)$ | $(k_1,0)$ | $(k_1,0)$ | $(0,k_1)$ |
| $u_2$ | $(k_2,0)$ | $(k_2,0)$ | $(0,k_2)$ | $(0,k_2)$ |
| $u_3$ | $(0,k_3)$ | $(0,0)$ | $(0,0)$ | $(0,0)$ |
| $u_4$ | $(k_4,0)$ | $(0,k_4)$ | $(k_4,0)$ | $(0,0)$ |
| $u_5$ | $(0,k_5)$ | $(0,k_5)$ | $(k_5,0)$ | $(0,k_5)$ |
| $u_6$ | $(0,k_6)$ | $(0,k_6)$ | $(0,k_6)$ | $(0,0)$ |
| $u_7$ | $(0,k_7)$ | $(0,k_7)$ | $(0,k_7)$ | $(0,0)$ |
| $u_8$ | $(0,0)$ | $(k_8,0)$ | $(0,k_8)$ | $(k_8,0)$ |

**Definition 3.4:** For any two bipolar soft multisets $(F_1,G_1,E)$ and $(F_2,G_2,E)$ over U. we say that $(F_1,G_1,E)$ is bipolar soft multiset of $(F_2,G_2,E)$ if
i)$F_1(e)$ is multisubset of $F_2(e)$, $\forall\, e \in E_1$ and ii) $G_1(f(e))$ is multisubset of $G_2(f(e))$, $\forall\, e \in E_1$

**Definition 3.6:** Two bipolar soft multisets $(F,G,E)$ and $(F_1,G_1,E)$ over a common universe U are said to be equal if
i)$(F,G,E)$ is a multisubset of $(F_1,G_1,E)$ and ii)$(F_1,G_1,E)$ is a multisubset of $(F,G,E)$

**Definition 3.7:** Let $(F,G,E),(F_1,G_1,E) \in BMS$. Then intersection of two bipolar soft multisets is denoted by
$(F,G,E) \cup^{\sim m} (F_1,G_1,E)$ defined by $(F \cup F_1)(e)=F(e) \cup F_1(e)$ and $(G \cup G_2)(f(e))=G(f(e)) \cap G_1(f(e))$, $\forall e \in E_1$

**Definition 3.8:** Union of any two bipolar soft multisets $(F_1,G_1,E)$ and $(F_2,G_2,E)$ denoted by $(F_1,G_1,E) \cap^{\sim m} (F_2,G_2,E)$ defined by $(F_1 \cap F_2)(e)=F_1(e) \cap F_2(e)$ and $(G_1 \cap G_2)(f(e))=G_1(f(e)) \cup G_2(f(e))$,for all $e \in E_1$

**Definition 3.9:** For any bipolar soft multiset $(F,G,E)$ $\forall e \in E_1$ $F(e)= \Phi$ and $G(f(e))=U$ then $(F,G,E)$ is called null bipolar soft set and denoted by $(\phi,U,E)$.

**Definition 3.10:** Absolute bipolar soft multiset of $(F,G,E)$ is denoted by $(U,\phi,E)$
and defined by $F(e)=U$ and $G(f(e))= \Phi$

**Definition 3.11:** The complement of a bipolar soft multiset (F,G,E) denoted by $(F,G,E)^{\tilde{c}^m}$ is a bipolar soft set over U such that $(F,G,E)^{\tilde{c}^m}$ =(H,K,E) where H(e)=G(f(e) and K(f(e))=F(e) for all $e \in E_1$.

**Example 3.12:** Let U={ $5/u_1, 6/u_2, 8/u_3, 7/u_4, 6/u_5, 6/u_6, 7/u_7, 5/u_8$} be an initial inverse and E={$e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8$} be a set of parameters. E can be bipartited by $E_1$={$e_1, e_3, e_5, e_7$} and $E_2$={$e_3, e_4, e_6, e_8$ }. Now, we define the bijective function f as $f(e_i)= \neg e_i (i=1,3,5,7)$. $\neg e_i$ means that " not $e_i$" for all i=1,3,5,7. We have,

$f(e_1)= \neg e_1=e_2$
$f(e_3)= \neg e_3=e_4$
$f(e_5)= \neg e_5=e_6$
$f(e_7)= \neg e_7=e_8$

we have the following bipolar soft multisets.

