A DEVELOPMENT OF SUPRA N-TOPOLOGY

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Abstract: The study of nearly open sets have a significant role in topological spaces. In this paper we introduce and study about weak form of open sets in supra *N*-topological spaces and obtain its characterisations. Also the notion of continuity is discussed and studied with respect to the introduced weak supra open sets.

Keywords: *N*-topology, supra *N*- topology, $N\mu$-αopen, $N\mu$-semi open $N\mu$-pre open, $N\mu$-β open.

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1. Introduction: The notions of nearly open sets namely α-open sets[9], semi-open sets[8], pre-open sets, β-open sets[1] in topological spaces paved a way to several interesting results and topics in topology. A.S. Mashhour et al [9] introduced the concept of Supra topological spaces and studied continuity in this space. J.C.Kelly [2] introduced the concept of bitopological spaces in which a non-empty set X is endowed with two arbitrary topologies. Later on many researchers introduced and devoloped various forms of open sets in this space namely $\tau_1\tau_2$[3],$\tau_{1,2}$ [4]. Lellis Thivagar et al in [6][5]., extended the concept of bitopological spaces to N topological spaces, in which a non-empty set X can be endowed with N arbitrary topologies. In this paper we introduce weak form of supra open sets in supra N-topological spaces and study its characterisations. The notion of continuity have also been studied and discussed with respect to the introduced weak open sets.

2. Preliminaries: In this section we have some basic definitions and concepts which will be helpful for a better understanding of this paper.

Definition 2.1 [8] Let X be a non-empty set and $P(X)$ denote the power set of X. A subclass $\mu \subseteq P(X)$, is said to be a supra topology on X if $X \in \mu$ and μ is closed under arbitrary union.

Definition 2.2 [3] A quasi-pseudo metric on a non-empty set X is a function $d_1 : X \times X \to R^+ \cup \{0\}$ such that
(i) $d_1(x,x) = 0$ for all $x \in X$.
(ii)$d_1(x,z)= d_1(x,y) + d_1(y,z)$ for all $x, y, z \in X$. where R^+ is the set of all positive real numbers.

Definition 2.3 [6] Let d_1 be a quasi-pseudo-metric on X, and let a function $d_2 : X \times X \to R^+ \cup \{0\}$ be defined by $d_2(x,y) = d_1(x,y)$ for all $x,y \in X$. Trivially d_2 is a quasi-pseudo-metric defined on X and we say that d_1 and d_2 are conjugate one another.

If d_1 is a quasi-pseudo-metric on X, then $B_{d_1}(x, k_1) = \{y : d_1(x,y) < k_1\}$, the open d_1-sphere with centre x and radius $k_1 > 0$. Classically, the collection of all $d_1$spheres forms a basis for a topology., the obtained topology be denoted by τ_1 and is called the quasi-pseudo-metric topology of d_1. Similarly we get a topology τ_2 for x, due to the quasi-pseudo-metric d_2.

Definition 2.4 [2] A non-empty set X equipped with two arbitrary topologies τ_1 and τ_2 is called a bitopological space and is denoted by (X,τ_1,τ_2).

Definition 2.5 [6] Let d_1 and d_2 be conjugate, quasi-pseudo-metrics on X and define a function $d_3 : X \times X \to R^+ \cup \{0\}$ by

$$d_3(x,y) = \frac{[2d_1(y,x) + d_2(y,x)]}{3}, \forall x,y \in X.$$

Then

1. $d_3(x,x) = \frac{[2d_1(x,x)+d_2(x,x)]}{3} = 0, \forall\ x \epsilon X.$
2. (ii)$d_3(x,z) = \frac{[2d_1(z,x)+d_2(z,x)]}{3} \le \frac{[2(d_1(z,y)+d_1(y,x))+d_2(z,y)+d_2(y,x))]}{3} = d_3(\mathrm{x},\mathrm{y}) + d_3(\mathrm{y},\mathrm{z})$, for all $x,y,z \in X$.

