ESTIMATION OF RESERVES OF LIMESTONE THROUGH KRIGING: USING THE BEST LINEAR UNBAISED ESTIMATION (BLUE)

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Abstract: Limestone deposit exhibit wide spatial variations both vertically and horizontally The erratic nature of the deposit makes the conventional method inadequate .The limestone deposits at Tummalapenta area belongs to Narji Limestone of kurnool Group.

The study area is situated 15 Km northwest of Tadpatri, Ananthapur district of Andhra Pradesh ,lie between15° 13' 03" and 15° 02'59" north latitudes and 78° 01'31" and 78° 01'22" east longitudes fall in the Topo sheet No 57J/6 .The average thickness of the litho units encountered is calcareous shale 2,5m,light grey limestone 3.5m grey limestone 4m ,and flaggy limestone 5m .Data from 156 drill holes ,covering 2500sq.km have been taken for this study. Variogram calculation ,modeling ad their validation ,kriging calculation of reserves ,classification of reserves and preparation of ore grade map have been employed for reserve estimation of these limestone to provide the best linear unbiased estimations (BLUE) .

Keywords: Geostatistics, Variogram, Estimation, Ordinary Kriging, Indicator Kriging.

1. Introduction: The cement grade limestone has been known in India since ages. It finds mention by different names in our scriptures dated as far back as mythological times. As a proof that limestone was produced in the country in those times there are evidences of numerous ancient works for limestone. The constituents of raw materials, such as calcium oxide (CaO), alumina (Al_2O_3), ferric oxide (Fe_2O_3), and silica (SiO_2), supplied to the plant be uniform or within permissible limits of variation. SO_3 has an adverse effect on kiln properties. As the SO_3 increases, more heat is required to run the kiln. Another important parameter that makes a cement plant cost effective is the lime saturation factor (LSF): a ratio of CaO and weighted sum of alumina, silica and ferric oxide. The LSF plays a vital role for cement production because it contains CaO, the primary constituent of cement. It has been found that kiln operation and cement quality is improved where the CaO in limestone is more than 44%. Thus, both high LSF and a low SO_3 contribute significantly to the cost-effectiveness of a cement plant. (Ingram and Daugherty, 1991)ES.

2. Kriging: Kriging is a term coined by G. Matheron in 1963 after the name of D.G. Krige. Kriging is based on a statistical model of a phenomenon instead of an interpolating function. It uses a model for a spatial continuity in the interpolation of unknown values based on values at neighboring points (Sunila et al., 2004). Kriging is called an optimal method because the interpolation weights are chosen to provide for the value at a given point the Best Linear Unbiased Estimate (BLUE)

There are several kriging techniques for different purposes such as ordinary kriging, simple kriging, universal kriging, indicator kriging, cokriging, point kriging, block kriging, disjunctive kriging, bayesian kriging and so on. In this research, the authors would like to focus on ordinary kriging as it is by far the most common type of kriging in practice.

Ordinary kriging is a variation of the interpolation technique, one that implicitly estimates the first order component of the data and compensates for this accordingly. This technique enables interpolation without the necessity of explicitly knowing the first order component of the data a priori (GIS dictionary, 1999). The basic equation used in ordinary kriging is as follows
$Z_v^* = \sum \lambda_i Z(x_i)$

Let Z be a random variable having N data values $Z(x_i)$, i=1 to N. The kriging estimate, denoted Z_v^*, over a volume V is defined as a linear combination of the neighboring information $Z(x_i)$, introducing the corresponding weights λ_i(Armstrong, 1998).

Review of Literature:

1. Delfiner, P. 1979. Basic introduction to geo statistics. Centre de Geostatistique (Fontainebleau) course CGMM-C78.
2. Demirel I.H, Sarac C, and Sen O, 2000, Geostatistical Reserve Estimation: A Case Study in the Canakci Coal Seam of Ermenek Basin, Turkey, *Energy Sources*, 22:925–933.

2.1. Types of Kriging: Depending on the stochastic properties of the random field different types of kriging apply. The type of kriging determines the linear constraint on the weights λ_i implied by the unbiasedness condition; i.e. the linear constraint, and hence the method for calculating the weights, depends upon the type of kriging.
Classical types of kriging are:

- Simple kriging assumes a known constant trend: $\mu(x) = 0$.
- Ordinary kriging assumes an unknown constant trend: $\mu(x) = \mu$.
- Universal kriging assumes a general linear trend model.
- Indicator kriging uses indicator functions instead of the process itself, in order to estimate transition probabilities.
- Disjunctive kriging is a nonlinear generalization of kriging.
- Lognormal kriging interpolates positive data by means of logarithms.

