ALGEBRAIC OPERATIONS ON PYTHAGOREAN FUZZY MATRICES

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Abstract: In this paper, we define the Pythagorean fuzzy matrix and their basic operations. We construct nA and A^n of a Pythagorean fuzzy matrix A and discuss them desirable properties. Also necessity and possibility operators of Pythagorean fuzzy matrices are defined and investigated their algebraic properties.

Keywords: Intuitionistic Fuzzy Matrix (IFM), Pythagorean Fuzzy Set (PFS), Pythagorean Fuzzy Matrix (PFM).

1. Introduction: The concept of intuitionistic fuzzy matrix (IFM) was introduced by Pal [12] and simultaneously by Im et.al [2] to generalize the concept of Thomason's [20] fuzzy matrix. Each element in an IFM is expressed by an ordered pair $\left\langle a_{ij}, a'_{ij} \right\rangle$. The sum $a_{ij} + a'_{ij}$ of each ordered pair is less than or equal to 1. Since the appearance of IFM in 2001, several researchers [9,10,19,25] have importantly contributed to the development of IFM theory and its applications, resulting in greater success from the theoretical and technological points of view. In Particular, [8] decomposition of intuitionistic fuzzy matrix is an interesting and important research topic in IFM theory that has been received more and more attention in recent years. In [2,3] the concept of the determination theory and the adjoint of a square IFM were studied. Also, they investigated their properties. Pal [11] introduced the Intuitionistic fuzzy determinant. Pal et al.[12] introduced the IFMs and studied several properties on it. Pal [4] defined some basic operations and relations of IFMs including maxmin, minmax, complement, algebraic sum, algebraic product etc. and proved equally between IFMs. Mondal and Pal [6] studied the similarity relations, together with invertibility conditions and eigenvalues of IFMs. Emam and Fndh [1] defined some kinds of IFMs, the max-min and min-max composition of IFMs. Also, they derived several important results of these compositions and construct an idempotent IFM from any given one through the min-max composition. Zhang [26] studied intuitionistic fuzzy value and introduced the concept of composition two intuitionistic fuzzy matrices. Silambarasan and Sriram [16,17] defined Hamacher operations on fuzzy matrices and investigated their algebraic properties, they Extend Hamacher operations to IFMs.

The multiple-criteria decision making (MCDM) is one of the most common activities in real life. The objective of the MCDM is to find the most desirable one from a finite set of alternatives with respect to the predefined attributes or criteria. Since its appearance, the IFS theory has gained more attention from researchers and has been widely applied in many fields such as pattern recognition, machine learning, decision making and market prediction. Yager [22,23,24] introduced the concept of the Pythagorean fuzzy set (PFS) and developed some aggregation operations for PFS. The PFS characterized by a membership degree and a nonmembership degree satisfying the condition that the square sum of its membership degree and nonmembership degree is equal to or less than 1, has much stronger ability than IFS to model such uncertain information in MCDM problems. Zhang and Xu [27] defined some novel operational laws of PFS and discuss its desirable properties.

In this paper the main objective is to introduce a Pythagorean fuzzy matrix and define some operations on PFMs and investigated their properties. The paper is organized as follows: In section 2 we first review the basic definitions of IFM and PFS. In section 3 we define the Pythagorean fuzzy matrix and their basic operations. We construct nA and A^n of a Pythagorean fuzzy matrix A and discuss them desirable properties. In section 4 Modal operators on PFMs are defined and investigated their algebraic properties.

2. Preliminaries: Several operators such as min-max, max-min, algebraic sum, algebraic product have been defined using T-norms and T-conorms. In this section, we first review the basic concept of Intuitionistic fuzzy matrix (IFM) and Pythagorean fuzzy set (PFS).

Definition 2.1 [12]: An intuitionistic fuzzy matrix (IFM) is a matrix of pairs $A=\left(\left\langle a_{ij},a'_{ij}\right\rangle\right)$ of a nonnegative real numbers $a_{ij},a'_{ij}\in[0,1]$ satisfying the condition $0\le a_{ij}+a'_{ij}\le 1$ for all i,j.

