ON MULTIGRANULAR ROUGH MULTISET

M. Gilbert Rani
Assistant Professor, Arul Anandar College, Karumathur

Abstract: After the identification of fuzzy set theory many researchers introduced so many sets as the modern mathematical tools to handle uncertain and imprecise data in problem solving at so many fields. Multigranular computing helps to partition the given data into the granules like classes, objects, clusters based on its indistinguishability, similarity and proximity relations. Rough set is a unigranular computing tool. Multigranular rough set was introduced to solve the complex problem in clustering, data mining and etc,. In this paper let us introduce multigranular rough multiset and investigate its properties

Keywords: Multigranulation Rough Set, Multi Set, Multigranular Rough Multiset, Rough Set.

Introduction: Roughest is an initiated by pawlak [13] which depends on a single equivalence relation so it supports an ungranulation with the multi equivalence relation the original roughtset theory has been extended to roughest model based on multigranulation. Multigranulation rough set introduced by Qianyetal [2]. Many of the researchers in computer science department strengthen the same concept . Multiset is a set with a number of occurrence of the elements existing in it. Multiset firstly counted by cerfet.al[8] in 1971. The multiset concept was validated in by manner Blizard. The combination of roughest and multiset put together in a single form called as rough multiset, it was deposited by K.P. Girishetal [1]. After that so many innovators in computer and mathe- matical science gave their own shares to rougmultiset. In this paper we introduce the combination of multigranulation rough set and roughmultiset, nothing but multigranulation rough multiset.

The rest of the paper deals the following: section 2 encapsulates the basic definition and needed results that help to understand the forth coming sections, section 3 opens its account with the introduction of multigranular rough multisets. Section 4 declares the conclusion of this paper.

Preliminary Ideas: In this section we recollect the needed definition and notation.
Definition2.1 [4] a multiset M drawn from the set X is represented by a function count M or C_M defined as C_M:X→N where N represents the set of non-negativeintergers.
Definition 2.2 [4] : A domain X is defined as a set of elements from which multisets are constructed. The multiset space $[X]^n$is the set of all multisets whose elements are in X such that no element in the multisets occurs more than times.
Definition 2.3 [4] : Let M_1 and M_2 be two msets drawn from a set X_1 then the Cartesian product of M_1 and M_2 is defined as
$M_1 X M_2 = \{(m/x, n/y) / mn : x \in^m M_1 , y \in^n M_2\}$,
Here the entry (m/x,n/y) /mn in M_1 XM_2 denotes x is repeated m times in M_1,y is repeated n times in M_2 and the pair (x, y) is repeated mn times in M_1 x M_2
Definition 2.4 [4]: A sub m set R of $M_1 X M_2$ is said to be an m set relation from M_1 to M_2 if every member ($^m/_x, ^n/_y$) of R has a count, the product of $C_1(x,y) and C_2(x,y)$. $^m/_x\, relatedto\, ^n/_y$is denoted by$m/x\ R\ n/y$.
Definition2.5 [1] : Let R be an m set relation on M. The post m- set of $x \in^m M$ is defined as

$$(m/x)R = \{n/y : there\, exists\, some\, k\, with (k/x)R(n/y)\}$$

and the pre- mset of $x \in^r M$ is defined as

$$R(r/x) = \{p/y : there\, exits\, some\, q\, with (p/y)R(q/y)\}$$

Definition 2.6 [1]: Let R be any binary m set relation on M in $[x]^m$. The m set $< ^n/_y >$ R is defined as the intersection of all post- msets containing y with nonzero multiplicity.

$$i.e., < n/y > R = \cap\{(m/x)R : y \in^n (m/x)R\}$$

Also, $R<n/y>$is the intersection of all
pre-msets containing y with nonzero multiplicity.

$$R<n/y>=\cap\{R(m/x): y\in^{n} R(m/x)\}$$

Definition 2.7 [16] : Let *K = (U, R)* be a knowledge base, *R* be a family of equivalence relations, $X\subseteq U$, and $P,Q\in R$ we define the lower approximation and upper approximation of X in U as

$$\underline{X}_{(P+Q)}=\{x:[x]_P\subseteq X \text{ or } [x]_Q\subseteq X\}$$

$$\overline{X}(P+Q)=(\underline{X^c}_{(P+Q)})^c$$

Multigranulation rough multiset: In this section let us see the new notion of rough set that is multigranulation rough multiset. As mentioned earlier, instead of a single equivalence class here we put more than one equivalence classes and at the same time multiset concept also merged here.

