ON $\widehat{\beta}g$ - SOFT SEMI OPEN SETS AND SOFT SEMI NEIGHBOURHOODS SETS IN SOFT TOPOLOGICAL SPACES

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Abstract: In this piece we carry on to investigate the properties of soft set semi open sets in soft topological spaces. The objective of this work is to describe the concept of $\widehat{\beta}$g- soft semi open sets and $\widehat{\beta}$g- soft semi-neighborhoods with few examples and prove that $\widehat{\beta}$g- soft semi nbhd need not be a $\widehat{\beta}$g- soft semi open topological spaces.

Keywords: Soft Semi-Open Sets ,Soft Semi-Neighborhoods, $\widehat{\beta}$g- Soft Semiopen – $\widehat{\beta}$g Soft Semi-Neighborhoods.

1. **Introduction:** Soft set theory concept has been introduced by Molodstov [8] this soft set content has been functional to many branches of applied mathematics such as Riemann Integration, operations research etc. The notion of a topological spaces for soft sets was formed by shabir and Naz [10] their work was showed the initial universe with a fixed set of parameters.

Levine introduced generalized closed sets in general topology .Kannan [7] established generalized soft closed and open sets in soft topological spaces.

In this piece we established a new category of sets namely $\widehat{\beta}$g- soft semi open sets in soft topological spaces. Furthermore , these research not only can form the speculative concept for further applications of soft topology on soft set but also lead to the enlargement of information.

2. Preliminaries:

2.1 Definition: [8]: A pair (F,E) is called a soft set over X_E if and only if F is a mapping of E into the set of all subsets of the set X_E.
In other words , the soft set is a parameterized family of subsets of the set X_E .
Each set F (e) , e $\in$ E. From this family may be considered as the set of *e*- approximate elements of the soft set.

2.1.1Example: A soft set (F,E) describes the attractiveness of the houses which Mr.Y is going to buy

- X_E – is the set of houses under thoughtfulness .
- E – is the set of parameters ,Each Parameter is a word or a sentence is given below
- {luxurious ,wooden, fine-looking, low cost ,in the green surrounding, fresh, in good repair, in bad repair }

To define a soft set means to point out a collection of houses in E. In this case the sets F (may be arbitrary) some of them may be empty, