NEW CLASSES OF NANO CONTRA OPEN FUNCTIONS

M. Lellis Thivagar
School of Mathematics/Professor & Chairperson/Madurai Kamaraj University,
Madurai, Tamilnadu,India
V.Sutha Devi
School of Mathematics/Research Scholar/Madurai Kamaraj University,
Madurai, Tamilnadu, India
V. Antonysamy
School of Mathematics/Research Scholar/Madurai Kamaraj University,
Madurai, Tamilnadu, India

Abstract: The purpose of this paper is to introduce the notion of nano contra open and nano contra closed functions in nano topological spaces. We also show that many results for nano contra open and nano contra closed functions can be extended to several weakly nano contra open and weakly contra closed functions. An attempt is made to compare these functions with the existing functions.

Keywords: Nano Topology, Nano Contra Open, Nano Weakly Contra Open.

Introduction: Ganster and Reily introduced and studied notion of LC-Continuous functions. Dontchev [2] presented a new notion of continuous function called Contra continuity, a stronger form of LC-Continuity. Contra open and contra closed functions are introduced by Baker in 1997. These new forms are used to extend several results in the literature. Nano topology explored by Lellis Thivagar et.al. can be described as a collection of nano approximations, a non-empty finite universe and empty set for which equivalence classes are buliding blocks. This is named as Nano topology, because of its size and what ever may be the size of universe it has atmost five elements in it. The elements of Nano topology are called the Nano open sets. He also continued to expose certain forms of nano contra continuity and nano Bi-contra continuity. In this paper we try to introduce the notion of nano contra open and nano contra closed functions in nano topological spaces. We also show that the many results for nano contra open and nano contra closed functions can be extended to several weakly nano contra open and weakly contra closed functions. An attempt is made to compare these functions with the existing functions.

Preliminaries: ***The following recalls necessary concepts and preliminaries required in the sequel of our work.***

Definition 2.1[4]: Let $\mathcal{U}$ be a non-empty finite set of objects called the universe $\mathcal{R}$ be an equivalence relation on $\mathcal{U}$ named as the indiscerniblity relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair $(\mathcal{U}, R)$ is said to be the approximation space. Let $X \subseteq \mathcal{U}$.

(i) The Lower appproximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is,

$L_R(X) = \{\bigcup_{x\in\mathcal{U}} \{R(x): R(x) \subseteq X\}\}$,

where R(x) denotes the equivalence class determined by x.

(ii) The Upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by

$$U_R(X) = \left\{\bigcup_{X\in\mathcal{U}} \{R(x): R(x) \cap X \neq \emptyset\}\right\}.$$

(iii) The Boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not -X with respect to R and it is denoted by $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2[4]: Let $\mathcal{U}$ be the universe, R be an equivalence relation on $\mathcal{U}$ and $\tau_R(X) = \{\mathcal{U}, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq \mathcal{U}$ and $\tau_R(X)$satisfies the following axioms.
(i) $\mathcal{U}$ and $\emptyset \in \tau_R(X)$.
(ii) The union of the elements of any subcollection$\tau_R(X)$ is in $\tau_R(X)$.
(iii) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.
That is, $\tau_R(X)$ forms a topology $\mathcal{U}$ called as the nano topology on $\mathcal{U}$ with respect to X. We call $(\mathcal{U}, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets. A set A is said to be nano closed if its complement is nano open.

Definition 2.3[4]: If $(\mathcal{U}, \tau_R(X))$ is a nano topological space with respect to X where $X \subseteq \mathcal{U}$ and if $A \subseteq \mathcal{U}$, then nano interior of A is defined as the union of all nano open subsets contained in A and its denoted by $\mathcal{N}Int(A)$. That is $\mathcal{N}Int(A)$ is the largest nano open subset contained in A.
The nano closure of A is defined as the intersection of all nano closed sets containing A and its denoted by $\mathcal{N}Cl(A)$. That is, $\mathcal{N}Cl(A)$ is the smallest nano closed set containing A.

Definition 2.4[6]: Let $(\mathcal{U}, \tau_R(X))$ and $(\mathcal{V}, \tau_{R'}(Y))$ be nano topological spaces.Then a mapping f: $(\mathcal{U}, \tau_R(X)) \to (\mathcal{V}, \tau_{R'}(Y))$ is nano continuous on $\mathcal{U}$ if the inverse image of every nano open set in $\mathcal{V}$ is nano open in $\mathcal{U}$.