$(F_1,G_1,E)$={<($e_1$,{$5/u_1$, $8/u_3$, $7/u_4$,}),
($e_2$,{$6/u_2$, $5/u_6$})>,
<($e_3$,{$6/u_2$, $6/u_5$, $7/u_7$}),
($e_4$,{$5/u_1$, $8/u_3$, $5/u_8$})>,
<($e_5$,{$8/u_3$, $7/u_4$}),
($e_6$, {$5/u_1$, $6/u_2$, $6/u_5$,$5/u_8$})>,
<($e_7$,{$6/u_5$, $5/u_6$, $7/u_7$, $5/u_8$}),
($e_8$,{$6/u_2$, $8/u_3$})>}

$(F_2,G_2,E)$={<($e_1$,{$5/u_1$, $6/u_2$, $7/u_4$,}),
($e_2$,{$8/u_3$, $6/u_5$,$5/u_6$, $7/u_7$})>,
<($e_3$,{$6/u_2$, $6/u_5$}),
($e_4$,{$5/u_1$, $8/u_3$, $7/u_4$,$5/u_8$})>,
<($e_5$,{$5/u_1$,$8/u_3$, $7/u_4$}),
($e_6$, {$6/u_2$, $6/u_5$,$7/u_7$,$5/u_8$})>,
<($e_7$,{$6/u_5$}),
($e_8$,{$6/u_2$, $8/u_3$,$7/u_4$})>}

$(F_3,G_3,E)$={<($e_1$,{$5/u_1$,$7/u_4$,}),
($e_2$,{$6/u_2$, $8/u_3$,$6/u_5$, $5/u_6$})>,
<($e_3$,{$6/u_2$, $6/u_5$}),
($e_4$,{$5/u_1$, $8/u_3$, $7/u_4$,$5/u_8$})>,
<($e_5$,{$8/u_3$}),
($e_6$, {$5/u_1$, $6/u_2$,$6/u_5$,$7/u_7$,$5/u_8$})>,
<($e_7$,{$6/u_5$,$5/u_8$}),
($e_8$,{$6/u_2$, $8/u_3$,$7/u_4$,$7/u_7$})>}

$(F_1,G_1,E) \cup^{\sim m} (F_2,G_2,E)$={($e_1$,{$5/u_1$, $6/u_2$, $8/u_3$,$7/u_4$},{$5/u_6$}),($e_3$,{$6/u_2$, $6/u_5$, $7/u_7$},{$5/u_1$, $8/u_3$, $5/u_8$}), ($e_5$,{$5/u_1$,$8/u_3$, $7/u_4$},{$6/u_2$, $6/u_5$,$5/u_8$}), ($e_7$,{$6/u_5$, $5/u_6$, $7/u_7$, $5/u_8$}, {$6/u_2$, $8/u_3$})}

$(F_1,G_1,E) \cap^{\sim m} (F_3,G_3,E)$={($e_1$,{$5/u_1$,$7/u_4$},{$6/u_2$, $8/u_3$,$6/u_5$, $5/u_6$}),($e_3$,{$6/u_2$, $6/u_5$},{$5/u_1$, $8/u_3$, $5/u_8$}), ($e_5$,{$8/u_3$},{$5/u_1$, $6/u_2$, $6/u_5$,$7/u_7$,$5/u_8$}), ($e_7$,{$6/u_5$,$5/u_8$},{$6/u_2$, $8/u_3$,$7/u_4$,$7/u_7$})}

Properties of bipolar soft set are also applicable for bipolar soft multiset.

**Defintion 3.12:** Let $(F_1,G_1,E),(F_2,G_2,E) \in BMS$. Then, and-product of bipolar soft multisets $(F_1,G_1,E)$ and $(F_2,G_2,E)$ is denoted by $(F_1,G_1,E) \wedge (F_2,G_2,E)$ defined by $(F_1 \wedge F_2)(e_1,e_2)=F_1(e_1) \cap F_2(e_2)$ and $(G_1 \wedge G_2)(f(e_1),f(e_2))=G_1(f(e_1)) \cup G_2(f(e_1),f(e_2))$for all $e_1,e_2 \in E_1$.

**Definition 3.13:** Or-product of two bipolar soft multisets $(F_1,G_1,E)$ and $(F_2,G_2,E)$ is denoted by $(F_1,G_1,E) \vee (F_2,G_2,E)$ defined by $(F_1F_2) \vee (e_1,e_2)=F_1(e_1) \cup F_2(e_2)$ and $(G_1 \vee G_2)(f(e_1),f(e_2))=$ $G_1(f(e_1)) \cap G_2(f(e_2))$ for all $e_1,e_2 \in E_1$.

**An Application Of Bipolar Soft Multiset In Decision Making:** In this section, let us see the applications of bipolar soft multiset in decision making.

**Definition 4.1:** Let $E=\{e_1,e_2.........e_n\}$ be a parameter set, $U=\{k_1/u_1,k_2/u_2........k_m/u_m\}$ be initial universe and (F,G,E) be a Bsm over U. Then score of an object, denoted by $S_i$. is computed as $S_i=C_i^+ - C_i^-$ Here $C_i^+$ and $C_i^-$ are computed form $C_i^+ = \sum_{j=1}^{n} a_{ij}$ and $C_i^- = \sum_{j=1}^{n} b_{ij}$

**Algorithm:**
Step1: Input the bipolar soft multiset (F,G,E)
Step 2: consider the bipolar soft multiset (F,G,E) and write it in tabular form.
Step 3: compute the score $S_i$ of $h_i$ $\forall$ i.
Step 4: Find $S_k=\max S_i$
Let us see how this algorithm works.

