ANTI L-FUZZY BI-IDEAL OF A RING

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Abstract: In this paper we introduce the notion of anti L-fuzzy bi-ideal in ring and give some Characterizations of anti L-fuzzy bi-ideals in rings.

Keywords: L-Fuzzy Set, L-Fuzzy Sub Ring, Fuzzy Bi- Ideal, L-Fuzzy Bi-Ideal, Anti L-Fuzzy Sub Ring, Anti L-Fuzzy Bi-Ideal.

Introduction: The idea of fuzzy subset μ of a set X was primarily introduced by L.A Zadeh [1].Garrett Birchof [2] introduced the concept of lattice theory. Further J.A Goguen [3] replaced the valuations set [0,1] by means of a complete lattice in an attempt to make a generalized study of fuzzy set theory by studying L-fuzzy sets. Lajos and Szasz [4] introduced the idea of bi-ideals in ring. Chelvam and Ganesan [5] has introduced the bi-ideal in near ring, later Kuroki [6] introduced the notion of fuzzy bi-ideals in semi groups and Liu [7] studied them in rings. A detail work about bi-ideal and fuzzy bi-ideals in a ring can be found by S.K. Datta [8]. Majumder and Sarder [9] explored on the idea of anti fuzzy bi-ideals in fuzzy group theory. In this paper, we introduced the notion of anti L-fuzzy bi-ideal of rings and investigate some properties.

***1. Preliminaries*:** In this section we include some elementary aspects that are necessary for this paper.

1.1 Definition: Let X is a non-empty set. A mapping $\mu : X \to [0,1]$ is called a fuzzy subset of X.

1.2 Definition: Let R be a ring .and μ be a fuzzy subset of R. μ is called a fuzzy ideal of R if

(i) $\mu(x-y) \geq \min\{ \mu(x), \mu(y) \}$

(ii) $\mu(xy) \geq \max\{ \mu(x), \mu(y) \}$

1.3 Definition: *Let R be a ring. A fuzzy set* μ of R is said to be fuzzy subring of R if,

(i) $\mu(x-y) \geq \min\{ \mu(x), \mu(y) \}$

(ii) $\mu(xy) \geq \min\{ \mu(x), \mu(y) \}$

1.4 Definition: Let μ be a fuzzy subset of a ring R. Imμ is defined as,
Imμ = $\{t \in [0,1] \backslash \mu(x) = t \text{ } for \text{ } some \text{ } x \in X\}$.

1.5 Definition: Let μ be a fuzzy subset of a ring R. The set $\mu_t = \{x \in R \backslash \mu(x) \leq t\}$ is called a lower level subset of μ.

1.6 Definition: A non empty fuzzy subset μ of a ring R is called an fuzzy bi-ideal of R if

(i) $\mu(x-y) \geq \min\{ \mu(x), \mu(y) \}$

(ii) $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$

(iii) $\mu(xyz) \geq \min\{\mu(x), \mu(z)\} \quad \forall x, y, z \in R$

1.7 Definition: Let R be a ring. A fuzzy set μ of R is said to be L-fuzzy subring of R if for R the following conditions are satisfied:

(i) $\mu(x-y) \geq \mu(x) \wedge \mu(y)$

(ii) $\mu(xy) \geq \mu(x) \wedge \mu(y)$

1.8 Definition: Let R be a ring and L be a lattice. A fuzzy set μ of R is said to be anti L-fuzzy subring of R if

(i) $\mu(x-y) \leq \mu(x) \vee \mu(y)$

(ii) $\mu(xy) \leq \mu(x) \vee \mu(y)$

1.9 Definition: Let R be a ring and L be a lattice. A fuzzy set μ of R is said to be L-fuzzy bi-ideal of R if

(i) $\mu(x-y) \geq \mu(x) \wedge \mu(y)$

(ii) $\mu(xy) \geq \mu(x) \wedge \mu(y)$

(iii) $\mu(xyz) \geq \mu(x) \wedge \mu(z) \ \forall x, y, z \in R$

2. Anti L-Fuzzy Bi-Ideal:

2.1 Definition:

Let R be a ring and L be a lattice. A fuzzy set μ of R is said to be an anti L-fuzzy bi-ideal of R if

(i) $\mu(x-y) \leq \mu(x) \vee \mu(y)$

(ii) $\mu(xy) \leq \mu(x) \vee \mu(y)$

(iii) $\mu(xyz) \leq \mu(x) \vee \mu(z) \ \forall x, y, z \in R$

Example:

Consider the fuzzy set μ of R by $\mu_A(x) = \begin{cases} 0.6 & if\ x\ is\ rational \\ 0.2 & if\ x\ is\ irrational \end{cases}$

Then μ_A is an anti L-fuzzy bi-ideal of R.

