

Proportioning Systems in Architecture

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Abstract: This paper is an attempt to establish the relationship between different proportioning systems and the use of module in architecture. The relationship between the two has always created a sense of order and has provided an aesthetic rationale among the various elements in architectural design. Thus the paper highlights the application of proportioning systems in architecture.

Keywords: Classical Orders, Fibonacci Series, Fractals, Golden Ratio, Modularity.

1. INTRODUCTION

In architecture, modularity can be referred to the construction of an object by joining together standardized units to form larger compositions, and/or to the use of a module as a standardized unit of measurement and proportion. The intent of all theories of proportion is to create a sense of order among the elements in a visual construction. According to Euclid, a ratio refers to the quantitative comparison of two similar things, while proportion refers to the equality of the ratios. Underlying any proportioning system therefore is a characteristic ratio, a permanent quality that is transmitted from one ratio to another. Thus proportioning systems establish a consistent set of visual relationships between the parts of a building, as well as between the parts and their whole.

In Greek architecture, the Golden mean (also known as the Golden rectangle, Golden section and Golden ratio, $\Phi=1.618\dots$) served as a canon for designing the buildings. A rectangle whose sides are proportioned according to the golden section is known as a golden rectangle. If a square is constructed on its smaller side, the remaining portion of the original rectangle would be a smaller but a similar golden rectangle. This operation can be repeated indefinitely to create a gradation of squares and golden rectangles. It is believed that it was used in the designing of the Parthenon and many other Greek buildings.

In Islamic architecture, the application of square as a generic unit brings relationship between all the parts from the smallest to the biggest dimension. It regulates not only totality in design but also achieves unity in the overall dimension. The elevation of Taj Mahal is a perfect square whereas that of Humayun's tomb's proportions are in the ratio of 1:2. The square was used not only in plan but also in elevation where rhythm of a façade is a compositional principle which regulates the elements of the design. Ancient Egyptian architects used grid system where the essential principle after all, was that objects with particular dimensions could be reproduced on a plane surface to any chosen scale - a

conception that would hardly have recommended itself to the constructors of portable shrines. On the contrary, the fact that the grid conforms to the convention used in art for standing figures, with 18 squares between the base and the top of the outer cornice, indicates that the drawing was used as a pattern for the drafting-out of the shrine in wall-scenes.

Indian temple construction was based on Vastupurushmandala where the form of the *purusha* (human) body was made to suit the abstract idea of the square, as the supreme geometric form. The *mandala* is actually a square subdivided into smaller squares in the form of a grid with each square unit clearly marking the areas of respective gods. The most commonly used *mandala* is the square divided into 64 and 81 squares. Mostly, the square of the *mandala* on its outer periphery is divided into 32 smaller squares, in accordance to the astrological calculation called *nakshatra* representing the constellations or the position of planets through which moon passes in its monthly path.

In Renaissance architecture the basic unit of dimension was the diameter of the column. From this module the dimensions of: the shaft, the capital, as well as the pedestal below and the entablature above, and down to the smallest detail, were derived. The system of spacing between columns – was also based on the diameter of the column. The dimensions of a structural component were used as a unit of measurement or standard for determining the proportions of the rest of the construction.

Swiss-born modern architect Le Corbusier (1887–1965) devised The Modular which is an anthropometric scale of proportions. It was developed as a visual bridge between two incompatible scales, the Imperial system and the Metric system. It is based on the height of an English man with his arm raised. He applied these concepts to his buildings namely Unite de habitation in Marseilles, Secretariat in Chandigarh, etc. He used a standardized, often interchangeable component of a system or construction that was designed for easy assembly or flexible use: a sofa consisting of two end modules and was based on golden ratio.

In recent times, the concept of fractals has been used where a geometric shape exhibits self-similarity across all scales. Quite a different form may serve for another, but from one basic idea all the formal elements of design are in each case derived and held well together in scale and character.

Today, proportioning systems present to us an opportunity to explore modularity in architecture, the use of which would help in creating architecture where all the formal elements of design are derived and held well together in scale and character.

2. AIM

The paper aims at the analysis of proportioning systems from the past and the present which would help the architect and designer today in realizing their role in expressing architectural thoughts. It is not likely to result in a definite yes or no but sow seeds of revolutionary thought for a revolutionary time and in the end bring us to a more mature and profound understanding of proportions in architecture.

