# ON σ-H- OPEN SETS AND σ-H- CONTINUOUS FUNCTIONS IN GTS WITH HEREDITARY CLASSES

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**Abstract:** In this paper we discuss the properties of σ-H- open sets and σ-H- continuous functions introduced in given the notions of σ-H- open and σ-H- closed functions in GTS with hereditary classes.
**Keywords:** σ-H- open sets, σ-H- continuous.

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**Introduction and Preliminaries:** In 2007, Csaszar [3] defined a nonempty class of subsets of a non empty set called hereditary class and studied modification of generalized topology via hereditary classes. The aim of the thesis is to extend the study of the properties of the generalized topologies via hereditary classes. A subfamily μ of ρ(X) is called a generalized topology (GT)[2] if $\phi \in \mu$ and μ is closed under arbitrary union. The pair (X,μ) is called a generalized topological space (GTS). Members of μ are called μ-open sets and its complement is called μ-closed set. The largest μ-open set contained in a subset A of X is denoted by $i_\mu(A)$ and is called the μ-interior of A [2]. The smallest μ-closed set containing A is called the μ-closure of A and is denoted by $c_\mu(A)$ [2]. A generalized topology μ is said to be a quasi topology if μ is closed under finite intersection. Let X be a nonempty set. A hereditary class H of X is a nonempty collection of subset of X such that $A \subset B, B \in H$ implies $A \in H$ [3]. A hereditary class of X is an ideal [7] if $A \cup B \in H$ whenever $A \in H$ and $B \in H$ . An ideal I in a topological space (X,τ) is said to be condense if $\tau \cap I = \{\phi\}$ . With respect to the generalized topology μ of all μ-open sets and a hereditary class H, for each subset A of X, a subset $A^*(H)$ or simply $A^*$ of X is defined by $A^* = \{x \in X \mid M \cap A \notin H$ for every M∊μ such that x∊M}[3] H is said to be μ-codense if $\mu \cap H = \{\phi\}$ [3]. If $c^*{}_\mu(A) = A \cup A^*$ for every subset A of X, with respect to μ and a hereditary class H of subsets of X, then $\mu^* = \{A \subset X \mid c^*{}_\mu(X - A) = X - A\}$ is a generalized topology [3], and $i^*{}_\mu(A)$ will denote the interior of A in (X, $\mu^*$). A subset A of a GTS (X,μ) with a hereditary class H is said to be $\alpha-$ open [3] (resp.σ-open [3], π-open [3], β-open [3]) (resp. $A \subset i_\mu(c_\mu(i_\mu(A))), A \subset c_\mu(i_\mu(A)), A \subset i_\mu(c_\mu(A)), A \subset c_\mu(i_\mu(c_\mu(A)))$. A subset A of a GTS (X,μ) with a hereditary class H is said to be α-H-open [3] (σ-H- open [3], π-H-open [3], β-H-open [3] H-open [3]) if

$$A \subset i_\mu(c_\mu{}^*(i_\mu(A))), resp. A \subset c^*{}_\mu(i_\mu(A)), A \subset i_\mu(c^*{}_\mu(A)), A \subset c_\mu(i_\mu(c_\mu{}^*(A))), A \subset i_\mu(A^*).$$

The family of all be α-H-open (resp.σ-H- open, π-H-open, β-H-open ) sets in a GTS (X,μ) with a hereditary class H is denoted by $\alpha_H(resp.\alpha_H, \Pi_{H,}\beta_H)$.

**On σ-H-Open Sets:**

μ-open → α-H-open → σ-H-open → σ-open
↓ ↓
H-open → π-H-open → π-open → β-open

The following examples show that μ-open set is not a H-open set and a H-open set is not a σ-H-open set. Therefore σ-H-open set and H-open sets are independent of each other.

**Example 1:** Let X={a,b,c} μ={Φ,{a},{a,c}} and H={Φ,{a}}. If A={a,c}, then $i_\mu(A)$={a,c}. Hence A is μ-open .But $A^*$ ={} and $i_\mu(A^*) = \{\phi\}$ ,which implies that A is not H-open.

**Example 2:** Let X ={a,b,c}, μ={Φ,{a},{b,c},{a,b},{a,b,c}} and H= ={Φ,{b}}.If A={a,c}, then $i_\mu(A^*) = \{a,b,c\}$ .Hence A is H-open. But $c^*{}_\mu i_\mu(A) = \{a\}$ ,which implies that A is not σ-H-open.

**Theorem 1:** Let (X,μ) be a GTS with hereditary class H and A be a subset of X. Then A is σ-H-open if and only if $c^*{}_\mu(A) = c^*{}_\mu(i_\mu(A))$.