2.2 Theorem: If A be an anti L-fuzzy subring of ring R, then,

(i) $\mu_A(0) \leq \mu_A(x)$

(ii) $\mu_A(-x) = \mu_A(x) \ \forall x, y \in R$

(iii) If R is ring with unity then $\mu_A(x) \geq \mu_A(1)$

Proof: For *x* in R and 0 is the identity element of R.

Now,

(i) $\mu_A(0) = \mu_A(x-x) \leq \mu_A(x) \vee \mu_A(x)$

$= \mu_A(x)$

$\mu_A(0) \leq \mu_A(x)$

(ii) $\mu_A(-x) = \mu_A(0-x) \leq \mu_A(0) \vee \mu_A(x)$

$= \mu_A(x)$

$\mu_A(-x) = \mu_A(x)$

For $x \neq 0$ in R and 1 is the identity element of R

(iii) $\mu_A(1) = \mu_A(xx^{-1}) \leq \mu_A(x) \vee \mu_A(x^{-1})$

$= \mu_A(x)$

$\mu_A(1) \leq \mu_A(x)$

2.3 Theorem: If A and B be Two anti L-fuzzy bi-ideal's of a ring R then A∩B isanti L-fuzzy bi-ideal of ring R.
Proof: Let A and B be two anti L-fuzzy bi-ideal of a ring R. Let $x, y \in A \cap B$ be any element.
Then

(i) $\mu_{A\cap B}(x-y) \leq \mu_{A\cap B}(x) \vee \mu_{A\cap B}(y)$

(ii) $\mu_{A\cap B}(xy) \leq \mu_{A\cap B}(x) \vee \mu_{A\cap B}(y)$

(iii) $\mu_{A\cap B}(xyz) \leq \mu_{A\cap B}(x) \vee \mu_{A\cap B}(z)$ for $z \in R$

Let $x, y \in A \cap B$ be any element Then,

(i) $\mu_{A\cap B}(x-y) \leq \mu_A(x-y) \vee \mu_B(x-y)$

$\leq \{\mu_A(x) \vee \mu_A(y)\} \vee \{\mu_B(x) \vee \mu_B(y)\}$

$= \{\mu_A(x) \vee \mu_B(x))\} \vee \{\mu_A(y) \vee \mu_B(y)\}$

$= \mu_{A\cap B}(x) \vee \mu_{A\cap B}(y)$

Thus, $\mu_{A\cap B}(x-y) \leq \mu_{A\cap B}(x) \vee \mu_{A\cap B}(y)$

(ii) $\mu_{A\cap B}(xy) \leq \mu_A(xy) \vee \mu_B(xy)$

$\leq \{\mu_A(x) \vee \mu_A(y)\} \vee \{\mu_B(x) \vee \mu_B(y)\}$

$= \{\mu_A(x) \vee \mu_B(x)\} \vee \{\mu_A(y) \vee \mu_B(y)\}$

$= \mu_{A\cap B}(x) \vee \mu_{A\cap B}(y)$

Thus, $\mu_{A\cap B}(xy) \leq \mu_{A\cap B}(x) \vee \mu_{A\cap B}(y)$

(iii) $\mu_{A\cap B}(xyz) \leq \mu_A(xyz) \vee \mu_B(xyz)$

$\leq \{\mu_A(x) \vee \mu_A(z)\} \vee \{\mu_B(x) \vee \mu_B(z)\}$

$= \{\mu_A(x) \vee \mu_B(x)\} \vee \{\mu_A(z) \vee \mu_B(z)\}$

$= \mu_{A\cap B}(x) \vee \mu_{A\cap B}(z)$

Thus, $\mu_{A\cap B}(xyz) \leq \mu_{A\cap B}(x) \vee \mu_{A\cap B}(z)$

Hence A∩B is a anti L-fuzzy bi-ideal of a ring R.

2.4 Theorem: If A is an anti L-fuzzy subring of R,then $\mu_A(x-y) = \mu_A(0)$ gives $\mu_A(x) = \mu_A(y)$ for x, y in R the identity 0 in R .
Proof: Let x and y in R the identity 0 in R

Now,

$$
\begin{aligned}
\mu_A(x) = \mu_A(x-y+y) &\leq \mu_A[x-y-(-y)] \\
&\leq \mu_A(x-y) \vee \mu_A(-y) \\
&= \mu_A(x-y) \vee \mu_A(y) \\
&= \mu_A(0) \vee \mu_A(y) \\
&= \mu_A(y)
\end{aligned}
$$

$$
\begin{aligned}
\mu_A(y) = \mu_A[x-(x-y)] &\leq \mu_A(x) \vee \mu_A(x-y) \\
&= \mu_A(x) \vee \mu_A(0)
\end{aligned}
$$

$= \mu_A(x)$

2.5 Theorem: If A is an anti L-fuzzy subring of a ring R. then $H = \{x/x \in R: \mu_A(x) = 1\}$ is either empty or is a subring of R.

