REMARKS ON A NEW FORM OF NANOCLOSED SETS

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Abstract: In this paper, the notions of $N\Lambda_a$ -sets and $N\Lambda_a$ -closed sets are introduced via nano a -open sets and nano a-closed sets. The notions of $N\Lambda_a$ -continuity , $N\Lambda_a$ -compact space and $N\Lambda_a$ -connected spaces are also introduced and several characterizations of them are obtained.

Keywords:, nano a -closed sets, nano open set, Nano topology, $N\Lambda_a$ -sets, $N\Lambda_a$ -closed sets.

MSC subject classification: 54B05, 54 C10.

Introduction: The concept of generalized closed sets plays a significant role in General Topology and they are the research topics of many Topologists worldwide. In 1970, Levine [8] introduced the concept of generalized closed sets as a generalization of closed sets in topological spaces. In 1986, Maki continued the work of Levine and Dunham on generalized closed sets by introducing the notion of a generalized Λ set in a topological space. Erdal Ekici introduced *a*-open sets. Lellis Thivagar introduced a new research area in topology namely Nano Topology[4], which was introduced in terms of approximations and boundary region of a subset of an universe using an equivalence relation. He also introduced the weak form of nano open sets namely nano *α*-open sets, nano semi-open sets, nano pre-open sets and nano generalized-closed sets. In this paper, we develop *N Λa* sets and *N Λa*-closed sets in nano topology and its properties are studied. Based on these sets, we introduce a new class of continuous function called nano *NΛa*-continuous function. As an application of *N Λa*-closed sets, *NΛa*-connected spaces are studied.

Preliminaries: In this section, we discuss some basic definitions which will be useful for this paper.

Definition 2.1 [2] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space and let $X \subseteq \mathsf{U}$.

1. The lower approximation of X with respect to R is the set of all objects, which can be for certainly classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \cup_{x\in\mathsf{U}}\{R(x) : R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by x.
2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \cup_{x\in\mathsf{U}}\{R(x) : R(x) \cap X \neq \emptyset\}$, where $R(x)$ denotes the equivalence class determined by x.
3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2 [3] Let U be an universe, R be an equivalence relation on U and $\tau_R(X) = \{\mathsf{U}, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq \mathsf{U}$. Then, $\tau_R(X)$ satisfies the following axioms :

1. U and $\phi \in \tau_R(X)$.
2. The union of the elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.
3. The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ forms a topology on U called the nano topology on U with respect to X. We call $(\mathsf{U}, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called nano-open sets and its complements are called nano closed sets.

Definition 2.3 [3] If $(\mathsf{U}, \tau_R(X))$ is a nano topological space with respect to X where $X \subseteq \mathsf{U}$ and if $A \subseteq \mathsf{U}$, then the nano interior of A is defined as the union of all nano-open subsets of A and is denoted by $\mathsf{N}int(A)$. That is, $\mathsf{N}int(A)$ is the largest nano-open subset of A.

The nano closure of A is defined as the intersection of all nano closed sets containing A and is denoted by $\mathsf{N}cl(A)$. That is, $\mathsf{N}cl(A)$ is the smallest nano closed set containing A.

Definition 3.1 [3] A subset A of a nano topological space $(\mathsf{U}, \tau_R(X))$ is called nano δ-closed if $A = \mathsf{N}cl_\delta(A)$ where $\mathsf{N}cl_\delta(A) = \{x \in \mathsf{U} : \mathsf{N}int(\mathsf{N}cl(U)) \cap A \neq \emptyset, U \in \tau_R(X), x \in \mathsf{U}\}$ and $x \in U$. The complement of nano δ-closed set is called nano δ-open set.

$\mathsf{N}\Lambda_a$**-Sets and** $\mathsf{N}V_a$**-Sets:** This section introduces some new types of sets in nano topology such as $\mathsf{N}\Lambda_a$-sets and $\mathsf{N}V_a$-closed sets and several properties of such notions are derived.

Definition 3.1 A subset A of a nano topological space $(\mathsf{U}, \tau_R(X))$ is called a nano a-open set if A $\subseteq \mathsf{N}int(\mathsf{N}cl(\mathsf{N}int_\delta(A)))$' The set of all nano a-open and nano a-closed sets are denoted as $\mathsf{N}aO(\mathsf{U}, \tau_R(X))\}$ and $\mathsf{N}aC(\mathsf{U}, \tau_R(X))\}$ respectively.