Definition 3.1:-Let *K = (M, R)* be a knowledge base, where M is a M set and R is an equivalence mset relation. Let X $\subseteq$ M, P, Q $\subseteq$R. Let $[m/x]p$ or $[m/x]Q$be the P- relative and Q – relative mset respectively. For any X$\subseteq$ M a pair of lower and upper m set approximations.

$$\underline{X}_{M(P+Q)}=\{m/x:[m/x]_P\subseteq X \text{ or } [m/x]_Q\subseteq X\}$$

$$\overline{X}P+Q=(\underline{X^c}_{M(P+Q)})^c$$

Example 3.2:
Let *K = (M,R)* be knowledge base.
Let M={3/x,9/y,7/z,11/r} be a m- set.
Let R be equivalence class and
R ={(3/x , 3/x)/ 9, (9/y , 9/y)/ 81
(7/z , 7/z)/ 49, (11/r, 11/r)/121,
(3/x , 9/y)/ 27, (3/x , 7/z)/ 21,
(3/x , 11/r)/ 33, (9/y , 3/x)/27, (7/z , 3/x)/ 21, (11/r , 3/x)/ 33}
P ={(3/x , 3/x)/ 9, (9/y , 9/y)/ 81, (3/x , 9/y)/ 27, (9/y , 3/x)/ 27, (3/x , 7/z)/ 21, (7/z , 3/x)/ 21}
Q={(7/z , 7/z)/49, (11/r , 11/r)/121, (3/x , 11/r)/33 , (11/r , 3/x)/33, (3/x , 3/x)/33}
Let X, Y $\subseteq M$
X = {3/x , 9/y , 7/z}
[3/x]P = {3/x , 9/y , 7/z}
[9/y]P = {9/y , 3/x},
[7/z]P = {7/z , 3/x}
[11/r]P =∅
[3/x]Q = {11/r}
[9/y]Q = ∅, [7/z]Q={7/z}
[11/r]Q = {$\frac{11}{r}$, $\frac{3}{x}$} =∅
Let X= {3/x , 9/y , 11/r},
Y = {7/z , 11/r}

$$\underline{X}_{m(P+Q)}=\{3/x,9/y,7/z\},\overline{X}^{m(P+Q)}=\{3/x,9/y\}$$

$$\underline{Y}_{m(P+Q)}=\{3/x,7/z\},\overline{Y}^{m(P+Q)}=\{3/x,7/z\}$$

Here $(\underline{X}_{m(P+Q)},\overline{X}^{m(P+Q)})$ is a multigranular rough multiset but not
$(\underline{Y}_{m(P+Q)},\overline{Y}^{m(P+Q)})$

The boundary region of A is defined as $B_{m(P+Q)}(A)=\overline{Y}^{m(P+Q)}-\underline{Y}_{m(P+Q)}$

The accuracy measure of the approximations is given by

$$\alpha(P+Q,A)=\frac{|\underline{X}_{m(P+Q)}|}{|\overline{X}^{m(P+Q)}|}$$

Properties of Approximations:
Let X and Y be two subsets of M. Then

$$(i)\underline{\phi}_{m(P+Q)}=\phi=\overline{\phi}^{m(P+Q)}$$

$$(ii)\underline{M}_{m(P+Q)}=M=\overline{M}^{m(P+Q)}$$

$$(iii)\underline{X}_{m(P+Q)}\subseteq X\subseteq\overline{X}^{m(P+Q)}$$

$$(iv)\underline{X}_{m(P+Q)}=(\overline{X^{c}}^{m(P+Q)})^{c}$$

$$(v)\overline{X}^{m(P+Q)}=(\underline{X^{c}}_{m(P+Q)})^{c}$$

$$(vi)X\subseteq Y\Rightarrow\underline{X}_{m(P+Q)}\subseteq\underline{Y}_{m(P+Q)}$$

