

SUPRA \mathcal{G} HOMEOMORPHISM IN TOPOLOGICAL SPACES

Usharani. S

Assistant Professor, Department of Mathematics,
Sri Vijay Vidyalaya College of Arts and Science, Dharmapuri

Abstract: In this paper, introduced the concept of contra supra \mathcal{G} -continuous functions and contra supra \mathcal{G} -irresolute, slightly supra \mathcal{G} continuous and obtained the basic properties of supra \mathcal{G} --Homeomorphism and supra \mathcal{G}^* - Homeomorphism. Also related the function with some other functions in supra topological spaces.

Keywords: contra supra \mathcal{G} -continuous functions, contra supra \mathcal{G} -irresolute, supra \mathcal{G} -Homeomorphism and supra \mathcal{G}^* -Homeomorphism, slightly supra \mathcal{G} continuous functions.

AMS Subject Classification: 54C05, 54C08, 54C10.

1. Introduction: In 1983, A.S Mashhour et al., [4] introduced the supra topological spaces and studied S^* -continuous functions and S^* -continuous functions. In 1996, Dontchev [3] introduced the notion of contra continuous functions. In 2007, Caldas, Jafari, [2] introduced a new class of functions called contra- β continuous functions. In 2013, L.Vidharani and M.Vigneshwaran[8] introduced and discussed about contra supra N -continuous functions.

I introduced the concept of contra supra \mathcal{G} -continuous functions and contra supra \mathcal{G} -irresolute and obtain the basic properties of supra \mathcal{G} - Homeomorphism and supra \mathcal{G}^* - Homeomorphism. Also related the function with supra slightly \mathcal{G} -continuous functions in supra topological spaces.

2. Preliminaries: Throughout this paper, (X, τ) , (Y, σ) , (Z, η) or X, Y, Z represent non - empty topological spaces on which no separations axioms are assumed unless otherwise mentioned. For a subset A of a spaces (X, τ) , $cl(A)$, and $int(A)$ denoted closure and interior of A respectively.

Definition: 2.1 A subfamily μ of X is said to be supra topology on X , if

- (i) $X, \emptyset \in \mu$
- (ii) if $A_i \in \mu$ for all $i \in J$, then $\bigcup A_i \in \mu$. The pair (X, μ) is called supra topological spaces.

The element of μ are called supra open sets in (X, μ) and the complement of supra open sets is called supra closed sets and it is denoted by μ^c .

Definition: 2.2 The supra closure of a set A is denoted by supra $cl(A)$ and defined as supra $cl(A) = \bigcap \{B : B \text{ is a supra closed and } A \subseteq B\}$. The supra interior of a set is denoted by supra $int(A)$, and defined as supra $int(A) = \bigcup \{B : B \text{ is supra open and } A \supseteq B\}$

Definition: 2.3 Let (X, τ) be a topological space and μ be a supra topology on X . We call μ a supra topology associated with τ , if $\tau \subseteq \mu$

Definition: 2.4 A subset A of a space X is called

- (i) supra semi open set [4], if $A \subseteq cl^\mu[int^\mu(A)]$
- (ii) supra α -open set [3], if $A \subseteq cl^\mu[cl^\mu(int^\mu(A))]$
- (iii) supra regular closed [], if $A = cl^\mu(int^\mu(A))$
- (iv) supra g -closed [5], if $cl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open in X
- (v) supra b -open [7], if $A \subseteq cl^\mu[int^\mu(A)] \cup (int^\mu(A))cl^{\mu\mu}(A)$

2. Contra Supra \mathcal{G} Continuous Functions:

Definition: 2.1 Let (X, τ) and (Y, σ) be two topological spaces and μ be an associated supra topology with τ . A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called contra supra \mathcal{G} continuous function if $f^{-1}(O)$ is \mathcal{G} closed in (X, τ) for every supra open set O in (Y, σ)

Examples: 2.2 Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{b\}, X\}$ and $\sigma = \{\phi, \{a, c\}, X\}$ we have, $\mathcal{G}(C(X)) = \{\phi, \{a, c\}, X\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then f is called contra supra \mathcal{G} continuous.

Theorem: 2.3 Every contra supra continuous is contra supra \mathcal{G} continuous function.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a contra supra continuous function. Let O be an supra open set (Y, σ) . Since, f is contra supra continuous. Then, $f^{-1}(O)$ is supra closed in (X, τ) . Therefore, f is contra supra \mathcal{G} continuous.

