ON NEW CLASSES OF SPLITTING GRAPHS

B.Rajendran
Department of Mathematics,
Mepco Schlenk Engineering College, Sivakasi - 626 005, Tamil Nadu, India
brrajendranbalusamy@gmail.com

J. Deepalakshmi
Department of Mathematics
Mepco Schlenk Engineering College, Sivakasi - 626 005, Tamil Nadu, India
jdeepa@mepcoeng.ac.in

G.Marimuthu
Department of Mathematics
The Madura College, Madurai - 625011,Tamil Nadu, India
yellowmuthu@yahoo.com

Abstract: This paper examines the extended concept of the splitting graph of a graph G in two ways called as Edge-Vertex Splitting Graph and Edge-Edge splitting graph. We obtained relations connecting a graph and its new classes of splitting graphs in the cases of number of vertices and edges, number of triangles, degree of the vertices, coloring number, covering number, independent number, planarity and non-planarity.

Keywords: Spitting graph, Edge-Vertex splitting graph, Edge-Edge splitting graph.

AMS Subject Classification Number: 05C75.

1. Introduction: Notations and terminologies are followed from [2]. In an undirected Graph $G = (V, E)$, for each vertex v of a graph G, take a new vertex v'. Join v' to all the adjacent vertices of v. The resulting graph is called splitting graph by Sampathkumar and Walikar [7]. Let us call this graph as Vertex-Vertex splitting graph of G and denote it by $S_{VV}(G)$.In [7],the authors obtained relations connecting a graph and its splitting graph in the cases of number of vertices, edges, number of triangles, degree of the vertices, coloring number, covering number, independent number, planarity and non-planarity. Using splitting graphs, Rajendran and Aravamudhan [6] derived a number of results in antipodal graphs. In [3], Gapobianco et.al studied some quadratic equations. They have given some solutions to the equation $G^2 = \bar{G}$. Rajendran and Aravamudhan [6] have given some more solutions for this equation and generalized the results to the equation $G^{d-1} = \bar{G}$ where d is the diameter of G which involves splitting graphs. In this view of splitting graphs, Line splitting graph has been introduced by Kulli et. al. [4]. Interestingly, Degree splitting graphs, Line degree Splitting graphs were explored in [1],[5]& [8].We addressed in getting new operated graph from the original graph in this similar sense. For each edge e_{ij} connecting two vertices v_i and v_j of a Graph G, add a new vertex v_{ij}. Join v_{ij} to v_i and v_j. The resulting graph is called Edge-Vertex splitting graph and is denoted by $S_{EV}(G)$. In section 2, $S_{EV}(G)$properties observed. In section 3, another graph by replacing an edge of a graph by cycle of length 4 introduced and studied. Such a resulting graph is called Edge-Edge splitting graph and denoted by $S_{EE}(G)$. Replacing an edge by a cycle on more vertices is similar to strengthening a specific skilled team with more people to work for same purpose in any service providing sector in any network analysis. Optimizing the efficiency of a network by enlarging the working circle is a point of keen interest and is omnipresent wall to wall. With this motivation, these two new classes of splitting graphs were introduced.

2. Edge-Vertex Splitting graph

Definition 2.1: Let $G = (V, E)$ be a simple, undirected graph. For every edge e connecting two vertices of G, introduce a vertex v_e and join it to the two end vertices of e. And call such a set of newly introduced vertices as V_E and a set of newly introduced edges as E_E.The Edge-Vertex Splitting graph is a graph on $V \cup V_E$ vertex set and its edge set is $E \cup E_E$. Here we replace an edge by a cycle of length 3.The graph obtained from G by replacing an edge by C_3 is called Edge-Vertex Splitting graph of G and is denoted by $S_{EV}(G)$.

The following observations were made from the definition of Edge-Vertex Splitting graph.

Proposition 2.2: If G is a (p,q) graph, then $S_{EV}(G)$ is $(p+q, 3q)$ graph.

Proposition 2.3: In $S_{EV}(G)$, for any vertex v, $deg(v)$ is twice its degree in G and degree of $v_e = 2$ always.

Proposition 2.4: If G has t triangles, then $S_{EV}(G)$ has $t + |E(G)|$ triangles.

Theorem 2.5: For a Edge-Vertex Splitting graph, $\chi(S_{EV}(G)) = \chi(G) + 1$.
Proof: As G is an induced subgraph of $S_{EV}(G)$, introduce the new color to the vertices of the set V_E. And all such V_E vertices are non adjacent. With that way, coloring number of $S_{EV}(G)$ is raised with value 1 from that of G.

Theorem 2.6: If G is Planar, then $S_{EV}(G)$ is Planar.
Proof: For a Planar Graph G, every block is planar. By the construction of $S_{EV}(G)$from G, it is planar with every face as triangle.

Theorem 2.7: If G is Eulerian, then $S_{EV}(G)$ is Eulerian graph.
Proof: The Eulerian circuit of G can be extended so which will cover the E_E edges and hence becomes an eulerian circuit of $S_{EV}(G)$.

Theorem 2.8: If G is Hamiltonian, then $S_{EV}(G)$ is Hamiltonian.
Proof: Let C be a hamiltonian circuit in G.It is an induced subgraph of $S_{EV}(G)$. Exactly removing the edge e that joins any two vertices u, v of C and adding the path connecting u, v via v_e will produce a hamiltonian circuit in $S_{EV}(G)$.