Proof: If no element satisfies this condition then H is empty.

If $x, y \in H$ then,

$\mu_A(x - y) \leq \mu_A(x) \vee \mu_A(y)$

$= 1 \vee 1$

$= 1$

$\mu_A(xy) \leq \mu_A(x) \vee \mu_A(y)$

$= 1 \vee 1$

$= 1$

Thus $x - y, xy \in H$

H is a subring of R.

Hence H is either empty or a subring of R.

2.6 Theorem: Let μ be a fuzzy subset of a ring R. μ is an anti L-fuzzy bi ideal of R iff its lower level setsμ_t's are bi ideals of R for all $t \in Im\ \mu$.

2.7 Theorem: Let μ be L-fuzzy set of R, then μ_A is an anti L-fuzzy bi-ideal of R iff μ_A^C is L-fuzzy bi-ideal of ring R.

Proof: Let μ_A be an anti L-fuzzy bi-ideal of R, then

(i) $\mu_A(x-y) \leq \mu_A(x) \vee \mu_A(y)$

(ii) $\mu_A(xy) \leq \mu_A(x) \vee \mu_A(y)$

(iii) $\mu_A(xyz) \leq \mu_A(x) \vee \mu_A(z)\ \forall\, x, y, z \in R$

To show that ,μ_A^C is L-fuzzy bi-ideal of R.

(i) $\mu_A(x-y) \leq \mu_A(x) \vee \mu_A(y)$

Imply that, $(1-\mu_A^C(x)) \vee (1 - \mu_A^C(y)) \geq 1- \mu_A^C(x - y)$

$1 - (\mu_A^C(x) \wedge \mu_A^C(y)) \geq 1- \mu_A^C(x - y)$

$\mu_A^C(x - y) \geq \mu_A^C(x) \wedge \mu_A^C(y)$

(ii) $\mu_A(xy) \leq \mu_A(x) \vee \mu_A(y)$

Imply that,$(1 - \mu_A^C(x)) \vee (1 - \mu_A^C(y)) \geq 1- \mu_A^C(xy)$

$1 - (\mu_A^C(x) \wedge \mu_A^C(y)) \geq 1- \mu_A^C(xy)$

$\mu_A^C(xy) \geq \mu_A^C(x) \wedge \mu_A^C(y)$

(iii) $\mu_A(xyz) \leq \mu_A(x) \vee \mu_A(z)$

Imply that,$(1-\mu_A^C(x)) \vee (1 - \mu_A^C(y)) \geq 1- \mu_A^C(xyz)$

$1 - (\mu_A^C(x) \wedge \mu_A^C(y)) \geq 1- \mu_A^C(xyz)$

$\mu_A^C(xyz) \geq \mu_A^C(x) \wedge \mu_A^C(z)$

Hence μ_A^C L-fuzzy bi-ideal of ring R.

Similarly, when μ_A^C is L-fuzzy bi-ideal of R. We can show that μ_A is an anti L-fuzzy bi-ideal of R.

Conclusion: In this paper, the definition, example and some theorems in anti L-fuzzy bi-ideals are given. Using these, various results can be developed under the topic anti L-fuzzy bi-ideal of a ring.

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References:

1. L. A. Zadeh, Fuzzy sets, Information and Control, 8 (1965), 338–353.
2. Garret Birkhof: Lattice Theory, American Mathematical Society Colloquium publications, Volume XXV.
3. J.A. Goguen : L-fuzzy sets J.Math.Anal.Appl.18,145-174 (1967).
4. S.Lajos, F.Szasz, Bi-ideals in associative rings,ActaSci.Math.,32(1971) 185-193.
5. T.T. Chelvam and N. Ganesan, "On Bi-Ideals of near rings, Indian J. pure and appl. Math.,18 (1987), 1002-1005.
6. N. Kuroki, "On fuzzy ideals and fuzzy bi-ideals in semi groups" Fuzzy Sets and Systems, 5,(1982), 203-215.
7. Wang-Jin Liu, Fuzzy invariant Subgroups and fuzzy ideals, Fuzzy sets and systems, 8 (1982) 133-139.
8. S.K. Datta, "On Bi-Ideals and Fuzzy Bi-Ideals of Rings", M.Phil. Thesis, under the guidance of Professor TapanDatta, Department of Pure Mathematics, University of Calcutta (1997).
9. S.K Majumder, S.K.Sarder, A note on anti fuzzy bi-ideal and anti fuzzy (1,2) – ideal, Communication.
10. Biswas.R, Fuzzy subgroups and Anti fuzzy subgroups, Fuzzy sets and systems,35, 121-124(1990).

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