Therefore, d_3 is a quasi-pseudo-metric on X which is called a Mean Conjugate (simply write M.C) of d_1, d_2 and d_1. For each $i = 1,2,3$, the quasi pseudo-metric d_i gives a topology τ_i whose base is $\{B_{di}(x,k_i)\}$, where $\{B_{di}(x,k_i) = \{y : d_i(x,y) < k_i\}$. Thus we define a non-empty set X equipped with three arbitrary topologies τ_1, τ_2, and τ_3 is called a tritopological space and is denoted by $(X,3\tau)$ or $(X, \tau_1, \tau_2, \tau_3)$. Generally, let d_1, d_2, $\cdots d_{N-1}$ be quasi-pseudo-metrics on X., d_1 and d_2 be conjugate and d_3, d_4,$\cdots d_{N-1}$
be M.C of d_1, d_2 and d_1; d_1, d_2, d_3 and d_1; $\cdots$; d_1, d_2, $\cdots$, d_{N-2} and d_1, respectively. Define a function $d_N : X \times X \to R^+ \cup \{0\}$ by

$$d_N(x,y) = \frac{[d_1(y,x) + \sum_{i=1}^{N-1} d_i(y,x)]}{N}, \forall x,y \in X$$

We can easily verify that d_N is a quasi-pseudo-metric on X. Also we note that for each N, $d_N(x,y)$ 6= $d_N(y,x)$ for all $x,y \in X$ and d_N is called a Mean Conjugate(simply write M.C) of $d_1,d_2,\cdots,d_{N-1}$ and d_1. For each i–1,2,$\cdots$,N, the quasi-pseudo metric d_i gives a topology τ_i whose basis is $\{B_{di}(x,k_i)\}$, where $B_{di}(x,k_i) = \{y : d_i(x,y) < k_i\}$. Thus we define a non-empty set equipped with N-arbitrary topologies $\tau_1,\tau_2,\cdots,\tau_N$ is called a N-topological space and is denoted by $(X,N\tau)$ or $(X,\tau_1,\tau_2,\cdots,\tau_N)$.

Definition 2.6 [6] Let X be a non-empty set, $\tau_1,\tau_2,\cdots,\tau_N$ be N-arbitrary topologies defined on X and let the collection $N\tau$ be defined by

$$N_\tau = \{S \subseteq X : S = (\cup_{i=1}^{N} A_i) \cup (\cap_{i=1}^{N} B_i), A_i, B_i \in \tau_i\}$$

satisfying the following axioms:
(i) $\emptyset$, $X \in N\tau$.
(ii) $\cup_{i=1}^{\infty} S_i \in N_\tau$ for all $S_i \in N_\tau$.
(iii)$\cap_{i=i}^{n} S_i \in N_\tau$ for all $S_i \in N_\tau$.
Then the pair $(X,N\tau)$ is called a N-topological space on X and the elements of the collection $N\tau$ are called $N\tau$ open sets on X. A subset A of X is said to be $N\tau$-closed on X if the complement of A is $N\tau$-open on X.

Definition 2.7 [5] Let X be a non-empty set. Let μ_1, μ_2, ..., μ_N be N-arbitrary supra topologies on X. Then the collection $N\mu$ defined on X as

$$N_\mu = \{ A \subseteq X : A = (\cup_{i=1}^{N} U_i) \cup (\cap_{i=1}^{N} V_i), U_i, V_i \in \mu_i\}$$

satisfying the following conditions:
(i) $\emptyset$, $X \in N\mu$
(ii) $\cup_{i=1}^{\infty} A_i \in N_\mu$ for all $A_i \in N_\mu$.
Now the pair $(X, N\mu)$ is called a supra N-topological space. The elements of the collection $N\mu$ are called supra $N\mu$-open sets and the complement of supra $N\mu$-open sets are called supra $N\mu$-closed sets.

Definition 2.8 [5] Let $(X, N\tau)$ be a N-topological space and $N\mu$ be a supra N-topology on X. We say that $N\mu$ is a supra N-topology associated with $N\tau$ if $N\tau \subseteq N\mu$.

3. Weak Form of Open Sets in Supra N-Topological Spaces: In this section we introduce weak form of supra $N\mu$-open sets and obtain its characterisations in the space.