**Proof:** Let A be σ-H-open, we have $A \subset c^*{}_\mu(i_\mu(A))$. Then $c^*{}_\mu(A) \subset c^*{}_\mu(c^*{}_\mu(i_\mu(A)))$, therefore, $c^*{}_\mu(A) \subset c^*{}_\mu(i_\mu(A)))$, since $A \subset c^*{}_\mu(A), i_\mu(A) \subset i_\mu(c_\mu{}^*(A)) \subset c^*{}_\mu(A) and c^*{}_\mu(i_\mu(A)) \subset c^*{}_\mu(c^*{}_\mu(A)) \subset c^*{}_\mu(A))$.Hence $c^*{}_\mu(A) = c^*{}_\mu(i_\mu(A))$. The converse is obvious.

**Theorem 2:** Let (X,μ) be a GTS with hereditary class H , A and B be a subsets of X. If A is a σ-H-open set and $A \subset B \subset c^*{}_\mu(A)$, then B is a σ-H-open set.

**Proof:** Since A is a σ-H-open, there exists μ-open set U such that $U \subset A \subset c^*{}_\mu(U)$. Then $U \subset A \subset B \subset c^*{}_\mu(A) \subset c^*{}_\mu(c^*{}_\mu(U)) = c^*{}_\mu(U)$ and hence $U \subset B \subset c^*{}_\mu(U)$ . Thus B is a σ-H- open set.

**Theorem 3:** Let (X, μ) be a quasi topological space with hereditary class H, A and B be subsets of X.

a) If $U_\sigma \epsilon \sigma_H$ for each σϵΔ, then $\bigcup\{U_\alpha : \alpha \in \Delta\} \in \sigma_H$ ,

b) If $A \epsilon \sigma_H$ and Bϵμ, then $A \cap B \in \alpha_H$ .

**Proof**: (a) Since $U_\sigma \epsilon \sigma_H$, we have $U_\alpha \subset c^*{}_\mu(i_\mu(U_\alpha))$ for each σϵΔ. Thus $\bigcup_{\alpha\in\Delta} U_\alpha \subset \bigcup_{\alpha\in\Delta} c^*{}_\mu(i_\mu(U_\alpha)) \subset \bigcup_{\alpha\in\Delta}\{(i_\mu(U_\alpha)) \subset \bigcup_{\alpha\in\Delta}\{(i_\mu(U_\alpha))^* \cup (i_\mu(U_\alpha))\} \subset (\bigcup_{\alpha\in\Delta}(i_\mu(U_\alpha)))^* \cup i_\mu(\bigcup_{\alpha\in\Delta} U_\alpha) \subset$ ${}_\alpha))^* \cup i_\mu(\bigcup_{\alpha\in\Delta} U_\alpha) = c^*{}_\mu(i_\mu(\bigcup_{\alpha\in\Delta} U_\alpha))$. This shows that $\bigcup_{\alpha\in\Delta} U_\alpha \in \alpha_H$ .

(b) Let $A \epsilon \sigma_H$ and Bϵμ. Then $A \subset c^*{}_\mu(i_\mu(A))$ and $B = i_\mu(B)$.Now,

$$A \cap B \subset c^*{}_\mu(i_\mu(A)) \cap B = ((i_\mu(A))^* \cup i_\mu(A)) \cap B = ((i_\mu(A))^* \cap B) \cup (i_\mu(A) \cap B) \subset (i_\mu(A) \cap B)^* \cup$$
$$(i_\mu(A) \cap B) = (i_\mu(A) \cap i_\mu(B))^* \cup (i_\mu(A)) \cap i_\mu(B)) = (i_\mu(A \cap B))^* \cup i_\mu(A \cap B) = c^*{}_\mu(i_\mu(A \cap B))$$

This shows that $A \cap B \in \alpha_H$ .

**Theorem 4:** Let (X,μ) be a GTS with hereditary class H and A be a subsets of X. If A is a σ-H-closed, then $i_\mu(c^*{}_\mu(A)) \subset A$.

**Proof:** Since A is a σ-H-closed , X-A is σ-H-open. Now, $X-A\subset c^{*}{}_{\mu}(i_{\mu}(X-A))\subset c_{\mu}(i_{\mu}(X-A))=X-i_{\mu}(c_{\mu}(A))\subset X-i_{\mu}(c^{*}{}_{\mu}(A))$. Therefore, $i_{\mu}(c^{*}{}_{\mu}(A))\subset A.$

**On σ-H-Continuous Function:** A function A functions f:(X,$\mu_1$,H)→(Y,$\mu_2$) is said to be σ-H-continuous [5](resp.σ- continuous[9] if $f^{-1}$(V) is σ-H- open (resp.σ-open) in(X,$\mu_1$,H) for each μ-open set V of (Y,$\mu_2$). A function f:(X,$\mu_1$,H)→(Y,$\mu_2$,I) is said to be H-irresolute if $f^{-1}$(V) is σ-H- open in (X,$\mu_1$,H) for each σ-I-open set V of (Y,$\mu_2$,I). It is obvious that continuity implies σ-H-continuity and σ-H-continuity implies σ-continuity.

