GENERALISED CONTINUITY AND ITS DECOMPOSITION

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Abstract. Some new topological sets are introduced and studied. Using the new notions, decompositions of generalised continuity are obtained.

1. Introduction: Different types of generalizations of continuous functions were introduced and studied by various authors in the recent development of topology. The decomposition of continuity is one of the many problems in general topology. Recently, various decompositions of continuity have been established, for example [2, 6, 8, 9, 10, 11, 12, 13]. In this paper, we obtain decompositions of g*-continuity in topological spaces using g^*_p-continuity, g^*_α-continuity, g^*_t-continuity and $g^*_{\alpha^*}$-continuity.

2. Preliminaries: Throughout this paper, space (X, □) (or simply X) always means topological space on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a space X. The closure of A and the interior of A are denoted by cl(A) and int(A), respectively.

Definition 2.1: A subset A of a topological space X is called

1. an α -open [5] if $A\subseteq$ int(cl(int(A))).
   The complement of an α -open set is called α -closed.
   The α -closure of a subset A of X, is denoted by αcl(A), is the intersection of all α -closed sets containing A.
2. preopen [4] if $A \subseteq$ int (cl(A));
   The complement of preopen set is called preclosed.

The preclosure of a subset A of X, is denoted by pcl(A), is the intersection of all preclosed sets containing A.

Definition 2.2: A subset A of a topological space X is called g-closed [3] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X. The complement of g-closed set is called g-open set.

Definition 2.3: [7] A subset A of a topological space X is called

1. g*-closed if cl(A) $\subseteq$ U whenever A U and U is g-open in X. The complement of g*-closed set is called g*-open.
2. g_α * -closed if α cl(A) $\subseteq$ U whenever A $\subseteq$U and U is g-open in X. The complement of g_α * -closed set is called g_α * -open.
3. g^*_p-closed if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g-open in X. The complement of g^*_p-closed set is called g^*_p-open.

Definition 2.4: A subset A of a topological space X is called

1. t-set [13] if int(A) = int(cl(A)).
2. α^*-set [1] if int(A) = int(cl(int(A))).
3. *η^{**}-set [7] if A = M ∩ N, where M is g-open and N is α-closed in X.*
4. η^{**}-set [7] if A = M ∩ N, where M is g^*_α -open and N is t-set in X.

The family of all η^{**}-sets (resp.η^{**}-sets) in a space X is denoted by η^{**}(X) (resp. η^{**} (X)).

Remark 2.5.[7]: The following statements are true.
1. Every α -closed set is g^*_α -closed but not conversely.
2. Every g^*_α -closed set is g^*_p-closed but not conversely.

Remark 2.6.[1]:
1. Every t-set is an α^*-set but not conversely.
2. An open set need not be an α^*-set .
3. The union of two α^* -sets need not be an α^*-set.
4. Arbitrary intersection of α^*-sets is an α^*-set.

Definition 2.7.[7]: *A function f : X →Y is called*
1. g^*_α -continuous if for each open set V of Y, $f^{-1}(V)$ is g^*_α -open in X.
2. g^*_p-continuous if for each open set V of Y, $f^{-1}(V)$ is g^*_p -open in X.
3. η^{**}-continuous if for each open set V of Y, $f^{-1}(V) \in \eta^{**}$ (X).
4. η^{**}-continuous if for each open set V of Y, $f^{-1}(V) \in \eta^{**}$ (X).

Recently, the following decompositions are established in [7].

Theorem 2.8: A function f : X →Y is α-continuous if and only if it is both g^*_α continuous and η^{**}-continuous.

Theorem 2.9: A function f : X →Y is g^*_α -continuous if and only if it is both g^*_p -continuous and η^{**}-continuous.

3. g^*_t -Sets and $g^*_{\alpha^*}$ -Sets

Definition 3.1: A subset S of a space X is called
1. an g^*_t -set if S = M ∩ N, where M is g^*-open in X and N is a t-set in X.
2. an $g^*_{\alpha^*}$ -set if S = M ∩ N, where M is g *-open in X and N is an α^*-set in X.

The family of all g^*_t -sets (resp. $g^*_{\alpha^*}$ -sets) in a space X is denoted by g^*_t (X) (resp. $g^*_{\alpha^*}$)

Proposition 3.2: Let S be a subset of X. Then
1. if S is a t-set, then $S \in g^*_t$ (X).
2. if S is an α^*-set, then $S \in g^*_{\alpha^*}(X)$.
3. if S is an g^*-open set in X, then $S \in g^*_t$ (X) and $S \in g^*_{\alpha^*}$ (X).

Proof: The proof is obvious.

Proposition 3.3: In a space X, every g^*_t -set is an $g^*_{\alpha^*}$-set but not conversely.

