CONNECTED DOMATIC NUMBER OF A GRAPH

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Abstract: In this paper, we will discuss about the definitions and basic concepts of graph theory along with some applications. Through, a series of applications, we will also present several different types of dominating sets. We also describe about the bounds for the domination number $\gamma(G)$.We also discuss some properties about the connected domatic number and give some bounds for the parameters. And we will also see the dominating set for connected graph & disconnected graph.

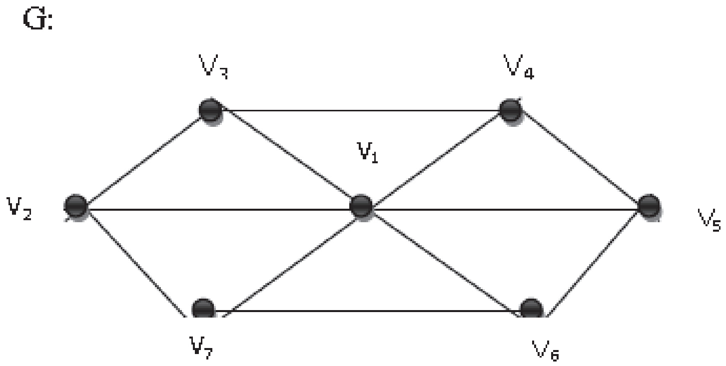
Keywords: Multiple Edges, Complete Bipartite Graph**,** Domatic Number.

Introduction: In this chapter, we discuss some properties about the connected domatic number and give some bounds for this parameter [2]. Throughout this chapter we consider only finite connected graphs without loops and multiple edges.

The connected domatic number of a graph is well defined only for connected graphs; in a disconnected graph there exists no connected dominating set and thus no connected domatic partition, while in every connected graph there exists at least one connected domatic partition, namely that which consists of one class.

Definition: A connected domatic partition of a graph G is a pertition of V(G), all of whose classes are connected dominating sets of G. The maximum number of classes of a connected domatic partition of G is called the **connected domatic number** of G and is denoted by $d_c(G)$.

Example: The connected domatic number of the following graph G is obtained as follows.



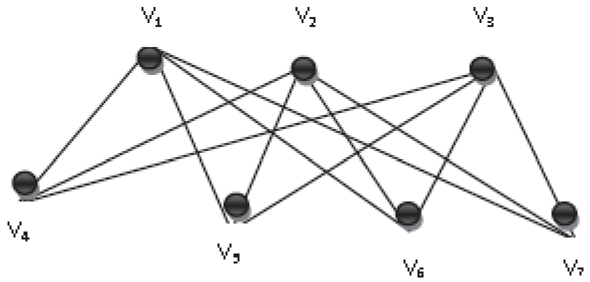
Figure

Here $\{\{v_1\}, \{v_2, v_3, v_4, v_5, v_6, v_7\}\}$ is a maximum connected domatic partition of G. Hence the connected domatic number $d_c(G) = 2$.

Remark: The following results can be easily seen.
i) For any complete graph K_n , $d_c(K_n) = n$.
ii) For any tree T, $d_c(T) = 1$, so that $d_c(K_{1,n-1}) = 1$.

iii) For any path P_n on n vertices,
$d_c(P_n)$ = 2, if n = 2
1, otherwise.
iv) For any cycle C_n of length n,
3, if n = 3
$d_c(C_n)$ = 2, if n = 4
1, otherwise.
v) For any complete bipartite graph $K_{r,s}$ $d_c(K_{r,s})$ = min {r, s}, r, s ≥2.

Example: Consider the complete bipartite graph $K_{3,4}$ given in Figure 3.2



Figure

Here {{v_1, v_4}, {v_2, v_5}, {v_3, v_6, v_7}} is a maximum connected domatic partition. Hence the connected domatic number $d_c(K_{3,4})$ =3.

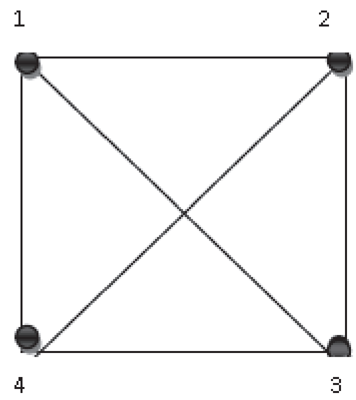
Lemma: Let G be a connected graph which is not a complete graph, let R be its vertex cut, let S be its connected dominating set. Then S∩R ≠ ∅.
Proof: Let G' = G – R and let C_1, C_2 … ,C_t be the connected components of the sub graph G' of G. It is clear that t ≥ 2.
Suppose S∩R = ∅. As the sub graph of G induced by S is connected, it is a sub graph of C_i for some i ∈ {1,2 … , t}. Let x ∈ C_j for j ≠i. Then x ∉ S and x has no neighbour in S, which is a contradiction.
Hence S ∩ R ≠ ∅.