Converse of the above theorem is need not be true. It is shown by example.

Examples: 2.4: Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, Y\}$ we have, $\mathcal{G}(C(X)) = \{\phi, \{c\}, \{a, c\}, \{b, c\}, X\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Since $f^{-1}\{b, c\} = \{b, c\}$ is contra supra \mathcal{G} continuous but not contra supra continuous. In this section, I introduce a new type of continuous functions called supra \mathcal{G} - continuous function and obtain some of their properties and characterizations.

Theorem: 2.5 Every contra continuous map is contra supra \mathcal{G} - continuous

Proof: Let (X, τ) and (Y, σ) be two supra topological spaces. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be contra continuous map and A is open in Y . then $f^{-1}(A)$ is an closed set in X . since μ is associated with τ , then $\tau \subset \mu$. Therefore, $f^{-1}(A)$ is supra closed set in X . Since every supra closed set is \mathcal{G}^μ - closed set Hence f is supra \mathcal{G} continuous.

The converse of the above theorem is not true.

Theorem: 2.6 Let (X, τ) and (Y, σ) be two topological spaces and μ be an associated supra topology with τ . Let f be a map from X into Y , then the following are equivalent:

- (i) f is a contra supra \mathcal{G} - continuous map.
- (ii) The inverse image of a supra closed set in Y is a supra \mathcal{G} - open set in X .
- (iii) The inverse image of a supra open set in Y is a supra \mathcal{G} - closed set in X .

Proof: (i) \Rightarrow (ii). Let A be a supra closed set in Y . Then $Y - A$ is supra open set in Y then $f^{-1}(Y - A) = X - f^{-1}(A)$ is a supra \mathcal{G} - closed set in X . It follows that $f^{-1}(A)$ is a supra \mathcal{G} -open subset of X

(ii) \Rightarrow (iii)

Let A be a supra open set in Y . Then $Y - A$ is supra closed set in Y then $f^{-1}(Y - A) = X - f^{-1}(A)$ is a supra \mathcal{G} - open set in X . It follows that $f^{-1}(A)$ is a supra \mathcal{G} -closed subset of X

(iii) \Rightarrow (i) It obviously true.

Theorem: 2.7 Let (X, τ) , (Y, σ) and (Z, ν) be three topological spaces, If a map $f: (X, \tau) \rightarrow (Y, \sigma)$ is contra supra \mathcal{G} - continuous and $g: (Y, \sigma) \rightarrow (Z, \nu)$ is a contra continuous map, then $g \circ f: (X, \tau) \rightarrow (Z, \nu)$ is contra supra \mathcal{G} - continuous.

Proof: Let F be any supra closed set in (Z, ν) . Since $g: (Y, \sigma) \rightarrow (Z, \nu)$ is contra continuous. $g^{-1}(F)$ is supra open in (Y, σ) . Since $f: (X, \tau) \rightarrow (Y, \sigma)$ is contra supra \mathcal{G} - continuous. $f^{-1}[g^{-1}(F)] = (g \circ f)^{-1}(F)$ is \mathcal{G} - supra closed in (X, τ) and so $g \circ f$ is contra supra \mathcal{G} - continuous.

3. Supra \mathcal{G} - Homeomorphism And Supra \mathcal{G}^* - Homeomorphism:

Definition: 3.1 A bijection $f: (X, \tau) \rightarrow (Y, \sigma)$ is called supra \mathcal{G} - Homeomorphism if f is both supra \mathcal{G} -continuous function and supra - open map

Definition: 3.2 A bijection $f: (X, \tau) \rightarrow (Y, \sigma)$ is called supra \mathcal{G}^* - Homeomorphism if f and f^{-1} are supra \mathcal{G} -irresolute.

we denoted the family of all supra \mathcal{G} - Homeomorphism (resp. supra \mathcal{G}^* - homeomorphism) of a supra topological space (X, τ) onto itself by $S\mathcal{G} - h(X, \tau)$ (resp. $S\mathcal{G}^* - h(X, \tau)$).

Theorem : 3.3 Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a bijective supra \mathcal{J} - continuous map. then, the following are equivalent.