**Theorem 5:** Let f:(X,$\mu_1$,H)→(Y,$\mu_2$) be σ-H-continuous and $f^{-1}(V^{*})\subset(f^{-1}(V))^{*}$ for each V∊$\mu_2$. Then f is H-irresolute.

**Proof:** Let B be σ-I-open set of (Y,$\mu_2$,I) there exists V∊$\mu_2$ such that $V\subset B\subset c^{*}{}_{\mu}(V)$. Therefore $f^{-1}(V)\subset f^{-1}(B)\subset f^{-1}(c^{*}{}_{\mu}(V))\subset c^{*}{}_{\mu}(f^{-1}(V))$. Since f is σ-H-continuous and V∊$\mu_2$ , $f^{-1}$(V) ∊$\sigma_H$ and $f^{-1}$(B) is σ-H- open in (X,$\mu_1$,H). Hence f is H-irresolute.

**On σ-H-Open and σ-H-Closed Functions:** A functions f: (X,$\mu_1$) → (Y,$\mu_2$,I) is called σ-H-open(resp. σ-H-closed) if for each U∊μ (resp.U is μ-closed ) f(U)∊$\sigma_H$ (Y,$\mu_2$,I) ( resp. f(U) is a σ-H-closed ).

**REMARK :** Let (X,μ) be a GTS with hereditary class H. Then the following hold.

(a) Every σ-H- open (resp σ-H- closed) function is σ-open (resp σ-closed).

(b) Every μ-open functions is σ-H-open.

The following examples shows that the converse of remark is not true.

**Example 1**: Let X={a,b,c}, $\mu_1$={ϕ,{a,b}}, $\mu_2$={ϕ,{a}} and I= {ϕ,{a}}.The identity function f:(X,$\mu_1$) → (Y,$\mu_2$,I) is σ-open, but it is not σ-H-open.

**Example 2**: Let X={a,b,c}, $\mu_1$={ϕ,{a,b}}, $\mu_2$={ϕ,{c}{b}{b,c}} and I={ϕ,{c}}.Define a function f: (X,$\mu_1$) → (Y,$\mu_2$,I) as follows, f(a)=a, f(b)=f(c)=b. Then f is σ-closed, but it is not σ-H-closed.

**Example 3:** Let X={a,b,c}, $\mu_1$={ϕ,{a,b}}, $\mu_2$={ϕ,{a}{c}{a,c}} and I={ϕ,{c}}.The identity function f: :(X,$\mu_1$) → (Y,$\mu_2$,I) is σ-H-open, but it is not σ-open.

**Theorem 6:** A function f: (X,$\mu_1$) → (Y,$\mu_2$,I) is σ-H-open if and only if for each x∊X and each neighborhood U of x, there exists V∊$\sigma_I$ containing f(x) such that $V\subset f(U)$.

**Proof**: Suppose that f is σ-H-open function. For each x∊X and each neighborhood U of x, there exists $U_o$∊μ such that x∊$U_o\subset U$ . Since f is σ-H-open V=f($U_o$)∊$\sigma_I$ and f(x)∊ $V\subset f(U)$ Conversely, let U be an μ-open set. For each x∊U, there exists $V_x$∊$\sigma_I$ (Y,$\mu_2$) such that f(x)∊ $V_x\subset f(U)$. Therefore ,f(U)= $\bigcup\{V_x \mid x\in U\}$ and f(U)∊$\sigma_I$ (Y,$\mu_2$). This shows that f is σ-H-open.

**References:**

1. A.Csaszar, Generalized open sets., Acta Math. Hungar., 1997,(1-2):65-87.
2. A.Csaszar, Generalized open sets in generalized topologies., Acta Math. Hungar.,2005,106:351-357.
3. A.Csaszar, Modification of Generalized topologies via hereditary classes., Acta Math, Hungar.,2007,115(29-36).
4. A.Csaszar, Remarks on quasi-topologies., Acta Math. Hungar.,2008,119(1-2):197-200.
5. E.Hatir and T.Noiri, On Decomposition of Continuity via Idealization, Acta Math. Hungar..96(4)(2002),341-349.

6. D.Jankovic, T.R.Hamlett. New topologies from old via ideals., Amer. Math. Monthly, 1990, 97(4):295-310.
7. D.Jankovic, T.R.Hamlett, Compatible extensions of ideals., Boll.U.M.I.,(7)6-B(1992),453-465.
8. K.Kuratowski. Topology, Vol. I, Academic Press, New York, 1996.
9. N.Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly,70(1963),36-41.
10. V.Renukadevi, K.Karuppayi. On Modifications of generalized topologies via hereditary classes., J.Adv. Res. Pure.Math.,2010, 2(2):14-20.

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