Definition 3.2 Let A be a subset of a nano topological space $(\mathsf{U}, \tau_R(X))$. We define the following subsets $\mathsf{N}\Lambda_a(A)$ and $\mathsf{N}V_a(A)$ as follows:

$\mathsf{N}\Lambda_a(A) = \cap\{G : A \subseteq G, G \in \mathsf{N}aO(\mathsf{U}, \tau_R(X))\}$ $\mathsf{N}V_a(A) = \cup\{H : H \subseteq A, H \in \mathsf{N}aC(\mathsf{U}, \tau_R(X))\}$

Lemma 3.3 Let A, B and $\{B_i : i \in I\}$ be subsets of a nano topological space $(\mathsf{U}, \tau_R(X))$. Then the following properties hold:

1. $A \subseteq \mathsf{N}\Lambda_a(A)$.
2. If $A \subseteq B$, then $\mathsf{N}\Lambda_a(A) \subseteq \mathsf{N}\Lambda_a(B)$.
3. $\mathsf{N}\Lambda_a(\mathsf{N}\Lambda_a(A)) = \mathsf{N}\Lambda_a(A)$.
4. If $A \in \mathsf{N}aO(\mathsf{U}, \tau_R(X))$, then $A = \mathsf{N}\Lambda_a(A)$ 5. $\mathsf{N}\Lambda_a(\cup_{i\in I} B_i) = \cup_{i\in I} \mathsf{N}\Lambda_a(B_i)$.
6. $\mathsf{N}\Lambda_a(\cap_{i\in I} B_i) \subseteq \cap_{i\in I} \mathsf{N}\Lambda_a(B_i)$.
7. $\mathsf{N}\Lambda_a(A^c) = (\mathsf{N}V_a(A))^c$ and $\mathsf{N}V_a(A^c) = (\mathsf{N}\Lambda_a(A))^c$.

Proof :

1. Let $x \notin \mathsf{N}\Lambda_a(A)$. Then there exists a $G \in \mathsf{N}aO(\mathsf{U}, \tau_R(X))$ such that $A \subseteq G$ and $x \notin G$. Hence, $x \notin A$ and so $A \subseteq \mathsf{N}\Lambda_a(A)$.
2. Let $x \notin \mathsf{N}\Lambda_a(B)$. Then there exists a $G \in (\mathsf{U}, \tau_R(X)))$ such that $B \subseteq G$ and $x \notin G$. Since $A \subseteq B$, we have $A \subseteq G$ and $x \notin G$ and hence $x \notin \mathsf{N}\Lambda_a(A)$.
3. From (i), $\mathsf{N}\Lambda_a(A) \subseteq \mathsf{N}\Lambda_a(\mathsf{N}\Lambda_a(A))$.On the other hand, if $x \in \mathsf{N}\Lambda_a(\mathsf{N}\Lambda_a(A))$, then for every nano a-open set G we have $\mathsf{N}\Lambda_a(A) \subseteq G$ and $x \in G$. By (i), $A \subseteq \mathsf{N}\Lambda_a(A) \subseteq G$, $x \in \mathsf{N}\Lambda_a(A)$ and so $\mathsf{N}\Lambda_a(\mathsf{N}\Lambda_a(A)) \subseteq \Lambda_a(A)$.
4. It is obvious from the definition of $\mathsf{N}\Lambda_a(A)$
5. Let $B = \cup_{i\in I}\{B_i\}$, then, $\mathsf{N}\Lambda_a(B_i) \subseteq \mathsf{N}\Lambda_a(B)$, for each $i \in I$ and hence $(\cup\{\mathsf{N}\Lambda_a(B_i) : i \in I\}) \subseteq \mathsf{N}\Lambda_a(B) = \mathsf{N}\Lambda_a(\cup\{B_i : i \in I\})$. On the other hand, suppose $x \notin \cup\{\mathsf{N}\Lambda_a(B_i) : i \in I\})$, then, $x \notin \mathsf{N}\Lambda_a(B_i)$, for each $i \in I$ and there exists a nano a-open set G_i such that $B_i \subseteq G_i$ and $x \notin G_i$, for each $i \in I$. We have $\cup_{i\in I} B_i \subseteq \cup_{i\in I} G_i$ and $\cup_{i\in I} G_i$ is a nano a-open set such that $x \notin \cup_{i\in I} G_i$. Thus, $x \notin \mathsf{N}\Lambda_a(\cup\{B_i : i \in I\})$ and so (v).
6. Assume that $B = \cap B_i$ and $B \subseteq B_i$, then $\mathsf{N}\Lambda_a(B) \subseteq \mathsf{N}\Lambda_a(B_i)$, for all $i \in I$ and hence $\mathsf{N}\Lambda_a(\cap_{i\in I} B_i) \subseteq \cap_{i\in I}\mathsf{N}\Lambda_a(B_i)$.
7. $(\mathsf{N}\Lambda_a(A))^c = X - \cup\{G \in \mathsf{N}aO((\mathsf{U}, \tau_R(X)) : A \subseteq G\}$
   $= \cap\{G^c \in \mathsf{N}aC((\mathsf{U}, \tau_R(X)) : G^c \subseteq A^c\} = (\mathsf{N}V(A^c))$. Similarly we can prove, $\mathsf{N}\Lambda_a(A^c) = (\mathsf{N}V_a(A))^c$
   The following example shows that the equality need not be true in (vi) of lemma 3.2. That is, $\mathsf{N}\Lambda_a(A \cap B) \neq \mathsf{N}\Lambda_a(A) \cap \Lambda_a(B)$.**Example 3.4** Let $\mathsf{U} = \{a,b,c,d\}$ with $\mathsf{U}/R = \{\{a,c\},\{b\},\{d\}\}$ and $X = \{a,b\}$. Then $\tau_R(X) = \{\varnothing, \mathsf{U}, \{b\}, \{a,c\}, \{a,b,c\}\}$. If $A = \{a,b\}$ and $B = \{b,c\}$. Then $\Lambda_a(A \cap B) \neq \{a,b,c\} = \mathsf{N}\Lambda_a(A) \cap \Lambda_a(B)$.