Definition 2.2 [25]: Let a set X be a universe of discourse A Pythagorean fuzzy set (PFS) P is an object having the form $P=\left(\left\langle x,P(\mu_p(x),\nu_p(x))\,|\,(x\in X)\right\rangle\right)$, Where the function $\mu_p:X\to[0,1]$ defines the degree of membership and $\nu_p:X\to[0,1]$ defines the degree of non-membership of the element $x\in X$ to P, respectively, and for every $x\in X$, it holds that $(\mu_p(x))^2+(\nu_p(x))^2\le 1$.

Definition 2.3 [19]: Let $A=\left(\left\langle a_{ij},a'_{ij}\right\rangle\right)$ and $B=\left(\left\langle b_{ij},b'_{ij}\right\rangle\right)$ be two intuitionistic fuzzy matrices of the same size $m\times n$, then

$(i)A\vee B=\left(\left\langle \max\left\{a_{ij},b_{ij}\right\},\min\left\{a'_{ij},b'_{ij}\right\}\right\rangle\right)$

$(ii)A\wedge B=\left(\left\langle \min\left\{a_{ij},b_{ij}\right\},\max\left\{a'_{ij},b'_{ij}\right\}\right\rangle\right)$

$(iii)A^C=\left(\left\langle a'_{ij},a_{ij}\right\rangle\right)$

$(iv)A\oplus B=\left(\left\langle a_{ij}+b_{ij}-a_{ij}b_{ij},a'_{ij}b'_{ij}\right\rangle\right)$ is called the algebraic sum of A and B.

$(v)A\odot B=\left(\left\langle a_{ij}b_{ij},a'_{ij}+b'_{ij}-a'_{ij}b'_{ij}\right\rangle\right)$ is called the algebraic product of A and B.

3. Main Results for Pythagorean Fuzzy Matrices:

Similar to the definition of IFM, in the following we introduce the definition of Pythagorean fuzzy matrices (PFMs).

Definition 3.1: A Pythagorean fuzzy matrix (PFM) is a pair $A=\left(\left\langle a_{ij},a'_{ij}\right\rangle\right)$ of nonnegative real numbers $a_{ij},a'_{ij}\in[0,1]$ satisfying the condition $a_{ij}^{\;2}+a'^{\,2}_{ij}\le 1$ for all i, j.

Remark 3.2: The main difference between intuitionistic fuzzy matrix (IFM)[19] and PFM is their different constraint conditions. According to their definitions, we know that the constraint condition of IFM is $0 \le a_{ij} + a'_{ij} \le 1$,whereas the constraint condition of PFM is ${a_{ij}}^2 + {a'_{ij}}^2 \le 1$. Because the fact that for any $a,b \in [0,1]$ if $a+b \le 1$ then $a^2+b^2 \le 1$, if one is an IFM, then it must also be a PFM, but not all PFM are the IFMs.

This is illustrated by the following example.

Example 3.3:

$$A = \begin{bmatrix} \langle 0.8, 0.5\rangle \langle 0.2, 0.4\rangle \\ \langle 0.1, 0.3\rangle \langle 0.5, 0.1\rangle \end{bmatrix}$$

Is not an IFM, but it A is a PFM.

On the basis of relationship between IFMs and PFMs, we define some novel operations of PFMs.

Definition 3.4: Given three PFMs, A, B and C of the same size, the basic operations are defined as follows:

$$(i) A \vee B = \left(\left\langle \max\left\{a_{ij}, b_{ij}\right\}, \min\left\{a'_{ij}, b'_{ij}\right\}\right\rangle\right)$$

$$(ii) A \wedge B = \left(\left\langle \min\left\{a_{ij}, b_{ij}\right\}, \max\left\{a'_{ij}, b'_{ij}\right\}\right\rangle\right)$$

$$(iii) A^C = \left(\left\langle a'_{ij}, a_{ij}\right\rangle\right)$$

$$(iv) A \oplus_P B = \left(\left\langle \sqrt{a_{ij}^2 + b_{ij}^2 - a_{ij}^2 b_{ij}^2}, a'_{ij} b'_{ij}\right\rangle\right)$$

$(v) A \odot_P B = \left(\left\langle a_{ij} b_{ij}, \sqrt{{a'_{ij}}^2 + {b'_{ij}}^2 - {a'_{ij}}^2 {b'_{ij}}^2}\right\rangle\right)$, where $+, -$ and $.$ are ordinary addition,subtraction and multiplication respectively.