Example 3.4: Let X = {a, b, c } with τ= { ϕ,{ a, b }, X }. Then {a, c } is $g^*_{\alpha^*}$-set but it is not an g^*_t -set.

Remark 3.5: The following Examples show that
1. the converse of Proposition 3.2 need not be true.
2. the concepts of g^*_t -sets and g^*_p-open sets are independent.
3. the concepts of $g^*_{\alpha^*}$ -sets and g^*_α -open sets are independent.

Example 3.6: Let X = {a, b, c} with □ = {Φ, {a}, {a, b}, X}. Then {a} is g^*_t -set but not a t-set and the set {a, b} is an -set but it is not an -set.

Example 3.7: Let X = {a, b, c} with = {Φ,{b}, {a, b}, X}. Then {a, c} is both g^*_t -set and $g^*_{\alpha^*}$ -set but it is not an g* -open set.

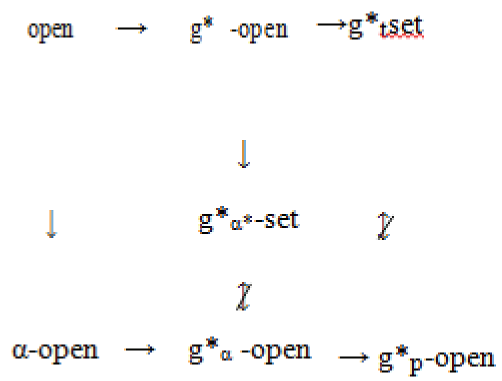
Example 3.8: In Example 3.4, the set {c} is an g^*_t -set but not a g^*_p-open set whereas the set {b, c} is a g^*_p-open set but not an g^*_t -set.

Example 3.9: Let X = {a, b, c} with □ = { Φ, {a}, {a, c}, X }. Then {b, c} is an $g^*_{\alpha^*}$ -set but not an g^*_α -open set whereas the set {a, b} is an g^*_α -open set but not an $g^*_{\alpha^*}$ -set.

Example 3.10: Let X = {a, b, c} with □={ Φ, {a} , X }. Then {a} is both $g^*_{\alpha^*}$ -set and g^*_t -set but it is not an g* -closed.

Remark 3.11: From the above discussions, we have the following diagram of implications where A → B (resp. A ↔B) represents A implies B but not conversely (resp. A and B are independent of each other).

Diagram I



Remark 3.12:
1. The union of two g^*_t -sets need not be an g^*_t set.
2. The union of two $g^*_{\alpha^*}$ sets need not be an $g^*_{\alpha^*}$ -set.

In Example 3.7, {b} and {c} are g^*_t -sets but {b}U{c} = {b, c} is not an g* -set.
In example 3.6 , {a} and {c} are *$g^*_{\alpha^*}$ -sets but {a}* U{c} = {a, c } is not an *$g^*_{\alpha^*}$ -sets*

Remark 3.13:
1. The intersection of any numbers of g^*_t -sets belongs to g^*_t (X).
2. The intersection of any numbers of $g^*_{\alpha^*}$ -sets belongs to $g^*_{\alpha^*}$ (X).

Lemma 3.14. [7]:

1. A subset S of X is g^*_α -open if and only if F ⊆ α int(S) whenever F ⊆ S and F is g-closed in X, where α int(S), denotes the α -interior of a subset S of X, is the union of all α-open subsets of X contained in S.
2. A subset S of X is g^*_p-open if and only if F⊆ pint(S) whenever F⊆ S and F is g-closed in X, where pint(S), denotes the pre interior of a subset S of X, is the union of all preopen subsets of X contained in S.
3. A subset S of X is g*-open if and only if F⊆ int(S) whenever F⊆ S and F is $g^*_{\alpha^*}$-closed in X.

Theorem 3.15: A subset S of X is g*-open in X if and only if it is both g^*_α -open and $g^*_{\alpha^*}$ -set in X.
Proof: Necessity. The proof is obvious.

Sufficiency. Let S be both g^*_α -open set and $g^*_{\alpha^*}$ -set. Since S is an $g^*_{\alpha^*}$ -set, S = A ∩ B, where A is g* -open and B is an α*-set. Assume that F ⊆ S, where F is g-closed in X. Since A is g *-open, F ⊆ int(A). Since S is g^*_α -open in X, by Lemma 3.14(3), F ⊆ int(S) = S ∩ int(cl(int(S))) = (A ∩ B) ∩ int(cl(int(A ∩ B))) ⊆ A ∩ B

∩ int(cl(int(A))) ∩int(cl(int(B))) = A∩B∩int(cl(int(A))) ∩ int(B) ⊆ int(B). Therefore, we obtain F ⊆ int(B) and hence F ⊆ int(A) ∩ int(B) = int(S). Hence S is g* -open.