- (i) f is an \mathcal{J} -open map.
- (ii) f is an \mathcal{J} -homeomorphism.
- (iii) f is an \mathcal{J} -closed map.

Proof: (i) (ii) : if f is a bijective supra \mathcal{J} -continuous map, suppose (i) holds. Let V be a supra closed in (X, τ) , then V^c is a supra open in (X, τ) . Since f is a \mathcal{J} - open map, $f(V^c)$ is a supra \mathcal{J} -open in (Y, σ) . Hence, $f(V)$ is supra \mathcal{J} -closed in (Y, σ) implies f^{-1} is supra \mathcal{J} -continuous. Therefore, f is a supra \mathcal{J} - homeomorphism.

(ii) (iii): Suppose f is a supra \mathcal{J} -homeomorphism and f is a bijective supra \mathcal{J} -continuous function, then from the definition 3.1, f^{-1} is a supra \mathcal{J} -continuous, implies f is a supra \mathcal{J} -closed map

(iii) (i) Suppose f is a supra \mathcal{J} - closed map, Let V be a supra open in (X, τ) , then, V^c is a supra closed in (X, τ) . since f is a supra \mathcal{J} -closed map, $f(V^c)$ is a supra \mathcal{J} - closed in (Y, σ) . Hence $f(V)$ is a supra \mathcal{J} - open in (Y, σ) . Therefore f is a supra \mathcal{J} -open map.

Remark: 3.4 The composition of two supra \mathcal{J} -homeomorphism need not be a supra \mathcal{J} - homeomorphism.

Example: 3.5 Let $X = Y = Z = \{a, b, c\}$ with $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$, $\sigma = \{\phi, \{a, b\}, Y\}$ and $\eta = \{\phi, \{a\}, \{a, b\}, Z\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be identity maps, Then f and g are supra \mathcal{J} - homeomorphism but their $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is not supra \mathcal{J} -homeomorphism.

Theorem: 3.6 Every supra \mathcal{J} -homeomorphism is supra \mathcal{J} -continuous.

Proof : It is obviously true from the definition 3.1.

The converse of the theorem need not be true.

Theorem: 3.7 Every supra \mathcal{J}^* -homeomorphism is supra \mathcal{J} -irresolute.

Proof : It is obviously from the definition. 3.2,

The converse of the theorem need not be true.

Theorem: 3.8 If $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$ are supra \mathcal{J}^* -homeomorphism, then the composition $g \circ f$ is also a supra \mathcal{J}^* -homeomorphism.

Proof : Let V be a supra \mathcal{J} -closed set in (Z, η) . Since g is a supra \mathcal{J}^* -homeomorphism g and g^{-1} are supra \mathcal{J} -irresolute, then $g^{-1}(V)$ is a supra \mathcal{J} -closed set in (Y, σ) . Now, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$. Since f is a supra \mathcal{J}^* -homeomorphism f and f^{-1} are supra \mathcal{J} -irresolute, then $f^{-1}(g^{-1}(V))$ is a supra \mathcal{J} -closed set in (X, τ) . Thus, $g \circ f$ is a supra \mathcal{J} -irresolute.

For a supra \mathcal{J} -closed set V in (X, τ) , $(g \circ f)(V) = g(f(V))$. By the hypothesis $f(V)$ is supra \mathcal{J} -closed set in (Y, σ) . Thus, $g(f(V))$ is supra \mathcal{J} -closed set in (Z, η) . Hence, $(g \circ f)^{-1}$ is supra \mathcal{J} -irresolute. Therefore, $g \circ f$ is supra \mathcal{J}^* - homeomorphism.

Theorem: 3.9 If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a supra \mathcal{J}^* -homeomorphism, then $\mathcal{J} - cl(f^{-1}(B)) = f^{-1}(\mathcal{J} - cl(B))$, for every $B \subset Y$ is a supra \mathcal{J} -closed.