Theorem: Let G be a connected graph which is not complete, let $d_c(G)$ be its connected domatic number, let k(G) be its vertex connectivity number. Then $d_c(G)$ ≤ k (G).
Proof: Let R be a vertex cut of G of the cardinality k(G) and let {V_1, V_2 …., V_d }be a connected domatic partition of G having d = $d_c(G)$ classes. By Lemma, we have V_i∩R ≠ ∅ for i = 1,2, …, d. Since V_1, V_2, …, V_d are pair wise disjoint,
the sets V_1 ∩ R, V_2 ∩ R,…,V_d ∩ R are also pair wise disjoint.
Therefore, d = $d_c(G)$ ≤ R = K(G)
$d_c(G)$ ≤ k(G).

Remark: For complete graph, the above theorem is not true, since $d_c(K_n)$ = n but k (K_n) = n – 1.

Example: The following graph in Figure illustrates the above result.

K_4:



In the above graph, $d_c(K_4)$ = {{1},{2},{3},{4}} but $k(K_n)=n-1= 4-1=3$.

Theorem: For an arbitrary positive integer q there exists a graph G such that $d(G) - d_c(G) = q$.

Proof: Let G' be a complete graph with vertices u, v, … , v_q' Let G'' be a complete graph with vertices u, v'', …, v_q''. The vertices u, v', … , v_q', v'', …, v_q''. are pair wise distinct and the vertex u is common to both G' and G'' Let G be the union of G' and G''. The set {u} is a vertex cut of G, therefore k (G) =1 and by Theorem 3.1, $d_c(G) = 1$. It is clear that {{u}, $\{v', v''\}$, …, $\{v_q', v_q''\}$} is the maximum domatic partition of G. Therefore the domatic number d(G) = q+1. Hence $d(G) - d_c(G)$ = q.

Theorem: Let G be a connected graph and let n_0 be the number of vertices of degree $n-1$. Then

$$d_c(G) \le \frac{1}{2}(n + n_0)$$

Theorem: Let G be a connected graph with at least three vertices. Let e = uv be an edge of G which is not a bridge. Let G'= G–e. If deg u = deg v = n–1 in G then $d_c(G') \ge d_c(G)-2$, otherwise $d_c(G') \ge d_c(G)-1$.

Theorem: Let G be a graph. Then $d_c(G) \le \delta(G) + 1$.

Corollary: If the domination number of graph G, $\gamma(G) \ge 2$ then $d_c(G) \le \delta(G)$.

Definition: A graph G is called **connected domatically full** if $d_c(G) = \delta(G)+1$.

Example: Any complete graph is connected domatically full graph.

Theorem: Let G be a graph such that both G and G are connected. Then $d_c(G) = d_c(G) \le n-1$.

Theorem: For any connected graph G with n vertices $\gamma_c(G) + d_c(G) \le n+1$ with equality if and only if G is complete.

Theorem: For any connected graph G with n vertices, $\gamma_c(G) . d_c(G) \le n$.

Definition: A connected graph G is said to be connected domatically critical, if $d_c(G-e) < d_c(G)$ for any edge e in G.

Theorem: Let G be a connected domatically critical graph, let $d_c(G) = d$. Then the vertex set of G is the union of pair wise disjoint sets $V_1, V_2, \ldots, V_d$ such that
i) the sub graph G_i of G induced by V_i is a tree for each $i = 1, 2, \ldots, d$.
ii) the sub graph G_{ij} of G with the vertex set $V_i \cup V_j$ and with the edge set consisting of all edges joining a vertex of V_i with a vertex of V_j is a forest, each of whose connected components is a star or a complete graph with two vertices for any i.j from the set $\{1,2 \ldots, d\}$ $i \neq j$.

Theorem: Let G be a connected domatically critical graph with $d_c(G) = d$. If G is regular of degree d–1, then G is isomorphic to K_d. If G is regular of degree d, then G_i is K_2 for each $i \in \{1,2 \ldots, d\}$ and G_{ij} consists of two connected components isomorphic to K_2 for any i,j from {1, 2, …, d), $i \neq j$.

Conclusion: In general, when studying subsets of a given type, we are interested in finding either a smallest or a largest such set in a graph. For instance, one considers such problems as finding the minimum cardinality of a dominating set or a cover or finding the maximum cardinality of an independent set or packing. In this dissertation, we described bounds for the domination number $\gamma(G)$.

References:

1. K.Appel and W. Hankin, "Every planar map is 4-colorable," Bulletin of the AMS, Volume 82 (1976), 711-712.
2. Beck, M. Bleicher and D. Crowe, Excursion into Mathematics, Worth Publishers, 1969 (ISBN 0-87901-004-5
3. N.Biggs, E. Lloyd, and R. Wilson, Graph Theory *1736-1936,* Clarendon Press Oxford, 1976 (ISBN 0-19-853901-0).
4. E.B.Dynkins and V. A. Uspenskii, Multicolor Problems, D.C.Heath and Company, Boston, 1963.
5. L.Euler, "Solutio Problematis Ad geometriam Situs Pertinentis," Commenrarii Academiae Scientiarum Imperialis Petropolitanae 8 (1736), pp. 128-140. O. Ore, The Four Color Problem, Academic Press, New York 1967.
6. K.H.Rosen, Discrete Mathematics and its Applications, Random House, NY, 1988. (ISBN 0-394-36768-5, QA39.2 R654)
7. L.Steen editor, For All Practical Purposes: Introduction to Contemporary Mathematics 3ed. W. H. Freeman and Company, New York 1994. (ISBN 0-7167-2378-6, QA7.F68 1994).

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