These operations are constructed in such a way that they produce PFMs. Since it is easy to prove that $a_{ij}^2 + b_{ij}^2 - a_{ij}^2 b_{ij}^2 + {a'_{ij}}^2 {b'_{ij}}^2 \le 1$ and $a_{ij}^2 b_{ij}^2 + {a'_{ij}}^2 + {b'_{ij}}^2 - {a'_{ij}}^2 {b'_{ij}}^2 \le 1$.Using expressions (iv) and (v), the following equations are obtained for any integer $n > 0$

$$nA = A \oplus_P A \oplus_P \ldots\ldots \oplus_P A = \left(\left\langle \sqrt{1-(1-a_{ij}^2)^n}, {a'_{ij}}^n\right\rangle\right)$$

$$A^n = A \odot_P A \odot_P \ldots\ldots \odot_P A = \left(\left\langle a_{ij}^n, \sqrt{1-(1-{a'_{ij}}^2)^n}\right\rangle\right)$$

It's easy to prove that nA and A^n is a PFM.

Theorem 3.5: Let A and B are two PFMs. If $n > 0$ are integer, then

(i) $n(A \oplus_P B) = nA \oplus_P nB$,

(ii) $(A \odot_P B)^n = A^n \odot_P B^n$.

Proof: $(i) n(A \oplus_P B) = \left(\left\langle \sqrt{1-(1-(a_{ij}^2 + b_{ij}^2 - a_{ij}^2 b_{ij}^2))^n}, (a'_{ij} b'_{ij})^n\right\rangle\right)$

$$= \left(\left\langle \sqrt{1-(1-a_{ij}^2)^n (1-b_{ij}^2)^n}, (a'_{ij} b'_{ij})^n\right\rangle\right) -- (3.1)$$

$$nA \oplus_P nB = \left(\left\langle \sqrt{1-(1-a_{ij}^2)^n + 1-(1-b_{ij}^2)^n - (1-(1-a_{ij}^2)^n)(1-(1-b_{ij}^2)^n)}, {a'_{ij}}^n {b'_{ij}}^n\right\rangle\right)$$

$$= \left(\left\langle \sqrt{1-(1-a_{ij}^2)^n (1-a_{ij}^2)^n}, (a'_{ij} b'_{ij})^n\right\rangle\right) -- (3.2)$$

Hence, from (3.1) and (3.2), we get the result (i).
(ii) It can be proved analogously.

Theorem 3.6: For any PFM A. If $n_1, n_2 > 0$ are integers,then
$(i) n_1 A \oplus_P n_2 A = (n_1 + n_2) A$
$(ii) A^{n_1} \odot_P A^{n_2} = A^{(n_1+n_2)}$
Proof:

$$(i) n_1 A \oplus_P n_2 A = \left(\left\langle \sqrt{1-(1-a_{ij}^2)^{n_1} + 1-(1-a_{ij}^2)^{n_2} - (1-(1-a_{ij}^2)^{n_1})(1-(1-b_{ij}^2)^{n_2})}, a_{ij}'^{n_1} b_{ij}'^{n_2} \right\rangle \right)$$
$$= \left(\left\langle \sqrt{1-(1-a_{ij}^2)^{n_1+n_2}}, (a_{ij}')^{n_1+n_2} \right\rangle \right)$$
$$= (n_1 + n_2) A$$

(ii) It can be proved analogously.

Theorem 3.7: For any PFM A. If $n > 0$ are integer, then
$(i)(A^C)^n = (nA)^C$
$(ii) n(A^C) = (A^n)^C$

Proof: $(i)(A^C)^n = \left(\left\langle a_{ij}'^{n}, \sqrt{1-(1-a_{ij}^2)^n} \right\rangle \right)$

$$(nA)^C = \left(\left\langle a_{ij}'^{n}, \sqrt{1-(1-a_{ij}^2)^n} \right\rangle \right)$$
$$\Rightarrow (A^C)^n = (nA)^C$$

(ii) It can be proved analogously.