Lemma 3.5 Let A, B and $\{B_i : i \in I\}$ be subsets of a nano topological space $(\mathsf{U}, \tau_R(X))$. Then the following properties hold:

1. $\mathsf{N}V_a(A) \subseteq A$.
2. If $A \subseteq B$, then $\mathsf{N}V_a(A) \subseteq \mathsf{N}V_a(B)$.
3. $\mathsf{N}V_a(\mathsf{N}V_a(A)) = \mathsf{N}V_a(A)$.
4. If $A \in \mathsf{N}aC(\mathsf{U}, \tau_R(X))$, then $A = \mathsf{N}V_a(A)$. 5. $\mathsf{N}V_a(\cap\{B_i : i \in I\}) = \cap\{\mathsf{N}V_a(B_i) : i \in I\})$
6. $\mathsf{N}V_a(\cup\{B_i : i \in I\}) \supseteq (\cup\{\mathsf{N}V_a(B_i) : i \in I\})$

Proof The proof follows trivially from the lemma 3.3.

Definition 3.6 A subset A of a nano topological space is said to be a $\mathsf{N}\Lambda_a$-set (resp. $\mathsf{N}V_a$-set) if $A = \mathsf{N}\Lambda_a(A)$ resp. $(A = \mathsf{N}V_a(A))$.

Lemma 3.7 Let A and $\{A_i : i \in I\}$ be subsets of a nano topological space $(\mathsf{U}, \tau_R(X))$. Then the following properties hold:

1. $\varnothing$ and U are $\mathsf{N}\Lambda_a$-sets and $\mathsf{N}V_a$-sets.
2. If $A \in \mathsf{N}aO(\mathsf{U}, \tau_R(X))$, then A is a $\mathsf{N}\Lambda_a$-set.

3. If A_i is a $\mathsf{N}\Lambda_a$-set for each $i \in I$, then $\cup_{i\in I} A_i$ is $\mathsf{N}\Lambda_a$-set.
4. If A_i is a Λ_a-set for each $i \in I$, then $\cap_{i\in I} A_i$ is $\mathsf{N}\Lambda_a$-set.
5. If $A \in \mathsf{N}aC(\mathsf{U}, \tau_R(X))$, then A is $\mathsf{N}V_a$-set.
6. If A_i is a $\mathsf{N}V_a$-set for each $i \in I$, then $\cup_{i\in I} A_i$ is $\mathsf{N}V_a$-set.
7. If A_i is a $\mathsf{N}V_a$-set for each $i \in I$, then $\cap_{i\in I} A_i$ is $\mathsf{N}V_a$-set.
8. A is a $\mathsf{N}\Lambda_a$-set if and only if A^c is is a $\mathsf{N}V_a$-set'

Proof Proof follows from lemma(3.3) and lemma(3.5)

Remark 3.8 From the above lemma, we can observe that the collection of all $\mathsf{N}\Lambda_a$-sets is a topology.