$$(vii)X\subseteq Y\Rightarrow\overline{X}^{m(P+Q)}\subseteq\overline{Y}^{m(P+Q)}$$

$$(viii)\underline{X\cap Y}_{m(P+Q)}\subseteq\underline{X}_{m(P+Q)}\cap\underline{Y}_{m(P+Q)}$$

$$(ix)\underline{X\cup Y}_{m(P+Q)}\supseteq\underline{X}_{m(P+Q)}\cup\underline{Y}_{m(P+Q)}$$

$$(x)\overline{X\cap Y}^{m(P+Q)}\subseteq\overline{X}^{m(P+Q)}\cap\overline{Y}^{m(P+Q)}$$

$$(xi)\overline{X\cup Y}^{m(P+Q)}\supseteq\overline{X}^{m(P+Q)}\cup\overline{Y}^{m(P+Q)}$$

Proof : From the definitions.

Example 3.3: Consider example 3.2 again. Here we give counter examples for inequalities.

$$X\cap Y=\{11/R\},\underline{X\cap Y}_{m(P+Q)}=\phi$$

$$\underline{X}_{m(P+Q)}\cap\underline{Y}_{m(P+Q)}=\{3/x,7/z\},$$

$$\overline{X\cap Y}^{m(P+Q)}=\phi,\overline{X}^{m(P+Q)}\cap\overline{Y}^{m(P+Q)}=\{3/x\},$$

And

$$\underline{X\cup Y}m(P+Q)=M,\underline{X}_{m(P+Q)}\cup\underline{Y}_{m(P+Q)}=\{3/x,9/y,7/z\}$$

Proposition 3.4: Let M be a mset& R be an equivalence relation $P,Q\in R$ .Then,

$$(i)\underline{(\underline{X}_{m(P+Q)})}_{m(P+Q)}\supseteq\underline{X}_{m(P+Q)}$$

$$(ii)X\subseteq\overline{(\underline{X}_{m(P+Q)})}^{m(P+Q)}$$

$$(iii)\overline{X}^{m(P+Q)}\subseteq\underline{(\overline{X}^{m(P+Q)})}_{m(P+Q)}$$

Example 3.5 Let us consider mset of balls(objects)

$$M=\{7/b_1,5/b_2,6/b_3,7/b_4,4/b_5,6/b_6,4/b_7,3/b_8\}$$

Which are of different colours (attributes){black,brown,white}, different types{basket ball, base ball, soccer ball} and differenttypes (attributes){costly,reasonable,cheap}. Thus , the information system *S=(M,A)* consists of $\sum_{i=1}^{8}k_i$ objects and 6 attributes. The given information table is

balls	Balls types	color	price	No.of items
b1	soccer	white	cheap	K1
b2	basket	brown	reasonable	K2
b3	soccer	white	costly	K3
b4	base	brown	reasonable	K4
b5	basket	black	cheap	K5
b6	base	white	resonable	K6
b7	basket	brown	cheap	K7
b8	base	white	reasonable	K8

From this information system we take A1={white soccer, brown basket}, A2={reasonable},then we have the following table,

balls	P	Q	D	C
b1	1	0	accept	K1
b2	0	1	accept	K2
b3	1	0	No	K3
b4	0	1	Accept	K4
b5	0	0	No	K5
b6	0	1	no	K6
b7	1	0	yes	K7
b8	0	1	yes	K8

From this
P={{k1/b1,k3,/b3,k7/b7},{k2/b2,k4/b4,k5/b5,
k6/b6,k8/b8}} and
Q={{k2/b2,k4/b4,k6/b6,k8/b8},{k1/b1,k3/b3,
k5/b5,k7/b7,k9/b9}}.
Then we find the lower and upper approximations using the respective formulae. Using generalized rules we can generate the rules using these approximations.
Conclusion: In this we introduced multigranular rough multiset and its application. In future we will give the brief algorithm to solve the problems in decision making using multigrannular rough multiset.

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