Theorem 3.8: Let A and B are two PFMs. If $n > 0$ are integer, then
$(i) n(A \vee B) = nA \vee nB$
$(ii)(A \vee B)^n = A^n \vee B^n$

Proof: $(i) n(A \vee B) = \left(\left\langle \sqrt{1-(1-\max\{a_{ij}^2, b_{ij}^2\})^n}, \min\{a_{ij}'^{n}, b_{ij}'^{n}\} \right\rangle \right)$---(3.3)

$$nA \vee nB = \left(\left\langle \sqrt{1-(1-a_{ij}^2)^n}, a_{ij}'^{n} \vee \sqrt{1-(1-a_{ij}^2)^n}, a_{ij}'^{n} \right\rangle \right)$$
$$= \left(\left\langle \max\left\{ \sqrt{1-(1-a_{ij}^2)^n}, \sqrt{1-(1-b_{ij}^2)^n} \right\}, \min\{a_{ij}'^{n}, b_{ij}'^{n}\} \right\rangle \right)$$
$$= \left(\left\langle \sqrt{1-(1-\max\{a_{ij}^2, b_{ij}^2\})^n}, \min\{a_{ij}'^{n}, b_{ij}'^{n}\} \right\rangle \right) --- (3.4)$$

From (3.3) and (3.4), we get the result (i).
(ii) It can be proved analogously.

The following theorem is obvious. The operations $\vee$ and $\wedge$ obey the De Morgan's laws.
Theorem 3.9: If A and B are two PFMs, then
$(i) A^C \vee B^C = (A \wedge B)^C$
$(ii) A^C \wedge B^C = (A \vee B)^C$.
The operations $\oplus_P$ and $\odot_P$ obey the De Morgan's laws.

Theorem 3.10: If A and B are two PFMs, then

$(i) A^C \oplus_P B^C = (A \odot_P B)^C$

$(ii) A^C \odot_P B^C = (A \oplus_P B)^C$

Proof: $(i) A^C \oplus_P B^C = \left(\left\langle \sqrt{a_{ij}'^2 + b_{ij}'^2 - a_{ij}'^2 b_{ij}'^2}, a_{ij} b_{ij} \right\rangle\right)$

$$(A \odot_P B)^C = \left(\left\langle \sqrt{a_{ij}'^2 + b_{ij}'^2 - a_{ij}'^2 b_{ij}'^2}, a_{ij} b_{ij} \right\rangle\right)$$
$$= A^C \oplus_P B^C$$

(ii) It can be proved analogously.

Theorem 3.11: If A and B are two PFMs, then

$(i)(A \vee B) \oplus_P (A \wedge B) = A \oplus_P B$

$(ii)(A \vee B) \odot_P (A \wedge B) = A \odot_P B$

Proof:

$$(i)(A \vee B) \oplus_P (A \wedge B) = \begin{pmatrix} \left\langle \sqrt{\max\left\{a_{ij}^2, b_{ij}^2\right\} + \min\left\{a_{ij}^2, b_{ij}^2\right\} - \max\left\{a_{ij}^2, b_{ij}^2\right\} \min\left\{a_{ij}^2, b_{ij}^2\right\}} \right\rangle, \\ \left\langle \min\left\{a_{ij}', b_{ij}'\right\} \max\left\{a_{ij}', b_{ij}'\right\} \right\rangle \end{pmatrix}$$
$$= \left(\left\langle \sqrt{a_{ij}^2 + b_{ij}^2 - a_{ij}^2 b_{ij}^2}, a_{ij}' b_{ij}' \right\rangle\right)$$
$$= A \oplus_P B$$

(ii) It can be proved analogously.

We discuss the Distributivity law in the case the operation of algebraic sum, algebraic product, $\vee$ and $\wedge$ are combined with each other.

Theorem 3.12: If A, B and C are two PFMs, then

$(i)(A \vee B) \wedge C = (A \wedge C) \vee (B \wedge C)$

$(ii)(A \wedge B) \vee C = (A \vee C) \wedge (B \vee C)$

$(iii)(A \vee B) \oplus_P C = (A \oplus_P C) \vee (B \oplus_P C)$

$(iv)(A \wedge B) \oplus_P C = (A \oplus_P C) \wedge (B \oplus_P C)$

$(v)(A \vee B) \odot_P C = (A \odot_P C) \vee (B \odot_P C)$

$(vi)(A \wedge B) \odot_P C = (A \odot_P C) \wedge (B \odot_P C)$

Proof: In the following, we shall prove (i),(iii),(v) and (ii),(iv),(vi) can be proved analogously.