Definition 3.9 A subset A of a nano topological space $(\mathsf{U}, \tau_R(X))$ is said to be $\mathsf{N}\Lambda_a$-closed if $A = T \cap C$, where T is $\mathsf{N}\Lambda_a$-set and C is nano a-closed set. family of all $\mathsf{N}\Lambda_a$-closed sets of a nano topological space $(\mathsf{U}, \tau_R(X)$ is denoted by $\mathsf{N}\Lambda_a C(\mathsf{U}, \tau_R(X))$.

Proposition 3.10 In a nano topological space $(\mathsf{U}, \tau_R(X))$, every $\mathsf{N}\Lambda_a$-set is $\mathsf{N}\Lambda_a$-closed.

Remark 3.11 The converse of the above proposition need not be true as shown in the following example.

Example 3.12 Let $\mathsf{U} = \{a,b,c,d\}$ with $\mathsf{U}/R = \{\{a\},\{b,c\},\{d\}\}$ and $X = \{a,c\}$. Then $\tau_R(X) = \{\varnothing, \mathsf{U}, \{b\}, \{a,c\}, \{a,b,c\}\}$. Then $\{b,c\}$ is $\mathsf{N}\Lambda_a$-closed but not nano a-closed. Here the set $\{a,d\}$ is $\mathsf{N}\Lambda_a$-closed but not $\mathsf{N}\Lambda_a$-set.

Theorem 0.13 The following statements are equivalent for a subset A of a nano topological space $(\mathsf{U}, \tau_R(X))$:

1. A is $\mathsf{N}\Lambda_a$-closed.
2. $A = T \cap \mathsf{N}acl(A)$, where T is a $\mathsf{N}\Lambda_a$-set.
3. $A = \mathsf{N}\Lambda_a(A) \cap \mathsf{N}acl(A)$.

Proof $(i) \Rightarrow (ii)$: Suppose A is $\mathsf{N}\Lambda_a$-closed. Then, $A = T \cap C$, where T is $\mathsf{N}\Lambda_a$-set and C is nano a-closed. Since $A \subseteq C$, $\mathsf{N}acl(A) \subseteq \mathsf{N}acl(C) = C$ and hence $A = T \cap C \supseteq T \cap \mathsf{N}acl(A) \supseteq A$. Thus, $A = T \cap \mathsf{N}acl(A)$.

$(ii) \Rightarrow (iii)$: Suppose $A = T \cap \mathsf{N}acl(A)$, where T is $\mathsf{N}\Lambda_a$-set. Since $A \subseteq T$, by $Lemma(3.2)\mathsf{N}\Lambda_a(A) \subseteq \mathsf{N}\Lambda_a(T) = T$ and so $A \subseteq \mathsf{N}\Lambda_a(A) \cap \mathsf{N}acl(A) \subseteq T \cap \mathsf{N}acl(A) = A$. Thus, $A = \mathsf{N}\Lambda_a(A) \cap \mathsf{N}acl(A)$.

$(iii) \Rightarrow (i)$: The proof follows from the fact that $\mathsf{N}\Lambda_a(A)$ is $\mathsf{N}\Lambda_a$-set and $\mathsf{N}acl(A)$ is nano a-closed.

Theorem 3.14 If A_i is $\mathsf{N}\Lambda_a$-closed for each $i \in I$, then $\cap\{A_i : i \in I\})$ is Λ_a-closed. **Proof** Since A_i is $\mathsf{N}\Lambda_a$-closed for each $i \in I$, there exists a $\mathsf{N}\Lambda_a$-set T_i and a nano a-closed set C_i such that $A_i = T_i \cap C_i$ and $\cap\{A_i : i \in I\} = \cap\{T_i \cap C_i : i \in I\} = (\cap\{T_i : i \in I\}) \cap (\cap\{C_i : i \in I\})$. By Lemma(3.7), $\cap\{T_i : i \in I\})$ is $\mathsf{N}\Lambda_a$-set and $\cap\{C_i : i \in I\}$ is nano a-closed and hence $\cap\{A_i : i \in I\}$ is $\mathsf{N}\Lambda_a$-closed.