Definition 3.1 Let $(X, N\mu)$ be a supra N-topological space and $A \subseteq X$. Then A is said to be

1. $N\mu$-α open if $A \subseteq N\mu$-int($N\mu$-cl($N\mu$-int(A))).
2. $N\mu$-semi open if $A \subseteq N\mu$-cl($N\mu$-int(A)).
3. $N\mu$-pre open if $A \subseteq N\mu$-int($N\mu$-cl(A).
4. $N\mu$-β open if $A \subseteq N\mu$-cl($N\mu$-int($N\mu$-cl(A))).

The set of all $N\mu$-α open (resp. $N\mu$-semi open, $N\mu$-pre open, $N\mu$-β open) sets of $(X,N\mu)$ is denoted by $N\mu$-$\alpha O(X,N\mu)$(resp. $N\mu$-$SO(X,N\mu)$, $N\mu$-$PO(X,N\mu)$, $N\mu$-$\beta O(X,N\mu)$).
The complement of a $N\mu$-α open (resp. $N\mu$-semi open, $N\mu$-pre open, $N\mu$-β open) set is called $N\mu$-α closed set (resp. $N\mu$-semi closed, $N\mu$-pre closed) and is denoted by $N\mu\alpha C(X, N\mu)$(resp.$N\mu$-$SC(X,N\mu)$, $N\mu$-$PC(X,N\mu)$, $N\mu$-$\beta C(X,N\mu)$).

Example 3.2 For $N = 2$, Let $X = \{a,b,c,d\}$ and $\mu_1 = \{\emptyset,X,\{b,c,d\},\{a,c,d\}\}$, $\mu_2 = \{\emptyset,X,\{a,c,d\},\{b,d\}$ be two supra topologies on X and $2\mu = \{\emptyset,X,\{d\},\{b,d\},\{c,d\},$
$\{a,c,d\},\{b,c,d\}\}$ be the supra 2μ topology on X. Then 2μ-$\alpha O(X,2\mu) = \{\emptyset, X, \{d\}, \{a,d\},$
$\{b,d\},\{c,d\},\{a,c,d\},\{a,b,d\},\{b,c,d\}\} = 2\mu$-$SO(X,2\mu) = 2\mu$-$PO(X,2\mu) = 2\mu$-$\beta O(X,2\mu)$.

Definition 3.3 Let $(X, N\mu)$ be a supra N-topological space and $A \subseteq X$. Then

1. $N\mu$-α (resp. $N\mu$-semi, $N\mu$-pre, $N\mu$-β) interior of A is the union of all $N\mu$-α (resp. $N\mu$-semi, $N\mu$-pre, $N\mu$-β) open sets contained in A and is denoted by $N\mu$-αint(A) (resp. $N\mu$-semi int(A), $N\mu$-pre int(A), $N\mu$-βint(A)).
2. $N\mu$-α (resp. $N\mu$-semi, $N\mu$-pre, $N\mu$-β) closure of A is the intersection of all $N\mu\alpha$ (resp. $N\mu$-semi, $N\mu$-pre, $N\mu$-β) closed sets containing A and is denoted by $N\mu\alpha$cl(A) (resp. $N\mu$-semi cl(A), $N\mu$-pre cl(A), $N\mu$-βcl(A)).

Remark 3.4 Let $(X, N\mu)$ be a supra N-topological space and $A \subseteq X$. Then

1. $N\mu$-α (resp. $N\mu$-semi, $N\mu$-pre, $N\mu$-β) interior of A is $N\mu$-α (resp. $N\mu$-semi, $N\mu$-pre, $N\mu$-β) open in $(X,N\mu)$.
2. $N\mu$-α (resp. $N\mu$-semi, $N\mu$-pre, $N\mu$-β) closure of A is $N\mu$-α (resp. $N\mu$-semi, $N\mu$-pre, $N\mu$-β) closed in $(X,N\mu)$.

Proposition 3.5 Let $(X, N\mu)$ be a supra N-topological space and $A \subseteq X$. Then the following hold:

1. $N\mu$-αint(A) $\subseteq A$.
2. A is $N\mu$-α open in $X \Leftrightarrow A = N\mu$-$\alpha$int($A$).
3. $A \subseteq B \Rightarrow N\mu$-$\alpha$int($A$) $\subseteq N\mu$-αint(B).