$$(i)(A \vee B) \wedge C = \left(\left\langle \min\left\{\max\left\{a_{ij}, b_{ij}\right\}, c_{ij}\right\}, \max\left\{\min\left\{a_{ij}', b_{ij}'\right\} c_{ij}'\right\} \right\rangle\right)$$
$$= \left(\left\langle \max\left\{\min\left\{a_{ij}, b_{ij}\right\}, \min\left\{a_{ij}, c_{ij}\right\}\right\}, \min\left\{\max\left\{a_{ij}', b_{ij}'\right\}, \max\left\{b_{ij}', c_{ij}'\right\}\right\} \right\rangle\right)$$
$$= \left(\left\langle \left\{\min\left\{a_{ij}, c_{ij}\right\}, \max\left\{a_{ij}', c_{ij}'\right\}\right\} \bigvee \left\{\min\left\{b_{ij}', c_{ij}'\right\}, \max\left\{b_{ij}', c_{ij}'\right\}\right\} \right\rangle\right)$$
$$= (A \wedge C) \vee (B \wedge C)$$

$$(iii)(A \vee B) \oplus_P C = \left(\left\langle \sqrt{\max\left\{a_{ij}^2, b_{ij}^2\right\} + c_{ij}^2 - \max\left\{a_{ij}^2, b_{ij}^2\right\} c_{ij}^2}, \min\left\{a_{ij}', b_{ij}'\right\} c_{ij}' \right\rangle\right)$$
$$= \left(\left\langle \sqrt{(1 - c_{ij}^2) \max\left\{a_{ij}^2, b_{ij}^2\right\} + c_{ij}^2}, \min\left\{a_{ij}' c_{ij}', b_{ij}' c_{ij}'\right\} \right\rangle\right) --- (3.5)$$

$$(A \oplus_P C) \vee (B \oplus_P C) = \left(\left\langle \max\left\{\sqrt{a_{ij}^2 + c_{ij}^2 - a_{ij}^2 c_{ij}^2}, \sqrt{b_{ij}^2 + c_{ij}^2 - b_{ij}^2 c_{ij}^2}\right\}, \min\left\{a'_{ij}c'_{ij}, b'_{ij}c'_{ij}\right\}\right\rangle\right)$$

$$= \left(\left\langle \max\left\{\sqrt{(1-c_{ij}^2)a_{ij}^2 + c_{ij}^2}, \sqrt{(1-c_{ij}^2)b_{ij}^2 + c_{ij}^2}\right\}, \min\left\{a'_{ij}c'_{ij}, b'_{ij}c'_{ij}\right\}\right\rangle\right)$$

$$= \left(\left\langle \sqrt{(1-c_{ij}^2)\max\left\{a_{ij}^2, b_{ij}^2\right\} + c_{ij}^2}, \min\left\{a'_{ij}c'_{ij}, b'_{ij}c'_{ij}\right\}\right\rangle\right) --- (3.6)$$

$$= (A \vee B) \oplus_P C$$

From (3.5) and (3.6), we get the result (iii).

$$(v)(A \vee B) \odot_P C = \left(\left\langle \max\left\{a_{ij}, b_{ij}\right\} c_{ij}, \sqrt{\min\left\{a_{ij}'^2, b_{ij}'^2\right\} + c_{ij}'^2 - \min\left\{a_{ij}'^2, b_{ij}'^2\right\} c_{ij}'^2}\right\rangle\right)$$

$$= \left(\left\langle \max\left\{a_{ij}, b_{ij}\right\} c_{ij}, \sqrt{(1-c_{ij}'^2)\min\left\{a_{ij}'^2, b_{ij}'^2\right\} + c_{ij}'^2}\right\rangle\right) -- (3.7)$$

$$(A \odot_P C) \vee (B \odot_P C) = \left(\left\langle \max\left\{a_{ij}c_{ij}, b_{ij}c_{ij}\right\}, \min\left\{\sqrt{a_{ij}'^2 + c_{ij}'^2 - a_{ij}'^2 c_{ij}'^2}, \sqrt{b_{ij}'^2 + c_{ij}'^2 - b_{ij}'^2 c_{ij}'^2}\right\}\right\rangle\right)$$

$$= \left(\left\langle \max\left\{a_{ij}, b_{ij}\right\} c_{ij}, \sqrt{(1-c_{ij}'^2)\min\left\{a_{ij}'^2, b_{ij}'^2\right\} + c_{ij}'^2}\right\rangle\right) -- (3.8)$$

$$= (A \vee B) \odot C.$$

From (3.7) and (3.8), we get the result (v).