Definition 3.15 A subset A of a nano topological space $(\mathsf{U},\tau_R(X))$ is said to be $\mathsf{N}\Lambda_a$-open if the complement of A is $\mathsf{N}\Lambda_a$-closed. The family of all $\mathsf{N}\Lambda_a$-open sets of a nano topological space $(\mathsf{U},\tau_R(X))$ is denoted by $\mathsf{N}\Lambda_a O(\mathsf{U},\tau_R(X))$

Theorem 3.16 The following statements are equivalent for a subset A of a nano topological space $(\mathsf{U},\tau_R(X)$:

1. A is $\mathsf{N}\Lambda_a$-closed.
2. $A=T\cup C$ where T is a $\mathsf{N}V_a$-set and C is nano a-open set.
3. $A=T\cup \mathsf{N}aint(A)$, where T is $\mathsf{N}V_a$-set. 4. $A=\mathsf{N}V_a(A)\cap \mathsf{N}aint(A)$

Proof $(i)\Rightarrow(ii)$: Suppose A is $\mathsf{N}\Lambda_a$-open, then $X-A=T\cap C$, where T is $\mathsf{N}V_a$-set and C is nano a-closed. Hence, $A=(X-T)\cup(X-C)$ where $(X-T)$ is a $\mathsf{N}V_a$-set and $(X-C)$ is a nano a-open set which implies (ii).

$(ii)\Rightarrow(iii)$: Suppose $A=T\cup C$, where T is $\mathsf{N}V_a$-set and C is nano a-open. Since $C\subseteq A$ and C is nano a-open, $C=\mathsf{N}aint(C)\subseteq \mathsf{N}aint(A)$ and hence, $A=T\cup C\subseteq T\cap \mathsf{N}aint(A)\subseteq A$. Thus, $A=T\cup \mathsf{N}int(A)$.

$(iii)\Rightarrow(iv)$: Suppose $A=T\cup \mathsf{N}aint(A)$, where T is $\mathsf{N}V_a$-set. Since $T\subseteq A$, $\mathsf{N}V_a(T)\subseteq \mathsf{N}V_a(A)$ and so $T\subseteq \mathsf{N}V_a(A)$. Hence, $A=T\cup \mathsf{N}aint(A)\subseteq \mathsf{N}V_a(A)\cup \mathsf{N}aint(A)\subseteq A$. Thus, $A=\mathsf{N}V_a(A)\cap \mathsf{N}aint(A)$.

$(iv)\Rightarrow(i)$: Suppose $A=\mathsf{N}V_a(A)\cap \mathsf{N}aint(A)$, Then $X-A=(X-\mathsf{N}V_a(A))\cap(X-\mathsf{N}aint(A))=\mathsf{N}\Lambda_a(X-A)\cap \mathsf{N}acl(X-A)$.Since $\mathsf{N}\Lambda_a(X-A)$ is $\mathsf{N}\Lambda_a$-set and $\mathsf{N}acl(X-A)$ is nano a-closed, $(X-A)$ is $\mathsf{N}\Lambda_a$-closed and hence A is $\mathsf{N}\Lambda_a$-open.

Proposition 3.17 The following hold for a subset A of a nano topological space $(\mathsf{U},\tau_R(X)$:

1. Every nano a-open set is $\mathsf{N}\Lambda_a$-open.
2. Every $\mathsf{N}V_a$-set is $\mathsf{N}\Lambda_a$-open.

Proof (i) Let A be a nano a-open set in a nano topological space $(\mathsf{U},\tau_R(X)$. Then $(X-A)$ is nano a-closed in U. By proposition $(X-A)$ is $\mathsf{N}\Lambda_a$-closed and hence A is $\mathsf{N}\Lambda_a$-open.

(ii) Let A be a $\mathsf{N}V_a$- set in $(\mathsf{U},\tau_R(X))$. Then, $A=A\cup\varnothing$ and so A is $\mathsf{N}\Lambda_a$-open.