Proof: Since f is a supra \mathcal{J}^* -homeomorphism, f and f^{-1} are supra \mathcal{J} irresolute. Let B be a supra \mathcal{J} -closed set in (Y, σ) . Since f is a supra \mathcal{J} -irresolute $f^{-1}(B)$ is supra \mathcal{J} -closed set in (X, τ) . Since, B is a supra \mathcal{J} -closed set $B = (\mathcal{J} - cl(B))$. Therefore, $f^{-1}(\mathcal{J} - cl(B))$ is supra \mathcal{J} -closed set in (X, τ) . Since $f^{-1}(B)$ is a supra \mathcal{J} -closed set, $\mathcal{J} - cl(f^{-1}(B)) = f^{-1}(B)$ is a supra \mathcal{J} - closed set in (X, τ) . Therefore, $\mathcal{J} - cl(f^{-1}(B)) = f^{-1}(\mathcal{J} - cl(B))$ is supra \mathcal{J} - closed set in (X, τ) .

Theorem: 3.10 If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a supra \mathcal{J}^* - homeomorphism, then $\mathcal{J} - cl(f(B)) = f(\mathcal{J} - cl(B))$, for every $B \subset X$ is a supra \mathcal{J} - closed.

Proof: Since f is a supra \mathcal{J}^* - homeomorphism, f and f^{-1} are supra \mathcal{J} irresolute. Let B be a supra \mathcal{J} -closed set in (X, τ) . Since f^{-1} is a supra \mathcal{J} irresolute $f(B)$ is supra \mathcal{J} - closed set in (Y, σ) . Since, B is a supra \mathcal{J} - closed set $B = (\mathcal{J} - cl(B))$. Therefore, $f(\mathcal{J} - cl(B))$ is supra \mathcal{J} - closed set in (Y, σ) . Since $f(B)$

is a supra \mathcal{G} - closed set, $\mathcal{G} - cl(f(B)) = f(B)$ is a supra \mathcal{G} - closed set in (Y, σ) . Therefore, $\mathcal{G} - cl(f(B)) = f(\mathcal{G} - cl(B))$ is supra \mathcal{G} - closed set in (Y, σ) .

4. Slightly Supra \mathcal{G} Continuous Functions:

Definition: 4.1 A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called slightly supra \mathcal{G} continuous at a point $x \in X$, if for each supra clopen subset V of Y containing $f(x)$, there exist supra \mathcal{G} open set U in x containing X such that $f(U) \subseteq V$. The function f is said to be slightly supra \mathcal{G} continuous if f is slightly supra \mathcal{G} continuous at each of its points.

Definition: 4.2 A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be slightly supra \mathcal{G} continuous, if the inverse image of every supra clopen set in Y is supra \mathcal{G} open in X .

Proposition: 4.3 For a function $f: (X, \tau) \rightarrow (Y, \sigma)$, the following statements are equivalent:

Def 4.1 and Def 4.2 are equivalent, if the inverse image of every supra clopen set in Y is supra \mathcal{G} open in X .

Proof: Suppose (i) holds. Let O be a supra clopen set in Y and $x \in f^{-1}(O)$. Then $f(x) \in O$ and thus there exists a supra \mathcal{G} - open set U_x such that $x \in U_x \subseteq f^{-1}(O)$ and $f^{-1}(O) = \bigcup_x U_x \in f^{-1}(O)U_x$. since, arbitrary union of supra \mathcal{G} - open set is supra \mathcal{G} - open, $f^{-1}(O)$ is supra \mathcal{G} - open in X and therefore f is slightly supra \mathcal{G} continuous. Suppose (ii) holds. Let $f(x) \in O$, where, O is a supra clopen set in Y . Since f is slightly supra \mathcal{G} continuous, $x \in f^{-1}(O)$ where $f^{-1}(O)$ is supra \mathcal{G} open in X . Let $U = f^{-1}(O)$. Then U is supra \mathcal{G} open in X , $x \in X$ and $F(U) \subseteq O$.

Theorem: 4.4 For a function $f: (X, \tau) \rightarrow (Y, \sigma)$, the following statements are equivalent.

(i) f is slightly supra \mathcal{G} continuous.

(ii) The inverse image of every supra clopen set O of Y is supra \mathcal{G} open in X .

(iii) The inverse image of every supra clopen set O of Y is supra \mathcal{G} closed in X .

(iv) The inverse image of every supra clopen set O of Y is supra \mathcal{G} clopen in X .

Proof: (i) (ii) Follows from the above proposition [5,3]

(ii)(iii) Let O be a supra clopen set in Y which implies O^c is supra clopen in Y . By (ii) $f^{-1}(O^c) = (f^{-1}(O))^c$ is supra \mathcal{G} open in X . Therefore $f^{-1}(O)$ is supra \mathcal{G} closed in X .