3. GOLDEN SECTION

The golden section can be defined as the ratio between two sections of a line, or the two dimensions of a plan figure, in which the lesser of the two is to the greater as the greater is to the sum of both. It can be expressed algebraically by the equation of two ratios: $\frac{a}{b} = \frac{b}{a+b}$

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The golden section has some remarkable algebraic and geometric properties that account for its existence in architecture as well as in the structures of many living organisms. Any progression based on the golden section is at once additive and geometrical.

A rectangle whose sides are proportioned according to the golden section is known as a golden rectangle. If a square is constructed on its smaller side, the remaining portion of the original rectangle would be a smaller but similar golden rectangle. This operation can be repeated indefinitely to create a gradation of squares and golden rectangles.

During this transformation, each part remains similar to all of the other parts, as well as the whole.

4. FIBONACCI SERIES

Here is a 'Fibonacci series'.

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144,..

If we take the ratio of two successive numbers in this series and divide each by the number before it, we will find the following series of numbers.

$$1/1 = 1$$

$$2/1 = 2$$

$$3/2 = 1.5$$

$$5/3 = 1.6666...$$

$$8/5 = 1.6$$

$$13/8 = 1.625$$

$$21/13 = 1.61538...$$

$$34/21 = 1.61904...$$

The ratio seems to be settling down to a particular value, which we call the golden ratio (Phi=1.618...).

In geometry, a *golden spiral* is a logarithmic spiral whose growth factor 'b' is related to ϕ , the golden ratio. Specifically, a golden spiral gets wider (or further from its origin) by a factor of ϕ for every quarter turn it makes.

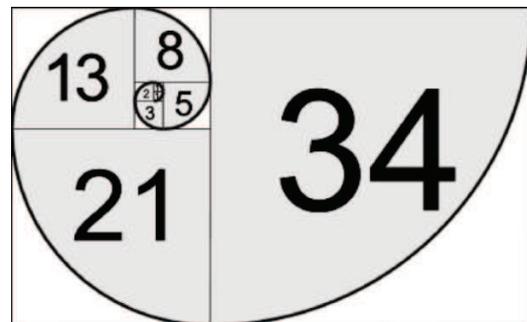


Fig. 1. Golden Spiral

Source:<http://www.mathsisfun.com/numbers/images/fibonacci-spiral.gif>

5. KEN

The traditional Japanese unit of measure, the shaku, was originally imported from china. It is almost equivalent to the English foot and visible into decimal units.

Another unit measure, the ken, was introduced in the latter half of Japan's Middle Ages. Although it was originally used simply to designate the interval between two columns and varied in size, the ken was soon standardized for residential architecture. Unlike the module of the classical orders, which was based on the diameter of a column and varied with the size of a building, the ken became an absolute measurement.

The ken, however, was not only a measurement for the construction of buildings. It evolved into an aesthetic module that ordered the structure, materials, and space of Japanese architecture. Two methods of designing with the ken modular grid developed that affected its dimension. In the inaka-ma method, the ken grid of 6 shaku determined the center-to-center spacing of columns.

Therefore, the standard tatami floor mat (3x6 shaku or 1/2x1ken) varied slightly to allow for the thickness of the columns. The width of the mat was the module.

6. CLASSICAL ORDERS

A classical order is one of the ancient styles of classical architecture, each distinguished by its proportions and characteristic profiles and details, and most readily recognizable by the type of column employed. From the 16th century onwards, architectural theorists recognized five orders. Each style has its proper entablature, consisting of architrave, frieze and cornice.

To the Greek and Roman, orders represented the perfect expression of beauty and harmony. The basic unit of dimension was the diameter of the column. From this module the dimensions of: the shaft, the capital, as well as the pedestal below and the entablature above, and down to the smallest detail, were derived. The system of spacing between columns – was also based on the diameter of the column.

Since the size of columns varied according to the size of the building, the Orders were not based on a fixed unit of measurement. Rather, the intention was to ensure that all parts of any one building were proportioned and in harmony with one another.

Vitruvius, in the time of Augustus, studied actual examples of the Orders and presented his “ideal” proportions for each in his treatise, *The Ten Books on Architecture*. Vignola re-edited these rules for the Italian Renaissance and his forms for the Orders are probably the best known today.

7. MODULAR

The Modular is an anthropometric scale of proportions devised by the Swiss-born French architect Le Corbusier (1887–1965). It was developed as a visual bridge between two incompatible scales, the Imperial system and the Metric system. It is based on the height of an English man with his arm raised.