Theorem 3.16: *A subset S of X is g -open in X if and only if it is both g^*_p-open and g^*_t -set in X.*
Proof: Similar to Theorem 3.15.

4. Decompositions of g* -Continuity:

Definition 4.1. A function f : X → Y is called

1. g^*_t -continuous if for each open set V of Y, $f^{-1}(V) \in g^*_t$ (X).
2. $g^*_{\alpha^*}$ -continuous if for each open set V of Y, $f^{-1}(V) \in g^*_{\alpha^*}$ (X).

Proposition 4.2. *For a function f : X → Y, the following implications hold:*

1. g *-continuity ⇨ g^*_t -continuity.
2. g*-continuity ⇨ $g^*_{\alpha^*}$ -continuity.
3. g*-continuity ⇨g -continuity ⇨ g^*_p-continuity.

The reverse implications in Proposition 4.2 are not true as shown in the following examples.

Example 4.3: Let X = Y = {a, b, c} with □= { ϕ, {b}, X} and σ = { ϕ, {b}, {a, c}, Y}.
Let f : X →Y be the identity function. Then f is g^*_t -continuous. However, f is neither g* -continuous nor g^*_p-continuous.

Example 4.4: Let X = Y = {a, b, c} with τ= { ϕ, {a}, {b}, {a, b}, X} and σ = { ϕ, {a}, {c}, {a, c}, Y}. Let f : X → Y be the identity function. Then f is g -continuous. However, f is neither $g^*_\alpha{}^*$ -continuous nor g^*_α -continuous.

The following Example 4.5 and Example 4.4 show that the concepts of $g^*_\alpha{}^*$ -continuity and g^*_α -continuity are independent.

Example 4.5: Let X = Y = {a, b, c} with = □ = {ϕ, {c},X} and σ = {ϕ , {a, c}, Y}. Let f : X → Y be the identity function. Then f is g^*_α -continuous but it is not $g^*_\alpha{}^*$ -continuous.
Examples 4.3 and 4.6 show that g_t -continuity and g_p-continuity are independent.

Example 4.6: Let X = Y = {a, b, c} with □= {ϕ , {a, b}, X} and σ = {ϕ , {a}, {b}, {a, b}, {a, c}, Y}. Let f : X → Y be the identity function. Then f is g^*_p-continuous but it is not g^*_t -continuous.

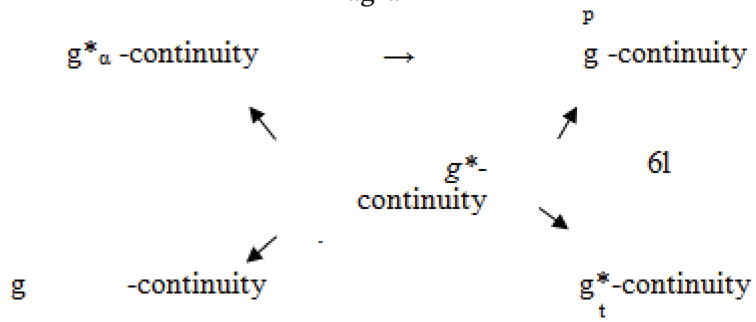
Example 4.7: Let X = Y = {a, b, c} with □ = { b},Y} . Let f : X → Y be the identity function. and g_p*continuous but it is not g*-continuous.
Φ , { b } , X } and σ = { Φ, {a, Then f is both g^*_α -continuous

Example 4.8: Let X = Y = {a, b, c} with □ = { Φ {a, b}, X} and σ = { Φ, {b}, {a, b}, {b, c}, Y}. Let f : X → Y be the identity function. Then f is g^*_p-continuous but it is not g^*_α -continuous.

Example 4.9: Let X = Y = {a, b, c} with □ = { Φ, {a, c}, X} and σ = { Φ , {a}, {a, b}, {a, c}, Y}. Let f : X →Y be the identity function. Then f is $g^*_{\alpha^*}$ -continuous but it is not g^*_t -continuous.

Remark 4.10: From the above discussions, we have the following diagram of im-plications. where A→ B (resp. A↔ B) represents A implies B but not conversely (resp. A and B are independent of each other).

Diagram II



Theorem 4.11: *A function f :* X→Y is g* -continuous if and only if it is both g^*_α -continuous and $g^*_{\alpha^*}$-continuous.
Proof: The proof follows immediately from Theorem 3.15.

Corollary 4.12: A function f : X → Y is g*-continuous if and only if it is g_p-continuous, g^*_p-continuous and g^*_t -continuous.
Proof: It follows from Theorems 2.9 and 4.11.

Theorem 4.13: A function f : X → Y is g* -continuous if and only if it is both g_{p^*}-continuous and g^*_t -continuous.
Proof: The proof follows immediately from Theorem 3.16.

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