(iii) (iv) By (ii) and (iii) $f^{-1}(O)$ is supra \mathcal{G} clopen in X .

(iv)(i) Let O be a supra clopen set in Y containing $f(X)$ by (iv) $U = f^{-1}(O)$ is supra \mathcal{G} clopen in X . Take $U = f^{-1}(O)$, Then $f(U) \subset O$. Hence, f is slightly supra \mathcal{G} continuous.

Theorem: 4.5 Every slightly supra continuous function is slightly supra \mathcal{G} continuous.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a slightly supra continuous functions. Let O be a supra clopen set in Y . Then $f^{-1}(O)$ is open in X . since, every supra open set is supra \mathcal{G} - open. Hence f is slightly supra \mathcal{G} continuous.

Remark: 4.6 The converse of the above theorem need not be true.

Theorem: 4.7 Every supra \mathcal{G} continuous function is slightly supra \mathcal{G} continuous.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a supra \mathcal{G} continuous function. Let O be a supra clopen set in Y . Then $f^{-1}(O)$ is supra open in X and supra \mathcal{G} - closed in X . Hence f is slightly supra \mathcal{G} continuous.

Remark: 4.8 The converse of the above theorem need not be true.

Theorem: 4.9 Every contra supra \mathcal{G} continuous is slightly supra \mathcal{G} continuous.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a contra supra \mathcal{G} continuous function. Let O be a supra clopen set in Y . Then $f^{-1}(O)$ is supra \mathcal{G} open in X . Hence, f is slightly supra \mathcal{G} continuous.

Remark: 4.10 The converse of the above theorem need not be true as can be seen from the following.

Theorem: 4.11 Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be functions. Then the following properties hold.

- (i) If f is supra \mathcal{G} - irresolute and g is slightly supra \mathcal{G} continuous. Then $(g \circ f)$ is slightly supra \mathcal{G} continuous.
- (ii) If f is supra \mathcal{G} - irresolute and g is supra \mathcal{G} continuous then $(g \circ f)$ is slightly supra \mathcal{G} continuous.
- (iii) If f is supra \mathcal{G} - irresolute and g is slightly supra continuous, then $(g \circ f)$ is slightly supra \mathcal{G} continuous.
- (iv) If f is supra \mathcal{G} continuous and g is slightly supra continuous, then $(g \circ f)$ is slightly supra \mathcal{G} continuous.

Proof:

- (i) Let O be a supra clopen set in Z , since, g is slightly supra \mathcal{G} continuous, $g^{-1}(O)$ is supra \mathcal{G} open in Y . since, f is supra \mathcal{G} irresolute, $f^{-1}(g^{-1}(O))$ is supra \mathcal{G} open in X . since $(g \circ f)^{-1}(O) = f^{-1}(g^{-1}(O)) \cdot g \circ f$ is slightly supra \mathcal{G} continuous.
- (ii) Let O be a supra clopen set in Z , since g is supra \mathcal{G} continuous, $g^{-1}(O)$ is supra open in Y , since f is supra \mathcal{G} irresolute, $f^{-1}(g^{-1}(O))$ is supra \mathcal{G} open in X . Hence $g \circ f$ is slightly supra \mathcal{G} continuous.
- (iii) Let O be a supra clopen set in Z . since g is slightly supra continuous, $g^{-1}(O)$ is supra open in Y . since f is supra \mathcal{G} - irresolute, $f^{-1}(g^{-1}(O))$ is supra \mathcal{G} open in X . Hence $g \circ f$ is slightly supra \mathcal{G} continuous.
- (iv) Let O be a supra clopen set in Z . since g is slightly supra continuous, $g^{-1}(O)$ is supra open in Y . since f is supra \mathcal{G} continuous, $f^{-1}(g^{-1}(O))$ is supra \mathcal{G} open in X . Hence $g \circ f$ is slightly supra \mathcal{G} continuous.

Theorem: 4.12 Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be functions. Then the following properties hold.

- (i) If f is slightly supra \mathcal{G} continuous and g is perfectly supra \mathcal{G} continuous, Then $(g \circ f)$ is supra \mathcal{G} - irresolute.
- (ii) If f is slightly supra \mathcal{G} continuous and g is contra supra continuous then $(g \circ f)$ is slightly supra \mathcal{G} continuous.
- (iii) If f is supra \mathcal{G} - irresolute and g is contra supra \mathcal{G} -continuous. Then $(g \circ f)$ is slightly supra \mathcal{G} continuous.