Proposition 3.6 Let $(X, N\mu)$ be a supra N-topological space and $A \subseteq X$. Then the following hold:

1. $A \subseteq N\mu$-αcl(A).
2. A is $N\mu$-α closed in $X \Leftrightarrow A = N\mu$-$\alpha$cl($A$).
3. $A \subseteq B \Rightarrow N\mu$-$\alpha$cl($A$) $\subseteq N\mu$-αcl(B).

Theorem 3.7 Let $(X, N\mu)$ be a supra N-topological space and $A \subseteq X$. Then the following hold:

1. $X\backslash N\mu$-αint(A) = $N\mu$-αcl($X\backslash A$).
2. $X\backslash N\mu$-αcl(A) = $N\mu$-αint($X\backslash A$).

Proof: Proof is straightforward.

Remark 3.8 Theorem 3.5,3.6,3.7 also hold for $N\mu$-semi, $N\mu$-pre and $N\mu$-β open sets.

Theorem 3.9 Let $(X, N\mu)$ be a supra N-topological space and $A,B \subseteq X$ and A be $N\mu$-α closed, then $N\mu$-αcl($A \cap B$) $\subseteq A \cap N\mu$-αcl(B).
Proof: Let $A \subseteq X$ be $N\mu$-α closed, then we have $A = N\mu$-αcl(A). Now $N\mu$-αcl($A \cap B$) $\subseteq N\mu$-αcl(A) $\cap$ $N\mu$-αcl(B) = $A \cap N\mu$-αcl(B) which completes the proof.

Remark 3.10 Theorem (3.9) also holds for $N\mu$-semi, $N\mu$-pre and $N\mu$-β closed sets.

Theorem 3.11 Let $(X, N\mu)$ be a supra N-topological space. Then the following hold:

1. Every supra $N\mu$-open set is $N\mu$-α open.

2. Every supra $N\mu$-open set is $N\mu$-semi open.
3. Every supra $N\mu$-open set is $N\mu$-pre open.
4. Every supra $N\mu$-open set is $N\mu$-β open.

Proof: Let A be a $N\mu$-supra open set of a supra N-topological space $(X, N\mu)$. Then by Proposition 3.5(i) we have $A = N\mu$-int(A).

1. Now $A = N\mu\text{-}int(A) \subseteq N\mu\text{-}cl(N\mu\text{-int}(A))$. Thus, $A \subseteq N\mu\text{-}cl(N\mu\text{-int}(A))$. Hence, $A = N\mu\text{-int}(A) \subseteq N\mu\text{-int}(N\mu)\text{-cl}(N\mu\text{-int}(A))$, and this completes the proof.
2. Similar to proof of (i).
3. We have by Proposition 3.6(ii) $A \subseteq N\mu\text{-cl}(A)$. Hence $A = N\mu\text{-int}(A) \subseteq N\mu\text{int}(N\mu\text{-cl}(A))$, which implies A is $N\mu$-pre open and this completes the proof of (iii).
4. we have $A = N\mu\text{-}int(A) \subseteq N\mu\text{-cl}(A) \subseteq N\mu\text{-cl}(N\mu\text{-int}(N\mu\text{-cl}(A)))$ and this completes the proof of (iv).

Remark 3.12 In Example (3.2), $\{a,d\}$ is $N\mu$-α open, $N\mu$-semi open, $N\mu$-pre open, and $N\mu$-β open, which is not 2μ-supra open.

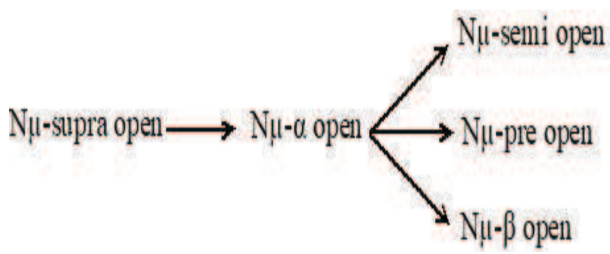
Theorem 3.13 In a supra N-topological space $(X,N\mu)$ the following hold:

1. Every $N\mu$-α open set is $N\mu$-semi open.
2. Every $N\mu$-α open set is $N\mu$-pre open.
3. Every $N\mu$-α open set is $N\mu$-β open

