OPTIMAL INVENTORY POLICIES FOR A MIDDLEMAN HANDLING PERISHABLE ITEMS IN MARKETING

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Abstract: A middleman is a person who connects consumers with retailers. The middleman purchase goods from a producer and then resell them to a supplier. A retailer is also an example of a middleman, as is a real estate agent. This paper deals with the development of deteriorating inventory model for a middle man, with constant demand and time dependent holding cost. In this model shortages are not allowed. A middleman's purpose in the marketing process is to be more cost efficient when transferring goods or services. So, the optimal total inventory cost and optimal order quantity is obtained with the help of calculus. Truncated Taylor's series is applied for finding a closed numeral optimal solution. Finally numerical example and sensitivity analysis is given to authorize the above model.

Keywords: Inventory Model, General, Preservation Technology, Time Dependent Holding Cost, Deterioration.

1. **Introduction:** The middleman does not produce anything but has extensive market knowledge. Wholesalers are a type of middleman. In traditional inventory models, demand rate is considered as either constant or time dependent. In case of demand rate is constant, the effects of variability of the holding cost on the total inventory cost functions of such model have also been considered. In real life, it is observed that demand for a particular product can be influenced by internal factors such as deterioration, price and variability. The main objective of inventory management is to minimize the total inventory carrying cost. Most of the items deteriorate with time. When the inventory undergoes decay or deterioration, there is a loss of original value of commodity that results in the decreasing usefulness from the original one. Certain products such as medicine, blood, green vegetables, radioactive chemicals, volatiles decrease under deterioration during their normal storage period. Thus while determining optimal inventory policy of that type of product; the loss due to deterioration can not be ignored. Various types of inventory models for deteriorating items were discussed by Agrawal and Jaggi [1], Roy Chowdhury and Chaudhuri [2], Padmanabhan and Vrat [3], Balkhi and Benkherouf [4] and Yang[5], Teng et el. [6] developed optimal pricing and ordering policy under permissible delay in payments by considering demand rate is a function of price. An EOQ model for perishable items with power demand and partial backlogging developed by Singh et al. [7]. Tripathy and Pradhan [8] developed an inventory model for Weibull deteriorating items with constant demand when delay in payments is allowed to retailer to settle the account against the purchases made. Dhandapani and Uthayakumar [9]present an inventory model integrating the advanced preservation technology to assimilate demand difference among different fruits. Liao et al. [10],Huang and Chung [11] developed optimal replenishment and payment policies in the EOQ model under cash discount and trade credit. Teng et al. [12] developed an inventory model for optimal pricing and ordering policy under permissible in payments.

For certain type of inventory, the consumption rate may be influenced by the stock levels, it means that consumption rate may go up and down with the on hand stock level. Hou [13] developed an inventory model for deteriorating items with stock dependent consumption rate and shortages under inflation and time discounting over a finite planning horizon and shown that total cost function is convex. Goh [14] developed inventory model by considering stock dependent demand rate and variable holding cost. Padmanabhan and Vrat [15] defined stock dependent consumption rate as a function of inventory level at any instant of time and developed models for nonsales environment, Sarker et al. [16] developed an order level lot size inventory model with inventory level dependent demand and deterioration level. Datta and Pal [17] presented an inventory system where the demand rate is influenced by stock level and selling price. Balkhi and Benkherouf [18] developed an inventory model for deteriorating items with stock dependent and time varying demand rate over a finite planning horizon. Other related articles on inventory model with a stock dependent demand rate have been performed by Pal et al. [19], Datta and Pal [20] and Baker and Urban [21].

In classical inventory EOQ model, holding cost is constant but in actual practice holding cost is not constant, it is a function of time or stock dependent. Tripathi [22] developed an inventory model for time varying demand and constant demand; and time dependent holding cost and constant holding cost. Alfares {23] developed an EOQ model by considering holding cost an increasing function of time spent in shortage. Ray and Chaudhuri [24] developed an inventory system with stock dependent demand rate and shortages. Goh [25] developed a model by considering holding cost variation over time as a continuous non-linear function.