4. Modal Operators on Pythagorean Fuzzy Matrices:

Pal [14] defined the necessity and possibility operators for an IFMs. Murugadas et al. [7] Studied the relations between □ and ◊ operators for IFMs. In this section, we define necessity and possibility operators for PFMs and proved their algebraic properties.

Definition 4.1: For every PFM A, the necessity $(\Box)$ and possibility $(\Diamond)$ operators are defined as follows, $\Box A = \left(\left\langle a_{ij}, \sqrt{1-a_{ij}^2}\right\rangle\right)$ and $\Diamond A = \left(\left\langle \sqrt{1-a_{ij}'^2}, a'_{ij}\right\rangle\right)$

Theorem 4.2: If A and B are two PFMs, then

$(i) \Box(A \vee B) = \Box A \vee \Box B$

$(ii) \Box(A \wedge B) = \Box A \wedge \Box B$

$(iii) \Diamond(A \vee B) = \Diamond A \vee \Diamond B$

$(iv) \Diamond(A \wedge B) = \Diamond A \wedge \Diamond B$

Proof: In the following, we shall prove (i), and (ii), (iii) , (iv) are proved analogously.

(i) $\Box(A \vee B) = \left(\left\langle \max\left\{a_{ij}, b_{ij}\right\}, \sqrt{(1-\max\left\{a_{ij}^2, b_{ij}^2\right\}}\right\rangle\right)$

$\Box A \vee \Box B = \left(\left\langle \max\left\{a_{ij}, b_{ij}\right\}, \min\left\{\sqrt{1-a_{ij}^2}, \sqrt{1-b_{ij}^2}\right\}\right\rangle\right)$

$= \left(\left\langle \max\left\{a_{ij}, b_{ij}\right\}, \sqrt{(1-\max\left\{a_{ij}^2, b_{ij}^2\right\}}\right\rangle\right) = \Box(A \vee B)$

Theorem 4.3: For any PFM A. If $n > 0$ are integer, then

$(i) \Box A^n = (\Box A)^n$

$(ii) \Diamond A^n = (\Diamond A)^n$

$(iii)\Box nA = n\Box A$

$(iv)\Diamond nA = n\Diamond A$

Proof: In the following, we shall prove (i), and (ii), (iii) , (iv) are proved analogously.

(i) $\Box A^n = \left(\left\langle a_{ij}^n, \sqrt{1-a_{ij}^{2n}}\right\rangle\right)$

$(\Box A)^n = \left(\left\langle a_{ij}^n, \sqrt{1-(1-(1-a_{ij}^{2})^n)}\right\rangle\right)$

$= \left(\left\langle a_{ij}^n, \sqrt{1-a_{ij}^{2n}}\right\rangle\right) = \Box A^n$

Theorem 4.4: If A and B are two PFMs, then

$(i)\Box(A \oplus_P B) = \Box A \oplus_P \Box B$

$(ii)\Diamond(A \oplus_P B) = \Diamond A \oplus_P \Diamond B$

Proof:

$$(i)\Box(A \oplus_P B) = \left(\left\langle \sqrt{a_{ij}^2 + b_{ij}^2 - a_{ij}^2 b_{ij}^2}, \sqrt{1-(a_{ij}^2 + b_{ij}^2 - a_{ij}^2 b_{ij}^2)}\right\rangle\right) --(4.1)$$

$$\Box A \oplus_P \Box B = \left(\left\langle \sqrt{a_{ij}^2 + b_{ij}^2 - a_{ij}^2 b_{ij}^2}, \sqrt{(1-a_{ij}^2)}\sqrt{(1-b_{ij}^2)}\right\rangle\right)$$

$$= \left(\left\langle \sqrt{a_{ij}^2 + b_{ij}^2 - a_{ij}^2 b_{ij}^2}, \sqrt{1-(a_{ij}^2 + b_{ij}^2 - a_{ij}^2 b_{ij}^2)}\right\rangle\right) --(4.2)$$