Proof:

1. Let A be a $N\mu$-α open set in $(X, N\mu)$. Then, $A \subseteq N\mu\text{-}int(N\mu\text{-}cl(N\mu\text{-}int(A)))$. We have, $N\mu\text{-int}(N\mu\text{-cl}(N\mu\text{-int}(A))) \subseteq N\mu\text{-cl}(N\mu\text{-int}(A))$. Thus, $A \subseteq N\mu\text{-cl}(N\mu\text{-int}(A))$ and this completes the proof of(i).
2. (iii) Similar to proof of (i).

Remark 3.14 We arrive at the following diagram:



Proposition 3.15 Let $(X, N\mu)$ be a supra N-topological space. Then, the intersection of a $N\mu$-open set and a $N\mu$-semi open (resp. $N\mu$-pre open, $N\mu$-β open) set is $N\mu$-semi open (resp. $N\mu$-pre open, $N\mu$-β open).

Theorem 3.16 Let $(X, N\mu)$ be a supra N-topological space and $A \subseteq X$. If A is both $N\mu$-β open and $N\mu$-semi closed then A is $N\mu$-semi open.
Proof: Since A is $N\mu$-β open and $N\mu$-semi closed we have, $A \subseteq N\mu\text{-cl}(N\mu\text{-int}(N\mu\text{-cl}(A)))$ and $N\mu\text{-int}(N\mu\text{-cl}(A)) \subseteq A$. Therefore we have, $N\mu\text{-int}(N\mu\text{-cl}(A)) \subseteq N\mu\text{-int}(A)$, which implies $N\mu\text{-cl}(N\mu\text{-int}(N\mu\text{-cl}(A))) \subseteq N\mu\text{-cl}(N\mu\text{-int}(A))$. Hence $A \subseteq N\mu\text{cl}(N\mu\text{-int}(N\mu\text{-cl}(A))) \subseteq N\mu\text{-cl}(N\mu\text{-int}(A))$ and this completes the proof.

Corollary 3.17: Let $B \subseteq X$ in a supra N-topological space $(X, N\mu)$. If B is both $N\mu$-β closed and $N\mu$-semi open then B is $N\mu$-semi closed.
Proof: Since B is $N\mu$-β closed and $N\mu$-semi open then $X\backslash B$ is $N\mu$-β open and $N\mu$-semi closed. Then by Theorem(3.16), $X\backslash B$ is $N\mu$-semi open which implies B is $N\mu$-semi closed and this completes the proof.

4. Continuous Functions on Weak Supra $N\mu$-Open Sets

In this section we introduce the notion of continuity with respect to weak form of supra $N\mu$-open sets and obtain its characterisations.

Definition 4.1 Let $(X, N\tau_1)$ and $(Y, N\tau_2)$ be two N topological spaces and let $N\mu_1$ be a supra N- topology associated with $N\tau_1$. A function $f : X \to Y$ is said to be

1. $N\mu$-α continuous if the inverse image of every $N\tau_2$-open set in Y is $N\mu_1$-α open in X.
2. $N\mu$-semi continuous if the inverse image of every $N\tau_2$-open set in Y is $N\mu_1$-semi open in X.
3. $N\mu$-pre continuous if the inverse image of every $N\tau_2$-open set in Y is $N\mu_1$-pre open in X.
4. $N\mu$-β continuous if the inverse image of every $N\tau_2$-open set in Y is $N\mu_1$-β open in X.