Theorem 2.9: For an Edge-Vertex Splitting graph, the edge chromatic number $\chi_1(S_{EV}(G)) = \chi_1((G)) + 2$.
Proof: In $S_{EV}(G)$, the edge set is $E \cup E_E$. Every edge in E incident with a path of length 2 containing edges of E_E.Hence the proof.

Theorem 2.10: The vertex independence number of an Edge-Vertex Splitting graph $\beta(S_{EV}(G)) = |E(G)|$.

Proof: The independent set of vertices in $S_{EV}(G)$ is V_E. These are the vertices introduced for every edge of G.Hence the proof.

3. *Edge-Edge Splitting graph*

Definition 3.1: Let $G = (V, E)$ be a simple, undirected graph. For every edge $e = uv$ connecting two vertices u, v of G, introduce an edge $e' = u'v'$ and join u to u', v to v'. Call a set of newly introduced vertices as $V' = \{u', v'/u, v \in V(G)\ and\ uv \in E(G)\}$ and a set of newly introduced edges as $E' = \{u'v', uu', vv'/\ uv \in E(G)\}$.The Edge-Edge Splitting graph is a graph on $V \cup V'$ vertex set and its edge set is $E \cup E'$. Here we replace an edge by cycle of length 4.The graph obtained from G by replacing an edge by C_4 is called Edge-Edge splitting graph of G and is denoted by $S_{EE}(G)$.

The following observations were made from definition of Edge-Edge Splitting graph.

Proposition 3.2: If G is a (p,q) graph, then $S_{EE}(G)$ is $(p+2q, q+3q)$ graph.
Proposition 3.3: In $S_{EE}(G)$, for any vertex $v \in V$, $deg(v)$ is twice its degree in $V(G)$ and degree of vertices in V' equal to 2 always.
With the similar argument as in the case of $S_{EE}(G)$,we can prove the following theorems.

Theorem3.4: If G is Planar, $S_{EE}(G)$ is Planar.

Theorem 3.5: If G is Hamiltonian, then $S_{EE}(G)$ is Hamiltonian.

Theorem 3.6: If G is Eulerian, then $S_{EE}(G)$ is Eulerian graph.

Theorem 3.7: For every graph G, the chromatic number $\chi(S_{EE}(G) = \chi(G)$.
Proof: The proper coloring of G can be extended to the proper coloring in $S_{EE}(G)$ by the following way. Let u, v be the adjacent vertices of $V(G)$ colored with colors i and j. Respected u', v' will be the vertices in V' which are at distance 1 and 2 from u. Hence coloring u' with the color j and v' with the color i, this extended proper coloring required no more new colors. Thus, $\chi(S_{EE}(G) = \chi(G)$.

Theorem 3.8: For a Edge-Edge Splitting graph, the edge chromatic number,

$$\chi_1\big(S_{EE}(G)\big) = \begin{cases} \chi_1(G) + 2, & if \quad \Delta(G) = 2. \\ \chi_1(G) + \Delta(G), & if \quad \Delta(G) > 2. \end{cases}$$

Proof: $\Delta(S_{EE}(G))$ is twice as that of G.With this observation, $\chi_1(G) + 2$ colors can applied when $\Delta(G) = 2$ and otherwise along $\chi_1(G)$ colors, an assignment of $\Delta(G)$ more colors to the edges of type uu' or vv' in $S_{EE}(G)$, one color to each edge, such that adjacent edges are of different colors achieved.

These two new classes of splitting graphs involve a replacement of an edge by cycles. Following literature properties of a graph related to cycles can be observed further on $S_{EV}(G)$,$S_{EE}(G)$. A graph is chordal if it does not contain an induced cycle of length at least four. A graph free from odd cycles will be a bipartite graph. A bipartite graph is chordal bipartite if it does not contain an induced cycle of length at least six. We record the following observations.

Observation 3.9: If G is chordal graph, then $S_{EV}(G)$ is chordal.

Observation 3.10: If G is bipartite, then $S_{EE}(G)$ is bipartite and it is chordal bipartite if and only if $G \cong C_4$or a tree or a complete bipartite graph.

4. Conclusion: We introduced two new classes of splitting graphs. Some important properties were studied. There is several extension of this work that could constitute future research related to these graphs.

References:

1. B. Basvanagoud and Roopa S.Kusugal, *On the line degree splitting graph of a graph*, Bull. Math.Sci and Appl.,18 (2017), 1-10.
2. Chartrand and Ping Zhang, *Introduction to Graph Theory*, Tata McGraw-Hill edition (2006).
3. M. Gapobianco, M. Karasinski and M. Randazzo, *On some quadratic graph equations*, Congressus Numeratium 35(1982), 149-156.
4. V.R. Kulli, M.S.Biradar, *The line-splitting graph of graph*, Acta Ciencia Indica. 28(3) (2002),317-322.
5. R.Ponraj and S.Somasundaram, *On the degree splitting graph of a graph*, Natl. Acad. Sci. Lett. 27(7-8) (2004), 275-278.
6. B.Rajendran, *Ph.D Thesis*,Madurai Kamaraj University, 1985.
7. E.Sampathkumar and H.B.Walikar, *On Splitting Graph of a Graph*, J.Karnatak Univ. Sci., 25(13) (1980), 13-16.
8. N.R.Santhi Maheswari and C.Sekar, *On the d_2-splitting graph of a graph*, Kragujevac Journal of Mathematics 36(2) (2012), 315-321.

***