Throughout this paper, $\mathcal{U}$ and $\mathcal{V}$ are non empty finite universes, $X \subseteq \mathcal{U}$ and $Y \subseteq \mathcal{V}$ and where R and R'are equivalence relations on $\mathcal{U}$ and $\mathcal{V}$ respectively.
$(\mathcal{U}, \tau_R(X))$ and $(\mathcal{V}, \tau_{R'}(Y))$ are the nano topological space with respect to X and Y respectively.

Nano Contra Open Functions: In this section we define nano contra open, nano slightly open and nano weakly closed functions and its characterisations were studied.
Definition 3.1: A function $f: \mathcal{U} \to \mathcal{V}$ is said to be nano contra open (resp. nano contra closed) if f(U) is closed (resp. open) for every open (resp. closed) subset U of $\mathcal{U}$.
Definition 3.2: A function $f: \mathcal{U} \to \mathcal{V}$ is said to be slightly nano open (resp. slightly nano closed) provided that, whenever A is nano clopen in $\mathcal{U}$, $f(A)$ is nano open in (resp. nano closed) in $\mathcal{V}$.

Definition 3.3: A function $f: \mathcal{U} \to \mathcal{V}$ is said to be weakly nano closed if $\mathcal{N}cl(f(U)) \subseteq f(\mathcal{N}cl(U))$ for every nano open set U in $\mathcal{U}$.
Example 3.4: Let $\mathcal{U} = \{a, b, c\}$ with $\mathcal{U}/R = \{\{a\}, \{b, c\}\}$ and let
$X = \{a, b\} \subseteq \mathcal{U}$, $\tau_R(X) = \{\mathcal{U}, \emptyset, \{a\}, \{b, c\}$. Let $\mathcal{V} = \{u, v, w\}$ with $\mathcal{V}/R' = \{\{u\}, \{v, w\}\}$ and $Y = \{u, v\} \subseteq \mathcal{V}$, $\tau_{R'}(Y) = \{\mathcal{V}, \emptyset, \{u\}, \{v, w\}$. Define $f: \mathcal{U} \to \mathcal{V}$ as $f(a) = u, f(b) = v, f(c) = w$. Here f is both nano contra open and slightly nano open.
Definition 3.5: A function $f: \mathcal{U} \to \mathcal{V}$ is said to be weakly nano contra open provided that for, every nano open subset U of $\mathcal{U}$ and every nano closed subset A of $\mathcal{U}$ with $A \subseteq U$, we have $\mathcal{N}cl(f(A)) \subseteq f(U)$.

Theorem 3.6: If $f: \mathcal{U} \to \mathcal{V}$ nano contra open then f is weakly nano contra open.
Proof: Assume $f: \mathcal{U} \to \mathcal{V}$ is nano contra open and let $A \subseteq U \subseteq \mathcal{U}$, where A is nano closed in $\mathcal{U}$ and U is nano open in $\mathcal{U}$. Then, since f(U) is nano closed, $\mathcal{N}cl(f(A)) \subseteq \mathcal{N}cl(f(U)) = f(U)$, which proves that f is weakly nano contra open.
Remark 3.7: The converse of the above theorem need not be true, which can be shown by the following example.

Example 3.8: Let $\mathcal{U} = \{a, b, c, d\}$ with $\mathcal{U}/R = \{\{a\}, \{b, c\}, \{d\}\}$ and let $X = \{a, c\} \subseteq \mathcal{U}$, $\tau_R(X) = \{\mathcal{U}, \emptyset, \{a\}, \{a, b, c\}, \{b, c\}$,$[\tau_R(X)]^c = \{\{\mathcal{U}, \emptyset, \{b, c, d\}, \{d\}, \{a, d\}\}$ Let $\mathcal{V} = \{x, y, z, w\}$ with $\mathcal{V}/R' = \{\{x\}, \{y\}, \{z, w\}\}$ and $Y = \{x, z\} \subseteq \mathcal{V}$, $\tau_{R'}(Y) = \{\mathcal{V}, \emptyset, \{x\}, \{x, z, w\}, \{z, w\}$,$[\tau_{R'}(Y)]^c = \{\{\mathcal{V}, \emptyset, \{y, z, w\}, \{y\}, \{x, y\}\}$ Define $f: \mathcal{U} \to \mathcal{V}$ as $f(a) = x, f(b) = z, f(c) = w \text{ and } f(d) = y$. Here f is weakly nano contra open but not nano contra open, since $f\{a\} = \{x\}, f\{a, b, c\} = \{x, z, w\}, f\{b, c\} = \{z, w\}$ are not nano closed in $\mathcal{V}$.