Example 4.2 For $N = 2$, Let $X = \{a,b,c,d\}$ and $\tau_1 = \{\emptyset,X\},\tau_2 = \{\emptyset,X,\{a,d\}\}$. Then $2\tau = \{\emptyset, X, \{a,d\}\}$ is the bi-topology on X. Let $\mu_1 = \{\emptyset, X, \{a,d\},\{b,d\},\{a,b,d\}\}$, $\mu_2 = \{\emptyset,X,\{a,c,d\},\{a,b,d\}$ be two supra topologies on X and $2\mu = \{\emptyset,X,\{d\},\{a,d\}$,
$\{b,d\},\{a,c,d\},\{a,b,d\}\}$ be the corresponding supra 2μ topology on X. Then we have 2μ is a supra bi-topology associated with 2τ. Now 2μ-$\alpha O(X,2\mu) = \{\emptyset, X, \{d\}, \{a,d\},\{b,d\},\{c,d\},\{a,c,d\},\{a,b,d\}$,
$\{b,c,d\}\}=2\mu SO(X,2\mu)=2\mu PO(X,N\mu)=2\mu\beta O(X,N\mu))$. Let $Y = \{a,b,c\}$ and $\mu_1 = \{\emptyset,Y,\{a\}\},\mu_2 = \{\emptyset,Y,\{a,b\},\{b,c\}\}$ be two supra topologies on X. Now the collection $2\mu = \{\emptyset, Y, \{a\}, \{a,b\}, \{b,c\}\}$ is a supra bi topology on Y. Define $f: X \to Y$ by $f(a) = c, f(b) = b = f(c) = f(d)$. Then f is 2μ-α continuous, 2μ-semi continuous, 2μ-pre continuous and 2μ-β continuous.

Theorem 4.3 Let $(X, N\tau_1)$ and $(Y, N\tau_2)$ be two N topological spaces and let $N\mu_1$ be a supra N- topology associated with $N\tau_1$. Let $f : X \to Y$ be a function. Then the following hold:
1. Every $N\mu$-continuous function is $N\mu$-α continuous.
2. Every $N\mu$-continuous function is $N\mu$-semi continuous.
3. Every $N\mu$-continuous function is $N\mu$-pre continuous.

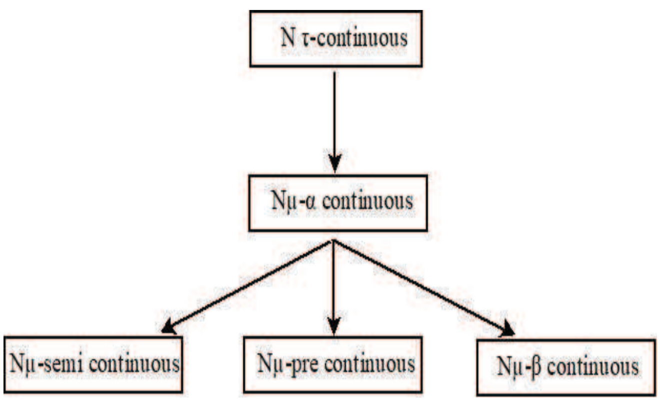
Proof:
1. Let $f: X \to Y$ be a $N\tau$-continuous function and V be a $N\tau_2$-open set in Y . Since f is $N\tau$-continuous, we have $f^{-1}(V)$ is supra $N\mu$-open in X. Since, every supra $N\mu$-open set is $N\mu$-α open we have $f^{-1}(V)$ is $N\mu$-α open in X and this completes the proof of (i).
2. Similar to proof of (i).

Remark 4.4 Since every $N\tau$-continuous function is $N\mu$-continuous, Theorem 3.19 also holds for $N\tau$-continuous functions.

Theorem 4.5 Let $(X, N\tau_1)$ and $(Y, N\tau_2)$ be two N topological spaces and let $N\mu_1$ be a supra N-topology associated with $N\tau_1$. Let $f : X \to Y$ be a function. Then the following hold:
1. Every $N\mu$-α continuous function is $N\mu$-semi continuous.
2. Every $N\mu$-α continuous function is $N\mu$-pre continuous.
3. Every $N\mu$-α continuous function is $N\mu$-β continuous.

Proof: Proof follows directly from Theorem 3.4.

Remark 4.6 We have the following diagram:



Theorem 4.7 Let $(X, N\tau_1)$ and $(Y, N\tau_2)$ be two N topological spaces and let $N\mu_1$ be a supra N-topology associated with $N\tau_1$. Let $f : X \to Y$ be a function. Then the following are equivalent:
1. f is $N\mu$-α continuous.