2. Notations and Assumptions:

2.1. Notations

The following notations are used throughout the manuscript:

K ordering cost per order
τ transportation cost per order
λ constant annual demand rate
$I(t)$ on-hand inventory level at time t
$h(t)$ time dependent holding cost of the item at time t, $h(t) = h.t$
Q order quantity
Q^* optimal order quantity
θ deterioration rate ($0 \leq \theta < 1$)
u Preservation technology cost for reducing deterioration rate in order to preserve the product, $u > 0$.
$m(u)$ Reduced deterioration rate due to use of Preservation technology.
ω Resultant deterioration rate, $\omega = 1 - m(u)$.
T cycle time
T^* optimal cycle time
TIC total inventory cost per cycle
TIC* optimal total inventory cost per cycle

2.2. Assumptions: The following assumptions are being made throughout the manuscript

1. The demand rate is constant.
2. The holding cost is time dependent i.e. $h(t) = h.t$.
3. There is no replacement or repair of deteriorated items in a given cycle but tocontrolthe
4. deterioration rate, preservationtechnologyis used.
5. Shortages are not allowed.
6. The inventory system under consideration deals with single item.
7. The planning horizon is infinite and lead time is zero.

3. Mathematical Formulations: The inventory under consideration is assumed to be constant demand rate with deterioration. Hence the rate of change of inventory level is governed by the following differential equation.

$$\frac{dI(t)}{dt} + \omega\theta . I(t) = -\lambda, 0 \le t \le T \qquad (1)$$

With the condition $I(T) = 0$. The solution of (1) is given by

$$I(t) = \frac{\lambda}{\omega\theta}\left(e^{\omega\theta\,(T-t)} - 1\right) \qquad (2)$$

The objective is to minimize the total inventory cost per unit time which contains three components (i) the ordering cost (ii) the holding cost per unit time and (iii) the transportation cost

(iii) Theordering costduring the period [0, T] is

ordering cost $= \frac{K}{T}$

(iv) The total holding cost per cycle is the integral of the product of the holding cost h(t) and inventory cost I(t) over the whole cycle T.The holding cost during the period [0, T] is

Inventory holding cost = $\frac{1}{T}\int_0^T H(t)\,.I(t)dt$

$$= \frac{1}{T}\int_0^T h\,t\,.I(t)dt$$

$$= \frac{h\lambda e^{\theta\omega T}}{\omega^3\theta^3 T} - \frac{h\lambda T}{2\omega\theta} - \frac{h\lambda}{\omega^2\theta^2} - \frac{h\lambda}{\omega^3\theta^3 T}$$

(v) Transportation cost = $\frac{\tau}{T}$

$TIC = \frac{1}{T}$ (Orderingcost + Inventory holding cost+ Transportation cost)

$$TIC = \frac{K}{T} - \frac{h\lambda T}{2\omega\theta} - \frac{h\lambda}{\omega^2\theta^2} - \frac{h\lambda}{\omega^3\theta^3 T} + \frac{h\lambda e^{\theta\omega T}}{\omega^3\theta^3 T} + \frac{\tau}{T} \qquad (3)$$

The optimal solution of (3) in obtained by solving $\frac{dTIC}{dT} = 0$. But it is difficult to find the solution of (3) in the present form. For finding closed form solution Truncated Taylor's series is used in exponential form i.e.$e^{\theta\omega T} \approx 1 + \theta\omega T + \frac{\omega^2\theta^2 T^2}{2} + \frac{\omega^3\theta^3 T^3}{6}$.We obtain

Total inventory cost

$TIC = \frac{K}{T} + \frac{h\lambda T^2}{6} + \frac{\tau}{T}$(4)

$$\frac{dTIC}{dT} = -\frac{K}{T^2} + \frac{h\lambda T}{3} \qquad (5)$$

$$\frac{d^2TIC}{dT^2} = \frac{2K}{T^3} + \frac{h\lambda}{3} > 0 \qquad (6)$$

Thus the optimal solution is obtained by putting

$$\frac{dTIC}{dT} = 0$$

We get optimal cycle time$T = T^* = \left[\frac{3K}{h\lambda}\right]^{\frac{1}{3}}$

Applying the condition I (0) = Q in (2) and using Truncated Taylor's series in exponential form i.e. $e^{\theta\omega T} \approx 1 + \theta\omega T + \frac{\omega^2\theta^2 T^2}{2} + \frac{\omega^3\theta^3 T^3}{6}$etc. We obtain Optimal order quantity

$$Q = Q^* = \lambda T\left(1 + \frac{\theta\omega T}{2} + \frac{\omega^2\theta^2 T^2}{6}\right)$$

4. Numerical Example: Let λ = 200 units/year, k = \$10, 000 per order, τ = \$300 per order, ω = \$3, θ = 0.3, h= \$10 per unit per year

$$T = T^* = \left[\frac{3K}{h\lambda}\right]^{\frac{1}{3}}$$

Putting the values in above equation, we get $T^* \approx$ 2.4662years and putting value of T^* in equation (4), we get TIC ≈ \$2149.0850.