**Example 4.2:** Let $U=\{2/u_1,3/u_2,7/u_3,5/u_4,6/u_5,8/u_6,7/u_7,6/u_8\}$ be the set of all available cars with available numbers.
Let $E=\{e_1,e_2,e_3,e_4,e_5,e_6,e_7,e_8\}$ $e_1$ stands "cheap", $e_2$ stands " expensive", $e_3$ stands "topend model" ,$e_4$stands "basic model", $e_5$stands "best resale", $e_6$stands " low resale price",$e_7$ stands " high engine capacity", $e_8$stands "low engine capacity".
Let $E_1=\{e_1,e_3,e_5,e_7\}$ and $E_2=\{e_2,e_4,e_6,e_8\}$ Note that we write to short (F(e),G(f(e))) as FG(e). The bipolar soft multiset is
(F,G,E)={<($e_1$,{$2/u_1$, $3/u_2$}),($e_2$,{$7/u_3$, $6/u_5$,$8/u_6$, $7/u_7$})> ,<($e_3$,{$3/u_2$,}),($e_4$,{$2/u_1$, $7/u_3$,$5/u_4$, $6/u_8$})>,<($e_5$,{$2/u_1$, $7/u_3$,$5/u_4$}),($e_6$,{$3/u_2$, $6/u_5$,$7/u_7$, $6/u_8$})>,<($e_7$,{$2/u_1$, $7/u_3$,$6/u_8$}),($e_8$,{6/u5, $8/u_6$})>}

| (F,G,E) | FG($e_1$) | FG($e_3$) | FG($e_5$) | FG($e_7$) |
|---|---|---|---|---|
| $u_1$ | (2,0) | (0,2) | (2,0) | (2,0) |
| $U_2$ | (3,0) | (3,0) | (0,3) | (0,0) |
| $U_3$ | (0,7) | (0,7) | (7,0) | (7,0) |
| $U_4$ | (0,0) | (0,5) | (5,0) | (0,0) |
| $U_5$ | (0,6) | (0,0) | (0,6) | (0,6) |
| $U_6$ | (0,8) | (0,0) | (0,0) | (0,8) |
| $U_7$ | (0,7) | (0,0) | (0,7) | (0,0) |
| $U_8$ | (0,0) | (0,6) | (0,6) | (6,0) |

| Now , | $C_i^+$ | $C_i^-$ | $S_i$ |
|---|---|---|---|
| $U_1$ | 4 | 4 | 0 |
| $U_2$ | 6 | 3 | 3 |
| $U_3$ | 7 | 14 | -7 |
| $U_5$ | 0 | 12 | -12 |
| $U_4$ | 5 | 5 | 0 |
| $U_6$ | 0 | 8 | -8 |
| $U_7$ | 0 | 14 | -14 |
| $U_8$ | 0 | 12 | -12 |

The buyer should buy $u_2$ which is 10 times better than others.

**Conclusion:** in this paper we introduced new version of soft set called as bipolar soft multiset and investigated its properties. Finally we saw the application of BSMS.

**References:**

1. L. A. Zadeh," Fuzzy sets", Inf. Control 8 (1965) 378-352.
2. Z. Pawlak, "Rough Sets", *Int. J. Inf. Comput. Sci.* 11 (1982) 341-356.
3. M. B. Gorzalzany, "A method of inference inapproximate reasoning based on interval-valued fuzzy sets", *Fuzzy sets and systems* 21 (1987) 1-17.
4. D. A. Moldtsov, "soft set theory : first result" *Comput. Math. Appl.* 37 (1999) 19-31.

5. Herawan, T, Mustafa, M. D. "On multi-soft set construction in information system", ICIC 2009LNAI, Springer, Heidelberg 5755 (2009) 101-110.
6. Babitha K. V., Sunil J. J. (2012) "On softmultiset" *Annals of Fuzzy Mathematics and Informatics* Volume 5, No.1, pp.35-44.
7. A.M. Ibrahim and H. M. Balami, "Application of soft multiset in decision making problems", *J.Nig. Ass. Of mathematical physics*.Vol. 25 (2013) pp 307- 311.
8. S. Alkhazaleh, A. R. Saleh, N. Hassan, "Soft Multiset Theory" *Applied Mathematical Sciences* vol. 5, 2011, no. 72, 3561- 3573.
9. Faruk Karaaslana and Serkan Karata¸s," A new approach to bipolar soft sets and its applications" arXiv:1406.2274v1 [math.GM] 6 Jun 2014.
10. Ke Gong, Zhi Xiao, Xia Zhang, "The bijective soft set with its operations", *computers and Mathematics with Applications* 60(2010) ,2270-2278.
11. P.K. Maji, R. Biswas, A.R. Roy," Soft set theory", *Comput. Math. Appl*. 45 (2003) 555–562.

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