2.2 Advantages of Kriging:

- The estimation technique is tailored by our interpretation of the geological environment as quantified in the semi-variogram model.
- The standard error provides estimation of how good or bad the estimator is.
- It combines the variance –distance relationship between or amongst the samples.
- It accommodates clustering and poor distribution of samples.
- Kriging automatically compensates for distribution and optimizes the weights that various samples get as well as ensuring that the weights add up to one.
- It allows for size and shape of the samples.
- It honors the data points because the locations of all points are known.

2.3 Disadvantages of Kriging:

- The estimation method is tailored by or interpretation of the geological environment and as quantified in the semi-variogram model. There is no way of determining whether the semi-variogram is right or not.
- You can only use it if there is no trend.
- The regression effect is present so long as you use a weighted average the high grades are over-estimated and the low grades are under-estimated.
- Highly skewed data causes problems.
- Smoothed the data.

3. Case Study Area: The study area, Tummalapenta limestone, is a part of sedimentary Cuddapah Basin. The Tummalapenta mine is situated 15km towards north-west of Tadpatri, Anantapur district of Andhra Pradesh, which lies between the north latitudes 15°03' 03'' & 15°02' 59''and east longitudes 78° 01' 31'' & 78° 01' 22'' and cover in the Survey of India toposheet No: 57J/6.

4. Cross-Validation: Cross validation is a method of validating the choice of a variogram model. Some of the data points are deleted and then using the selected variogram model these deleted points are estimated. True and estimated values are then compared in such a way that the model can be accepted or rejected. The main difficulty with any estimation technique is that it is almost impossible to see if the estimated value and the actual value are close to one another or not. For the cross-validation of a model to confirm the, goodness of fit, the average error statistic must be approximately equal to zero and the standard deviation error must be approximately equal to one.

Figures 1 shows the cross validation test to validate variogram models of SO_3 variables of the deposit. The base map (upper left corner) shows locations of the tested data points. The pattern dimension is proportional to the value of the true value Z. The scatter plot (upper right corner) is of the true values Z against the estimated values Z^*.This should be closed to the first bisector. The histogram (lower left corner) is of the standardized estimation errors and it gives an idea of the unbiasedness (median of the histogram) and the quality of the estimate. It also helps locating the outliers which are outside the two vertical lines corresponding to the threshold value. The scatter plot (lower right corner) is of the standardized estimation errors versus the estimated value Z^*. Since the two variables are theoretically independent. This should have no preferential shape. The threshold value (applied on the standardized estimation error) is represented as two horizontal lines.