Proof:

- (i) Let O be a supra \mathcal{G} - open in Z . since g is perfectly supra \mathcal{G} continuous, $g^{-1}(O)$ is supra open and supra closed in Y . since f is slightly supra \mathcal{G} continuous, $f^{-1}(g^{-1}(O))$ is supra \mathcal{G} - open in X . Hence $g \circ f$ is supra \mathcal{G} - irresolute.
- (ii) Let O be a supra clopen in Z . since g is contra supra continuous, $g^{-1}(O)$ is supra open and supra closed in Y . since f is slightly supra \mathcal{G} continuous, $f^{-1}(g^{-1}(O))$ is supra \mathcal{G} - open in X . Hence $g \circ f$ is slightly supra \mathcal{G} continuous.
- (iii) Let O be a supra clopen in Z . since g is supra contra \mathcal{G} -continuous, $g^{-1}(O)$ is supra \mathcal{G} - open and supra \mathcal{G} - closed in Y . since f is supra \mathcal{G} - irresolute, $f^{-1}(g^{-1}(O))$ is supra \mathcal{G} - open and supra \mathcal{G} - closed in X . Hence $g \circ f$ is supra \mathcal{G} continuous.

Theorem: 4.13 If the function $f: (X, \tau) \rightarrow (Y, \sigma)$ is slightly supra \mathcal{G} and (X, τ) is $T_{1/2}$ space, then f is slightly supra continuous.

Proof: Let O be a supra clopen in Y . since, g is slightly supra \mathcal{G} continuous, $f^{-1}(O)$ is supra \mathcal{G} open in X , since X is $T_{1/2}$ space, $f^{-1}(O)$ is supra open in X . Hence f is slightly supra continuous.

Theorem: 4.14 If the function $f: (X, \tau) \rightarrow (Y, \sigma)$ is slightly supra \mathcal{G} continuous and (Y, σ) is locally indiscrete space, then f is supra \mathcal{G} continuous.

Proof: Let O be an open subset of Y . since (Y, σ) is a locally indiscrete space. O is closed in Y . since f is slightly supra \mathcal{G} continuous, $f^{-1}(O)$ is supra \mathcal{G} - open in X . Hence, f is supra \mathcal{G} continuous.

References:

1. Arockiarani.I and M.Trinita Pricilla, On Supra T - closed sets, *Int.J.of Mathematics Archive*, 2(8), (2011), 1376 - 1380
2. Caldas. M and Jafri.S, Some properties of Contra- β -continuous functions, *Mem.Fac.Sci.Koch.Univ.(Math)*22 (2001), 19-28.

4. Devi.R, S.Sampath Kumar and J.Caldas, On Supra α - open set and $S\alpha$ - continuous functions. *Gen.Mathematics*, Vol 16, Nr, 2, (2008), 77-84
5. J. Dontchev, T. Noiri, contra " semicontinuous functions, *Mathematica. Pannonica*, 10/2 (1999), 159 " 168
6. Mashhour.A.S, A.A.Allam, E.S.Mahmoud and F.H.Khedr, On supra topological spaces. *Ind.J.Pure and Appl. Mathematics*, 14 (4), (1983), 502 - 510
7. Ravi.O, G.Ramkumar and M.Kamaraj, On supra g - closed set. *Int.J.of Advances and Applied Mathematics*, 1 (12), (2011), 52-66
8. Sayed.O.R and T.Noiri, On Supra b - open sets and supra b - continuity on topological spaces. *Euro.J. of Pure and Applied Mathematics*, 3(2), (2010), 295 - 302
9. Vidyarani.L and M.Vigneshwaran, Contra supra N - continuous functions in supra topological space, *Journal of global research in mathematical archives*. Vol 1, No 9, Sep 2013.
10. Usharani.S and S.Padmasekaran, Further Study of Topological Spaces. *Ph.D thesis, Periyar University, Salem*, Dec 2016.
11. Usharani.S, Supra continuous \tilde{g} in topological spaces, *Int. J. of Math And Appln*, 6(2-B) 2018, 45 - 50.