Theorem 3.9: If $f: \mathcal{U} \to \mathcal{V}$ is weakly nano contra open, then f is slightly nano closed.
Proof: Assume U is a nano clopen subset of $\mathcal{U}$. Then, since f is weakly nano contra open, $\mathcal{N}cl(f(U)) \subseteq f(U)$, which proves that f(U) is nano closed and that f is slightly nano closed.

Theorem 3.10: If the function $f: \mathcal{U} \to \mathcal{V}$ is nano closed, then f is weakly nano contra open.
Proof: Assume $f: \mathcal{U} \to \mathcal{V}$ is nano closed and let $A \subseteq U \subseteq \mathcal{U}$, where A is nano closed in $\mathcal{U}$ and U is nano open in $\mathcal{U}$. Since f(A) is nano closed, $\mathcal{N}cl(f(A)) = f(A) \subseteq f(U)$ and hence f is weakly nano contra open.
Remark 3.11: The converse of the above theorem need not be true as shown in the following example.

Example 3.12: Let $\mathcal{U} = \{a, b, c, d\}$ with $\mathcal{U}/R = \{\{a, d\}, \{b\}, \{c\}\}$ and let $X = \{a, c\} \subseteq \mathcal{U}$, $\tau_R(X) = \{\mathcal{U}, \emptyset, \{c\}, \{a, c, d\}, \{a, d\}$, $[\tau_R(X)]^c = \{\{\mathcal{U}, \emptyset, \{a, b, d\}, \{b\}, \{b, c\}\}$. Let $\mathcal{V} = \{x, y, z, w\}$ with $\mathcal{V}/R' = \{\{x\}, \{y\}, \{z\}, \{w\}\}$ and $Y = \{x, w\} \subseteq \mathcal{V}$, $\tau_{R'}(Y) = \{\mathcal{V}, \emptyset, \{x, w\}$, $[\tau_{R'}(Y)]^c = \{\{\mathcal{V}, \emptyset, \{y, z\}\}$ Define $f: \mathcal{U} \to \mathcal{V}$ as $f(a) = x, f(b) = y, f(c) = z \text{ and } f(d) = w$. Here f is weakly nano contra open but not nano closed.
Remark 3.13: A function $f: \mathcal{U} \to \mathcal{V}$ which is weakly nano contra open are also nano closed under some conditions is given in the following theorem.

Theorem 3.14: If the function $f: \mathcal{U} \to \mathcal{V}$ is weakly nano contra open and if for every nano closed subset F of $\mathcal{U}$ and every $y \in \mathcal{V}$ such that $f^{-1}(y) \subseteq \mathcal{U} - F$ there exists an nano open set U in $\mathcal{U}$ such that $F \subseteq U$ and $f^{-1}(y) \cap U = \emptyset$, then f is nano closed.
Proof: Let F be a nano closed set in $\mathcal{U}$ and suppose $y \notin f(F)$. Then we have $f^{-1}(y) \subseteq \mathcal{U} - F$. Hence there exists an nano open set U in $\mathcal{U}$ such that $F \subseteq U$ and $f^{-1}(y) \cap U = \emptyset$. Since f is weakly nano contra open, $\mathcal{N}cl(f(F)) \subseteq f(U)$. Since $f^{-1}(y) \cap U = \emptyset$, $y \notin f(U)$ and hence $y \notin \mathcal{N}cl(f(F))$. Thus $\mathcal{N}cl(f(F)) = f(F)$ and $f(F)$ is nano closed.

Weakly Nano Contra Closed Functions: In this section we define weakly nano contra closed functions and its properties were dealt.
Definition 4.1: A function $f: \mathcal{U} \to \mathcal{V}$ is said to be weakly nano contra open provided that for, every nano open subset U of $\mathcal{U}$ and every nano closed subset A of $\mathcal{U}$ with $A \subseteq U$, we have $f(A) \subseteq \mathcal{N}int(f(U))$.