Theorem 3.18 If A_i is $\mathsf{N}\Lambda_a$-open. For each $i\in I$, then $\cup\{A_i : i\in I\}$ is $\mathsf{N}\Lambda_a$-open.

Proof Since A_i is $\mathsf{N}\Lambda_a$-open, for each $i\in I$, $X-A_i$ is $\mathsf{N}\Lambda_a-closed$ for each $i\in I$. Then, $\cap\{X-A_i : i\in I\}$ is $\mathsf{N}\Lambda_a$-closed which implies $X-\cup\{A_i : i\in I\}$ is $\mathsf{N}\Lambda_a$-closed and so $\cup\{A_i : i\in I\}$ is $\mathsf{N}\Lambda_a$-open.

Definition 3.19 Let $(\mathsf{U},\tau_R(X)$ be a nano topological space and $A\subseteq X$. A point $x\in X$ is called a $\mathsf{N}\Lambda_a$-cluster point of A if for every $\mathsf{N}\Lambda_a$-open set U of X containing x such that $A\cap U\neq\varnothing$. The set of all $\mathsf{N}\Lambda_a$-cluster points of A is called the $\mathsf{N}\Lambda_a$ closure of A and is denoted by $\mathsf{N}\Lambda_a$ cl(A).

The proof of the following lemma is straight forward as lemma 3.2.

Lemma 3.20 Let A and B be subsets of a nano topological space $(\mathsf{U}, \tau_R(X)$. Then the following properties hold:

1. $A \subseteq \mathsf{N}\Lambda_a$ cl(A).
2. $\mathsf{N}\Lambda_a cl(A) = \cup\{F : A \subseteq F$ and F is $\mathsf{N}\Lambda_a$ closed}.
3. If $A \subseteq B$, then $\mathsf{N}\Lambda_a$ cl(A) $\subseteq \mathsf{N}\Lambda_a$ cl(B).
4. A is $\mathsf{N}\Lambda_a$ closed if and only if $A = \mathsf{N}\Lambda_a$ cl(A).
5. $\mathsf{N}\Lambda_a$ cl(A) is $\mathsf{N}\Lambda_a$ closed.

$\mathsf{N}\Lambda_a$ -Continuous Functions.

Definition4.1 A function $f : (\mathsf{U}, \tau_R(X)) \to (\mathsf{V}, \tau_{R'}(Y))$ is called $\mathsf{N}\Lambda_a$ continuous if $f^{-1}(V)$ is $\mathsf{N}\Lambda_a$ -open subset of $(\mathsf{U}, \tau_R(X))$, for every nano open set V of $(\mathsf{V}, \tau_{R'}(Y))$.

Theorem 4.2 For a function $f : (\mathsf{U}, \tau_R(X)) \to (\mathsf{V}, \tau_{R'}(Y))$, the following are equivalent:

1. f is $\mathsf{N}\Lambda_a$ -continuous.
2. $f^{-1}(V)$ is $\mathsf{N}\Lambda_a$ closed subset of $(\mathsf{U}, \tau_R(X))$, for every nano closed set V of $(\mathsf{V}, \tau_{R'}(Y))$.
3. For each $x \in X$ and for nano open set V of $(\mathsf{V}, \tau_{R'}(Y))$ containing $f(x)$, there exists a $\mathsf{N}\Lambda_a$ open set U of $(\mathsf{U}, \tau_R(X))$ containing x such that $f(U) \subseteq V$.
4. $f(\mathsf{N}\Lambda_a$ cl(A)) $\subseteq \mathsf{N}cl(f(A))$ for each subset A of $(\mathsf{U}, \tau_R(X))$.

Proof $(i) \Leftrightarrow (ii)$: Let V be any nano closed subset of $(\mathsf{V}, \tau_{R'}(Y))$. Then $Y - V$ is nano open in $(\mathsf{V}, \tau_{R'}(Y))$. Since f is $\mathsf{N}\Lambda_a$ continuous, $f^{-1}(Y - V) = X - f^{-1}(V)$ is $\mathsf{N}\Lambda_a$ open in $(\mathsf{U}, \tau_R(X))$ and so $f^{-1}(V)$ is $\mathsf{N}\Lambda_a$ closed in $(\mathsf{U}, \tau_R(X))$.We can similarly prove the converse.