Le Corbusier began his study in 1942, and published the Modular: a harmonious measure to the human scale universally applicable to architecture and mechanics. The basic grid consists of three measures, 113, 70, and 43 centimetres, proportioned according to the golden section, Where: $a / b = b / (a + b)$

His Modular system was based on a man 6 feet (183 centimeters) tall. In the 'red' series this height is divided in golden section by the height of the navel. The 'blue' series is based on the height of the raised hand, above the ground and above the groin. Dividing these distances by the golden

section extends the two series, which are related to basic human postures.

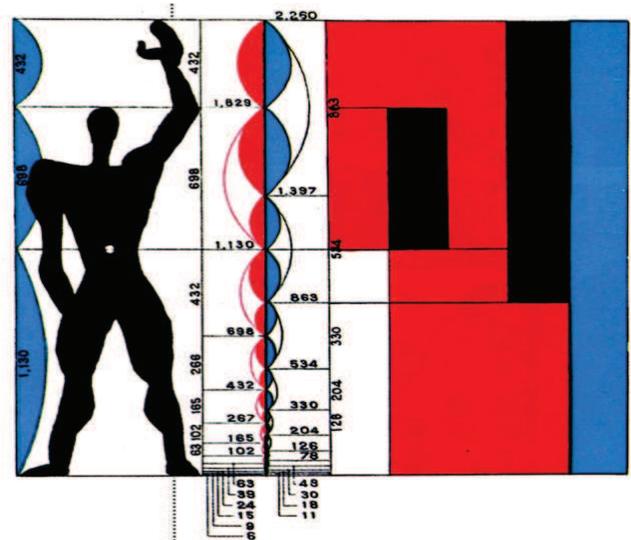


Fig. 2: The modular dimensions

Source: www.wharferj.files.wordpress.com/2010/12/45_le-corbusier-le-modular-dor.jpg

8. FRACTALS

A fractal is a geometric shape that exhibits self-similarity across all scales.

If architecture stands for continuing the development from the protecting caves over the fallen down tree as a first shelter to buildings made of timber or stones and up to modern interpretations of nature like Frank Lloyd Wright's examples, then architecture, natural materials, time and the structure of nature may still be a unity. In this way, nature, as we have seen in the sections before, offers characteristics of fractal geometry rather than such of Euclidean geometry. So maybe because of similarities between nature and architecture, with regard to material and structure, some of the fractal attributes can also be found in buildings, in their elevations and ground floors organization:

"Quite a different form may serve for another, but from one basic idea all the formal elements of design are in each case derived and held well together in scale and character".

9. EGYPTIAN ARCHITECTURE

It is well known that representations of the human figure in ancient Egyptian art usually conformed to highly stylized principles in which the proportions between the different parts of the human body were determined by a set of fixed laws constituting a Canon of Proportions. Egyptian artists were

thereby able to make use of a conventional system of proportion which was found to be aesthetically pleasing, while also rendering their subjects in idealized forms which may or may not have been faithful to the exact proportions of the persons in question. The Egyptian Canon of Proportions was maintained over many centuries through the medium of the artist's grid, in which the different parts of the human body corresponded to different squares in the grid. This grid system was not merely a copying device which made it possible to render a particular scene on any chosen scale, but rather a complete system of proportions by means of which the human figure could in theory be correctly represented.

From the Middle Kingdom onwards, ancient Egyptian artists used square grids to design human figures and to establish the general layout of two-dimensional scenes. They do not appear to have been tied by any strict rule, and were perfectly able to reproduce the same proportions without the help of grids.

10. GREEK ARCHITECTURE

The initiation of the Golden Section refers to the Ancient Greek. At that time, Pythagoras developed his concept, which stated the relation between some harmonic numbers and the universe. Greek believed that human body and temples were related to Higher Universal Order. They noticed the accordance of the Golden Section System with the proportion of human body. As a result, their buildings were designed on the bases of The Golden Section. Similarly, Gothic Architects followed their footsteps.

Leonardo did an entire exploration of the human body and the ratios of the lengths of various body parts. "Vitruvian man" illustrates that the human body is proportioned according to the Golden Ratio. The exterior dimensions of the Parthenon in Athens, built in about 440BC, form a perfect Golden Rectangle.