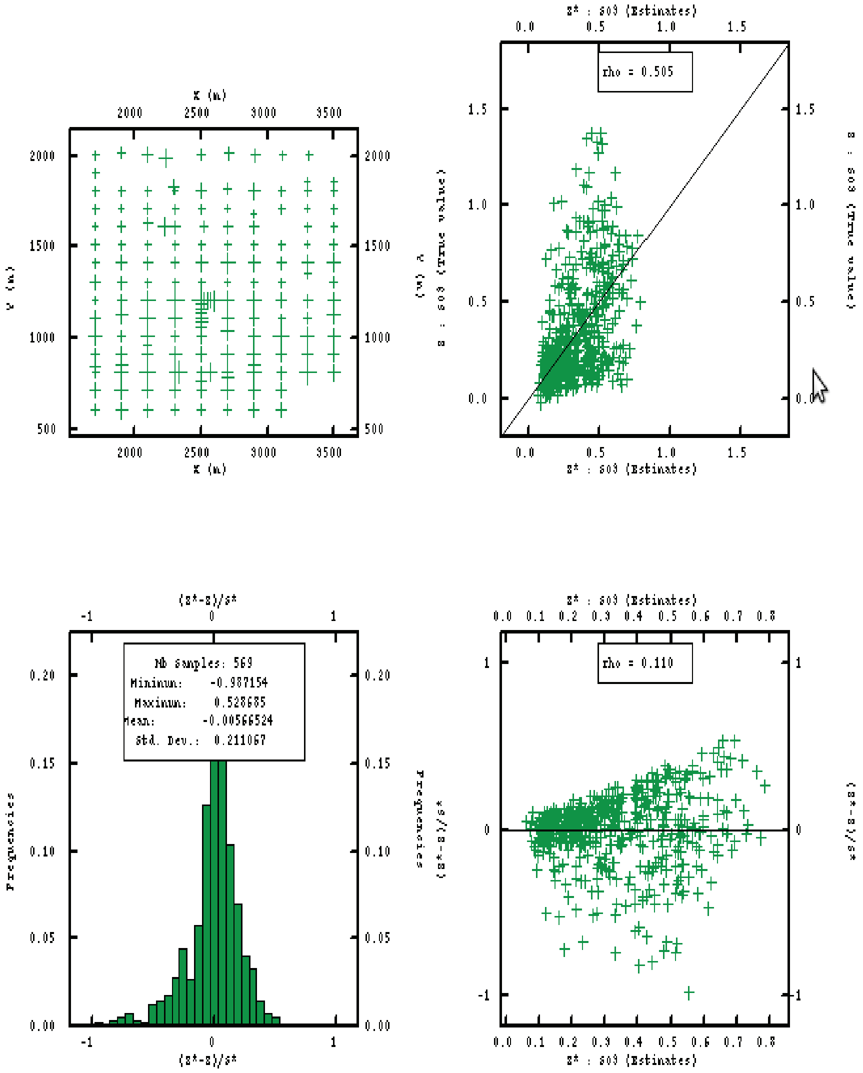

Figure 1: Cross Validation Test for Variogram Model of SO_3 Variable

4. 1 Estimation by Ordinary Kriging: The variables LSF, SO_3 are estimated by ordinary kriging using above fitted variogram models. The kriging was done for all the eight benches covering maximum.

The distribution of kriged values of LSF and SO_3 are presented in Figures. The distribution of high grade values of LSF along with low grade values of SO_3 can found in the part of the 343m (1^{st}) bench. The low grade values of LSF along with high grades values of SO_3 can found in the part of the 343m (1^{st}) bench. In the 2^{nd} bench the high grade values of SO_3 are present in the southern part of the study area. The high grade values of SO_3 extend from southern part to northeastern part of the area from fourth to bench.

4.2 Ore Reserve Estimation: For the purpose of estimating reserves, the area was divided into 2400 blocks of 100x100x 9 m each. The 100 x 100 x 9 m blocks were considered to be the selective mining unit on which the production was carried out. The neighborhood for kriging is range of the variogram for X, Y directions and 8m for Z direction. Each block is estimated using 8 nearest samples, after discretizing it into 25 sub blocks. After applying cutoff grades, the total volume was calculated by multiplying thickness with area of the blocks. The bulk density of limestone was taken as 2.5 tons/m^3 and in this case the total tonnage was determined for various cutoff grades is presented.

Tonnage (MT) at various cutoff grades of LSF and SO_3

LSF/SO_3	0 - 0.15	0.15 - 0.25	0.25 - 0.5	0.5 - Ceiling
90 - 110	24.3	23.18	63.68	53.55
90 - 120	36.68	39.6	104.18	76.73
120 - 125	7.65	12.82	17.1	9.68
120 - Ceiling	40.28	49.72	68.4	20.03
90 - Ceiling	76.95	89.33	172.35	96.75

5 .**Result and Conclusion**: LSF has strong positive correlation with CaO, and strong negative correlation with SiO_2 and Al_2O_3. Fe_2O_3 and SO_3does not show good correlation with all other variables irrespective of positive or negative. A high negative correlation between CaO and Al2O3 is observed if cut-off grade of CaO is 45%, and beyond this cut-off value there not much correlation is present. The distribution of high grade values of LSF along with low grade values of SO_3 can found in the part of the 343m (1st) bench. The low grade values of LSF along with high grades values of SO_3 can found in the part of the 343m (1st) bench. In the 2nd bench the high grade values of SO_3 are present in the southern part of the study area. The high grade values of SO_3 extend from southern part to northeastern part of the area from third to eight benches. The ore reserves are classified as proved, probable, possible and inferred based on the relative kriging estimation error at 90% confidence level. Most of the blocks (99%) estimated are inferred as proved and probable.

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