$(i) \Leftrightarrow (iii)$: Let $x \in X$ and V be any open set of $(\mathsf{V}, \tau_{R'}(Y))$ such that $f(x) \in V$, then $x \in f^{-1}(V)$ and $f^{-1}(V)$ is $\mathsf{N}\Lambda_a$ -open in $(\mathsf{U}, \tau_R(X))$. Let $U = f^{-1}(V)$,then $x \in U$ and $f(U) \subseteq V$. Conversely let V be a nano open set in $(\mathsf{V}, \tau_{R'}(Y))$ and $x \in f^{-1}(V)$, then $f(x) \in V$ and there exists a $\mathsf{N}\Lambda_a$ -open set U_x in $(\mathsf{U}, \tau_R(X))$ such that $x \in U_x$ and $f(U_x) \subseteq V$. Therefore, we have $x \in U_x \subseteq f^{-1}(V)$ and hence $f^{-1}(V) = \cup\{U_x : x \in U_x\}$. Therefore, $f^{-1}(V)$ is $\mathsf{N}\Lambda_a$ -open in $(\mathsf{U}, \tau_R(X))$.

$(ii) \Leftrightarrow (iv)$: Let A be any subset of $(\mathsf{U}, \tau_R(X))$ and $\mathsf{N}cl(f(A))$ is nano closed in $(\mathsf{V}, \tau_{R'}(Y))$, $f^{-1}(\mathsf{N}cl(f(A)))$ is $\mathsf{N}\Lambda_a$ -closed in $(\mathsf{U}, \tau_R(X))$. Now $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(\mathsf{N}cl(f(A)))$ which implies $f(\mathsf{N}\Lambda_a - \mathsf{N}cl(f(A))) \subseteq f(f^{-1}(\mathsf{N}cl(f(A)))) \subseteq \mathsf{N}cl(f(A)))$ which implies (iv). Conversely let V be a nano closed subset of $(\mathsf{V}, \tau_{R'}(Y))$. Then, $f^{-1}(V) \subseteq X$. By (iv), $f(\mathsf{N}\Lambda_a \mathsf{N}cl(f(A))) \subseteq \mathsf{N}cl(f(f^{-1}(V))) \subseteq \mathsf{N}cl(V) = V$ which implies $\mathsf{N}\Lambda_a cl(f^{-1}(V)) \subseteq f^{-1}(V)$ and so $f^{-1}(V) = \mathsf{N}\Lambda_a - \mathsf{N}cl(f(A))$. Hence, $f - 1(V)$ is $\mathsf{N}\Lambda_a$ -open in $(\mathsf{U}, \tau_R(X))$, which implies (ii).

$(iv) \Leftrightarrow (v)$ Let B be a nano closed subset of $(\mathsf{V}, \tau_{R'}(Y))$. By,iv), $f(\mathsf{N}\Lambda_a cl(f^{-1}(B))) \subseteq \mathsf{N}cl(f(f^{-1}(B))) \subseteq \mathsf{N}cl(B)$ .Hence, $\mathsf{N}\Lambda_a - \mathsf{N}cl(f^{-1}(B))) \subseteq f^{-1}(\mathsf{N}cl(B))$ which implies (v). Conversely let A be a subset of $(\mathsf{U},\tau_R(X))$. Take $B=f(A)$. Then by (v), $\mathsf{N}\Lambda_a cl(A)\subseteq \mathsf{N}\Lambda_a cl(f^{-1}(B)))\subseteq f^{-1}(\mathsf{N}cl(B))\subseteq f^{-1}(\mathsf{N}cl(f(A)))$. Hence, $f(\mathsf{N}\Lambda_a cl(A))\subseteq \mathsf{N}cl(f(A))$ which implies (iv).

Theorem 4.3 Let $f:(\mathsf{U},\tau_R(X))\to(\mathsf{V},\tau_{R'}(Y))$. If f is nano a continuous, then f is nano Λ_a continuous. **Proof** Let V be a nano closed subset of $(\mathsf{V},\tau_{R'}(Y))$. Since f is nano a-continuous, $f^{-1}(V)$ is nano a-closed in $(\mathsf{U},\tau_R(X))$.

Remark 4.4 Composition of two $\mathsf{N}\Lambda_a$ continuous functions is not $\mathsf{N}\Lambda_a$ continuous in general as shown by the following example.