11. ISLAMIC ARCHITECTURE

When considering ideal forms, the circle and square and have been used as a source of completeness in architecture around the world (Critchlow 1999). On occasion, the equilateral triangle can be considered idyllic as well. These have been given remarkable prominence because of their simplicity. They are believed to be perfect shapes due to the regularity of the distance from the perimeter to the center. This perfection has been linked to eternity and divinity. The circle and the square can also be related to the human figure proportioning system as first seen in the drawing of the Vitruvian man by Marcus Vitruvius Pollio. Variations are often created with these geometric forms such as the octagon, hexagon and pentagon. These polygon variations are more commonly found in Mughal architecture as a direct derivation of the circle or sphere.

According to Islam, human works of art are inherently flawed compared to the work of God; thus, it is believed by many that to attempt to depict in a realistic form any animal or person is insolence to God. The limited possibilities have been explored by artists as an outlet to artistic expression, and have been cultivated to become a positive style and tradition, emphasizing the decorative function of art, or its religious functions via non-representational forms such as Geometric patterns, floral patterns, and arabesques.

It is supposed to be much more, to be a representation of the ideal, through its instantiation of harmony and order, and its reference to the nexus between the finite and the infinite.

12. HINDU ARCHITECTURE

The Hindu philosophy was among the first to relate the human figure as the basis of a system of proportion, the form of the *purusha* (human) body was made to suit the abstract idea of the square, as the supreme geometric form.

The basic form of the *Vastupurushamandala* is the square and square is the important and ideal geometric form in Hindu philosophy, which represents the earth. All the necessary forms like the triangle, hexagon, octagon and circle, etc can be derived from the square. The four sides of the square represent the four cardinal directions.

The square also symbolizes the order, the completeness of endless life and the perfectness of life and death. According to Hindu beliefs, man's everyday life is also governed by the number four as in four classes (*varnas*), four phase of life, four great eras, four head of Brahma (the creator God), the four Vedas, etc. Similarly, the circle represents the universe and is considered as the perfect shape, without any beginning and end, suggesting timelessness and infinity, a typically heavenly feature.

The *mandala* is actually a square subdivided into smaller squares in the form of a grid with each square unit clearly marking the areas of respective gods. The most commonly used *mandala* is the square divided into 64 and 81 squares. Mostly the square of the *mandala* on its outer periphery are divided into 32 smaller squares, in accordance to the astrological calculation called *nakshatra* representing the constellations or the position of planets through which moon passes in its monthly path. The closed polygon of thirty-two squares is symbolical to the recurrent cycles of time as calculated by the movements of the moon (Gosai 2002-2008). The four directions lie outside the *mandala* which represents the meeting of the earth and the universe as well as the movement of the sun from east to west and its rotation to the northern and southern hemispheres. The central portion of the *mandala* is the place for the Brahma, the creator, and the rest squares are the position of other Gods as per their relevance.

13. RENAISSANCE ARCHITECTURE

The basic axiom of the Renaissance architect is the conviction that the architecture is a science and that every part of a building has to be integrated into the same system of mathematical relation. This relationship between mathematics and architecture has been well documented in the History of Art from the point of view of the symbolic proportions, but not as often as from the point of view of the technical procedures. The proportion was not only a beauty problem, but also a practical method to improve new construction skills. The Renaissance tried to extract and codify the system of proportions in the orders as used by the ancients, believing that with analysis a mathematically absolute ideal of beauty would emerge.

Attention to the proportion of open spaces and building masses was an integral part of city building in the great cultural periods of the past. Studies by *Camilio Sitte* indicated that certain relationships between space and buildings were recognized in the medieval town. From these observations he estimated that the minimum dimension of a plaza should be equal to the height of the principal building facing upon it and the maximum distance should not exceed twice the height of the building. He considered that the length of a plaza should not exceed three times the width.

The architects of the Renaissance, believing that their buildings had to belong to the higher order, returned to the Greek mathematical system of proportions. As the Greeks conceived music to be geometry translated into sound, Renaissance architects believed that architecture was mathematics translated into spatial units. Applying Pythagoras, theory of mean to the ratios of the intervals of the Greek musical scale, they developed an unbroken progression of ratios that formed the base for the proportions of their architecture. These series of ratios manifested themselves not only in the dimensions of a room or a facade, but also in the interlocking proportions of a sequence of spaces or an entire plan.

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