5. Sensitivity Analysis: To find sensitivity analysis, the effect of parameters 'h', 'k', 'θ', 'λ' on optimal solution, the set of values of 'h', 'k', 'θ', 'λ' are assumed to as 'h' = 20, 40, 50, 70, 100, 200, 300, 'k'= 400, 500, 1000, 2000, 3000, 4000, 5000, 6000, 7000, 8000, 9000, 10, 000, 'θ'= 0.60, 0.65, 0.70, 0.75, 0.80, 0.85, 0.90 and 'λ' = 20, 50, 100, 400, 600, 800, and 1000. Meanwhile the other parameter's values follow those data mentioned above in the numerical example. The results of sensitivity analysis are given in Table 1, 2, 3 and 4.

Table: 1 Variation of optimal solution of $T = T^*, TIC = TIC^*$and $Q = Q^*$ with the variation of the holding cost parameter h, keeping all the parameters same as in example.

h	$T = T^*$	$Q = Q^*$	$TIC = TIC^*$
11	2.3891	1445.6269	1946.3469
12	2.3208	1445.6269	2143.6088
13	2.2597	1445.6269	2340.8707
14	2.2046	1445.6269	2538.1326
15	2.1544	1445.6269	2613.7300
16	2.1086	1445.6269	2932.6565
17	2.0664	1445.6269	3129.9184

Table: 2 Variation of optimal solution of $T = T^*, TIC = TIC^*$and $Q = Q^*$ with the variation of the ordering cost K, keeping all the parameters same as in example.

K	$T = T^*$	$Q = Q^*$	$TIC = TIC^*$
8000	2.2894	1445.6269	1338.1207
8300	2.3177	1445.6269	1459.7654
8600	2.3453	1445.6269	1581.4100
8900	2.3722	1445.6269	1703.0546
9200	2.3986	1445.6269	1824.6992
9500	2.4244	1445.6269	1946.3438
9800	2.4497	1445.6269	2067.9884

Table: 3 Variation of optimal solution of $T = T^*, TIC = TIC^*$and $Q = Q^*$ with the variation of deterioration rate θ, keeping all the parameters same as in example.

θ	$T = T^*$	$Q = Q^*$	$TIC = TIC^*$
0.60	2.4662	3219.6381	2149.0850
0.65	2.4662	3593.6995	2149.0850
0.70	2.4662	3990.3648	2149.0850
0.75	2.4662	4409.6339	2149.0850
0.80	2.4662	4851.5068	2149.0850
0.85	2.4662	5315.9836	2149.0850
0.90	2.4662	5803.0642	2149.0850

Table: 4 Variation of optimal solution of $T = T^*, TIC = TIC^*$and $Q = Q^*$ with the variation of λ, keeping all the parameters same as in example.

λ	$T = T^*$	$Q = Q^*$	$TIC = TIC^*$
20	5.3133	144.5627	3973.7277
50	3.9149	361.40672	3669.6206
100	3.1072	722.81344	3162.7754
125	2.8845	903.51679	2909.3528
150	2.7144	1084.2202	2655.9302
175	2.5785	1264.9235	2402.5076
200	2.4662	1445.6269	2149.0849

Based on the results we can make the following conclusions,

1. Based on the observations found from table 1, we can conclude that with increase in holding cost parameter h, decrease in optimal cycle time $T = T^*$and $TIC = TIC^*$ but $Q = Q^*$is a constant function.
2. From table 2, an increase in ordering cost k results increase in optimal cycle time $T = T^*$ and optimal total inventory cost $TIC = TIC^*$ but $Q = Q^*$is a constant function.
3. From table 3, an increase in deterioration rate θ results increase in optimal order quantity $Q = Q^*$. but optimal cycle time $T = T^*$and optimal total inventory cost $TIC = TIC^*$ are constant functions.

4. From table 4, an increase in demand rate λ decreases in optimal cycle time $T = T^*$ and optimal total inventory cost $TIC = TIC^*$ but increase in optimal order quantity $Q = Q^*$.

6. Conclusion and Future Research: In this paper, we developed an inventory model for a middle man in marketing who handles the deteriorating items with constant demand rate and time dependent holding cost. Shortages are not allowed and cycle time is infinite. Total inventory cost (minimum) is obtained by using calculus. Numerical example is given for determining the minimum total inventory cost. Sensitivity analysis is specified for determining the total inventory cost, optimal order quantity and optimal cycle time and it is observed that with the increase in holding cost 'h', total inventory cost 'TIC' increases. For managerial point of view, the variation is quite sensitive with the variation of parameters. This model is valid for short time inventories. The model presented in this study can be extended in different ways. The model can be extended by involving shortages. This model can also extend for time dependent deteriorating items.

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