Example4.5 Let $\mathsf{U}=\{a,b,c,d\}$ with $\mathsf{U}/R=\{\{a,b\},\{c,d\}$ and $X=\{a,b,c\}$. Then $\tau_R(X)=\{\varnothing,\mathsf{U},\{a,b\},\{c,d\},\}$. Let $\mathsf{V}=\{1,2,3,4\}$ with $\mathsf{V}/R'=\{1,2\},\{3,4\},\}$ and $Y=\{1,2\}$. Then $\tau_{R'}(Y)=\{\varnothing,\mathsf{U},\{1,2\}\}$.$\mathsf{W}=\{a,b,c,d\}$ with $\mathsf{W}/R''=\{\{a,c\},\{b\},\{d\}$ and $Z=\{a,b\}$. Then $\tau_{R''}(Z)=\{\varnothing,\mathsf{W},\{b\},\{a,c\},\{a,b,c\}\}$. Define $f:(\mathsf{U},\tau_R(X))\to(\mathsf{V},\tau_{R'}(Y))$ by $f(a)=3,f(b)=4,f(c)=1,f(d)=2$. Define $g:(\mathsf{V},\tau_{R'}(Y))\to(\mathsf{W},\tau_{R''}(Z))$ by $g(1)=a,g(2)=c,g(3)=d=g(4)$. Then f and g are both $\mathsf{N}\Lambda_a$-continuous. But $g\circ f:(\mathsf{U},\tau_R(X))\to(\mathsf{W},\tau_{R''}(Z))$ is not $\mathsf{N}\Lambda_a$ continuous since $(g\circ f)^{-1}(\{a,c\})=\{b,c\}$ which is not $\mathsf{N}\Lambda_a$-open in $(\mathsf{U},\tau_R(X))$.

Applications: In this section the concept of $\mathsf{N}\Lambda_a$-connected spaces are introduced and their properties are studied.

Definition 5.1 A nano topological space $(\mathsf{U},\tau_R(X))$ is said to be $\mathsf{N}\Lambda_a$-connected if U cannot be written as a disjoint union of two non empty $\mathsf{N}\Lambda_a$-open sets of U.

Theorem 2.2 For a nano topological space $(\mathsf{U},\tau_R(X))$, the following are equivalent:

1. $(\mathsf{U},\tau_R(X))$ is $\mathsf{N}\Lambda_a$ connected.
2. The only subsets of U which are both $\mathsf{N}\Lambda_a$ open and $\mathsf{N}\Lambda_a$ closed are $\varnothing$ and U.

Theorem 2.3 Every $\mathsf{N}\Lambda_a$ connected space is nano a connected.

Proof Let $(\mathsf{U},\tau_R(X))$ be a $\mathsf{N}\Lambda_a$-connected space. Suppose $(\mathsf{U},\tau_R(X)$ is not nano a connected. Then $X=A\cup B$, where A and B are disjoint non empty nano a-open sets in $(\mathsf{U},\tau_R(X))$. Since every nano a-open sets is $\mathsf{N}\Lambda_a$ open, A and B are disjoint non empty nano a open sets in $(\mathsf{U},\tau_R(X))$, a contradiction to the fact that $(\mathsf{U},\tau_R(X))$ is $\mathsf{N}\Lambda_a$ connected. Hence $(\mathsf{U},\tau_R(X))$ is nano a-connected.

Remark 2.4 The following example shows that the converse of the above theorem is not true in general.

Example 2.5 Let $\mathsf{U}=\{a,b,c,d\}$ with $\mathsf{U}/R=\{\{a\},\{b,c\},\{d\}\}$ and $X=\{a,c\}$. Then $\tau_R(X)=\{\varnothing,\mathsf{U},\{a\},\{b,c\},\{a,b,c\}\}$. Then $(\mathsf{U},\tau_R(X))$ is a nano a connected but not $\mathsf{N}(\Lambda_a)$ connected since $\mathsf{U}=\{a,d\}\cup\{b,c\}$ where $\{a,d\}$ and $\{b,c\}$ are disjoint non-empty $\mathsf{N}\Lambda_a$-open sets in U.

Conclusion: Here we developed a generalized closed set in nano topology called $N\Lambda a$-closed set and characterized its properties. Further we introduced a nano $N\ \Lambda a$-continuous function. Apart from this we analyzed some properties of $N\Lambda a$-connected spaces by using $N\Lambda a$-closed sets with suitable examples.

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