Theorem 4.2: If $f: \mathcal{U} \to \mathcal{V}$ is nano contra closed, then f is weakly nano contra closed.
Proof: Assume $f: \mathcal{U} \to \mathcal{V}$ is nano contra closed and let $A \subseteq U \subseteq \mathcal{U}$, where A is nano closed in $\mathcal{U}$ and U is nano open in $\mathcal{U}$. Since f(A) is nano open in $\mathcal{V}$, $f(A) = \mathcal{N}int(f(A)) \subseteq \mathcal{N}int(f(U))$. Hence f is weakly nano contra closed.

Theorem 4.3: If $f: \mathcal{U} \to \mathcal{V}$ is nano open, then f is weakly nano contra closed.
Proof: Assume $f: \mathcal{U} \to \mathcal{V}$ is nano open and let $A \subseteq U \subseteq \mathcal{U}$, where A is nano closed in $\mathcal{U}$ and U is nano open in $\mathcal{U}$. Since f(U) is nano open, $f(A) \subseteq f(U) = \mathcal{N}int(f(U))$. Hence f is weakly nano contra closed.

Theorem 4.4: If $f: \mathcal{U} \to \mathcal{V}$ is weakly nano contra closed, then f is slighty nano open.
Proof: Assume U is a nano clopen subset of $\mathcal{U}$. Then, since f is weakly nano contra closed, $f(U) \subseteq \mathcal{N}int(f(U))$. Therefore $f(U)$ is nano open and hence f is slightly nano open.

Remark 4.5: We have also investigated under which conditions weakly nano contra closed functions are nano open.

Theorem 4.6: If $f: \mathcal{U} \to \mathcal{V}$ is weakly nano contra closed and U is an nano open subset of $\mathcal{U}$ that is a union of nano closed sets, then, $f(U)$ is nano open.
Proof: Assume $f: \mathcal{U} \to \mathcal{V}$ is weakly nano contra closed. Let U be an nano open subset of $\mathcal{U}$ such that $U = \bigcup_{\alpha \in \mathcal{A}} F_\alpha$, where for every $\alpha \in \mathcal{A}$, F_α is a nano closed subset of $\mathcal{U}$. Since f is weakly nano contra closed $f(F_\alpha) \subseteq \mathcal{N}int(f(U))$ for every $\alpha \in \mathcal{A}$. Thus $f(U) = \bigcup_{\alpha \in \mathcal{A}} f(F_\alpha) \subseteq \mathcal{N}int f(U)$, which proves that $f(U)$ is nano open.

Theorem 4.7: If $f: \mathcal{U} \rightarrow \mathcal{V}$ is nano contra closed and U is nano open subset of $\mathcal{U}$ that is a union of nano closed sets, then $f(U)$ is nano open.

Relationships Between Weakly Nano Contra Open And Weakly Nano Contra Closed Functions: In this section we establish that weak nano contra openness and weak nano contra closedness are independent.

Remark 5.1: The concept of weakly nano contra open and weakly nano contra closedness are independent to each other which can be revealed by the following example.

Example 5.2: Let $\mathcal{U} = \{a, b\}$ with $\mathcal{U}/R = \{\{a\}, \{b\}\}$ and let $X = \{a\} \subseteq \mathcal{U}$, then $\tau_R(X) = \{\mathcal{U}, \emptyset, \{a\}\}$ and Define $f: \mathcal{U} \rightarrow \mathcal{U}$ as $f(a) = f(b) = a$. Then f is nano open and hence weakly nano contra closed, but f is not weakly nano contra open. Also $\{b\} \subseteq \mathcal{U}$, but $\mathcal{N}cl(f\{b\})$Ú$\mathcal{U}$. Therefore weakly nano contra closed does not imply weakly nano contra open.

Example 5.3: Let $\mathcal{U} = \{a, b\}$ with $\mathcal{U}/R = \{\{a\}, \{b\}\}$ and let $X = \{a\} \subseteq \mathcal{U}$, then $\tau_R(X) = \{\{\mathcal{U}, \emptyset, \{a\}\}$ and Define $f: \mathcal{U} \rightarrow \mathcal{U}$ as $f(a) = f(b) = b$. Then f is nano closed and hence weakly nano contra open, but f is not weakly nano contra open. Also $f(\{b\}) \subseteq \mathcal{N}int(f(\mathcal{U}))$, f is not weakly nano contra closed. Hence weakly nano contra open does not imply weakly nano contra closed.

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