From (4.1) and (4.2), $\Box(A \oplus_P B) = \Box A \oplus_P \Box B$

$$(ii)\Diamond(A \oplus_P B) = \left(\left\langle \sqrt{1-a_{ij}'^2 b_{ij}'^2}, a_{ij}' b_{ij}'\right\rangle\right) --(4.3)$$

$$\Diamond A \oplus_P \Diamond B = \left(\left\langle \sqrt{(1-a_{ij}'^2)+(1-b_{ij}'^2)-(1-a_{ij}'^2)(1-b_{ij}'^2)}, a_{ij}' b_{ij}'\right\rangle\right)$$

$$= \left(\left\langle \sqrt{1-a_{ij}'^2 b_{ij}'^2}, a_{ij}' b_{ij}'\right\rangle\right) --(4.4)$$

From (4.3) and (4.4), $\Diamond(A \oplus_P B) = \Diamond A \oplus_P \Diamond B$.

Theorem 4.5: If A and B are two PFMs, then

$(i)\Box(A \odot_P B) = \Box A \odot_P \Box B$

$(ii)\Diamond(A \odot_P B) = \Diamond A \odot_P \Diamond B$

Proof:

$$(i)\Box(A \odot_P B) = \left(\left\langle a_{ij} b_{ij}, \sqrt{1-a_{ij}^2 b_{ij}^2}\right\rangle\right) --(4.5)$$

$$\Box A \odot_P \Box B = \left(\left\langle a_{ij} b_{ij}, \sqrt{(1-a_{ij}^2)+(1-b_{ij}^2)-(1-a_{ij}^2)(1-b_{ij}^2)}\right\rangle\right)$$

$$= \left(\left\langle a_{ij} b_{ij}, \sqrt{1-a_{ij}^2 b_{ij}^2}\right\rangle\right) --(4.6)$$

From (4.5) and (4.6), $\Box(A \odot_P B) = \Box A \odot_P \Box B$

(ii) It can be proved analogously.

Theorem 4.6: If A and B are two PFMs, then

$(i)\left(\Box(A^C \oplus_P B^C)\right)^C = \Diamond A \odot_P \Diamond B$

$(ii)\left(\Box(A^C \odot_P B^C)\right)^C = \Diamond A \oplus_P \Diamond B$

Proof:

$$(i)(A^C \oplus_P B^C) = \left(\left\langle \sqrt{a_{ij}'^2 + b_{ij}'^2 - a_{ij}'^2 b_{ij}'^2}, a_{ij} b_{ij} \right\rangle\right)$$

$$\Box(A^C \oplus_P B^C) = \left(\left\langle \sqrt{a_{ij}'^2 + b_{ij}'^2 - a_{ij}'^2 b_{ij}'^2}, \sqrt{1-(a_{ij}'^2 + b_{ij}'^2 - a_{ij}'^2 b_{ij}'^2)} \right\rangle\right)$$

$$\left(\Box(A^C \oplus_P B^C)\right)^C = \left(\left\langle \sqrt{1-(a_{ij}'^2 + b_{ij}'^2 - a_{ij}'^2 b_{ij}'^2)}, \sqrt{a_{ij}'^2 + b_{ij}'^2 - a_{ij}'^2 b_{ij}'^2} \right\rangle\right)$$

$$= \lozenge A \odot_P \lozenge B$$

(ii) It can be proved analogously.

The proof of the following theorem follows from the theorem 4.4.

Theorem 4.7: If A and B are two PFMs, then

$$(i)\left(\lozenge(A^C \oplus_P B^C)\right)^C = \Box A \odot_P \Box B$$

$$(ii)\left(\lozenge(A^C \odot_P B^C)\right)^C = \Box A \oplus_P \Box B.$$

5. Conclusion: In this article, we define the Pythagorean fuzzy matrix and studied basic operations are maximized, minmax, algebraic sum, algebraic product and then define nA and A^n of a Pythagorean fuzzy matrix A and discuss them desirable properties. Also necessity and possibility operators of Pythagorean fuzzy matrices are defined and investigated their algebraic properties.

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