2. The inverse image of every $N\tau_2$ closed set in Y is $N\mu$-α closed in X.
3. $N\mu$-αcl$(f^{-1}(B)) \subseteq f^{-1}(N\tau_2 cl(B))$, for every set B in Y.
4. $f(N\mu$-αcl$(A)) \subseteq N\tau_2 cl(f(A))$, for every set A in X.
5. $f^{-1}(N\tau_2$-int$(B)) \subseteq N\mu$-αint$(f^{-1}(B))$, for every set B in Y.

Proof: (i)⇒(ii) Let V be $N\tau_2$-closed in Y. Then $Y \setminus V$ is $N\tau_2$-open in Y. Now by our assumption, $f^{-1}(Y \setminus V) = X \setminus f^{-1}(V)$ is $N\mu$-α open in X, and thus we have $f^{-1}(V)$ is $N\mu$-α closed in X.
(ii)⇒(iii) Let B be any set in X. Therefore $N\tau_2 cl(B)$ is $N\tau_2$-closed in Y and thus by our assumption, $f^{-1}(N\tau_2 cl(B))$ is $N\mu$-α closed in X. Now, $f^{-1}(N\tau_2 cl(B)) = N\mu\alpha$cl$(f^{-1}(N\tau_2 cl(B))) \supseteq N\mu$-$\alpha$cl$(f^{-1}(B))$.
(iii)⇒(iv) Let A be any set in X. Then $f(A) \subseteq Y$ and hence by our by our assumption, $f^{-1}(N\tau_2 cl(B)) \supseteq N\mu$-$\alpha$cl$(f^{-1}(f(A))) \supseteq\supseteq N\mu$-$\alpha$cl$(A)$. Thus we get $(N\tau_2 cl(B) \supseteq f(N\mu\alpha$cl$(A))$.
(iv)⇒ (v) Assume $f(N\mu$-αcl$(A)) \subseteq N\tau_2 cl(f(A))$, for every set A in X. Then, $N\mu\alpha$cl$(A)) \subseteq f^{-1}(N\tau_2 cl(f(A)))$. Now, $X \setminus N\mu$-αcl$(A)) \supseteq X \setminus f^{-1}(N\tau_2 cl(f(A)))$ and $N\mu\alpha$-int$(X \setminus A) \supseteq f^{-1}(N\tau_2 int(Y \setminus f(A)))$. Thus, $N\mu$-α-int$(f^{-1}(B)) \supseteq f^{-1}(N\tau_2 int(B))$, where $B = Y \setminus f(A)$. Thus we have, $N\mu$-α-int$(f^{-1}(B)) \supseteq f^{-1}(N\tau_2 int(B))$, for every set B in Y.
(v)⇒ (i) Let V be $N\tau_2$ open in Y. Then by our assumption, $f^{-1}(N\tau_2$-int $(V)) \subseteq N\mu$-αint$(f^{-1}(V))$, and thus $f^{-1}(V) \subseteq N\mu$-αint$(f^{-1}(V))$. But always we have $N\mu\alpha$-int$(f^{-1}(V)) \subseteq f^{-1}(V)$. Hence we have $f^{-1}(V) = N\mu$-αint$(f^{-1}(V))$, which implies $f^{-1}(V)$ is $N\mu$-α open in X and this completes the proof.

Theorem 4.8 Let $(X,N\tau_1)$ and $(Y,N\tau_2)$ be two N topological spaces and let $N\mu_1$ be a supra N-topology associated with $N\tau_1$. Let $f : X \to Y$ be a function. Then the following are equivalent:

1. f is $N\mu$-β continuous.
2. The inverse image of every $N\tau_2$ closed set in Y is $N\mu$-β closed in X.
3. $N\mu$-βcl$(f^{-1}(V)) \subseteq f^{-1}(N\tau_2 cl(V))$, for every set V in Y.
4. $f(N\mu$-βcl$(U)) \subseteq N\tau_2 cl(f(U))$, for every set U in X.
5. $f^{-1}(N\tau_2$-int$(V)) \subseteq N\mu$-βint$(f^{-1}(V))$, for every set V in Y.

Proof: Similar to proof of Theorem 4.7.

5. Conclusion: The ideas of weak open sets in N supra topological spaces are introduced and studied. The notion of continuity has also been discussed with respect to the introduced weak open sets. This work can be further extended to study several other weak and strong forms of open sets in this space which will hopefully bear interesting results in the area of research.

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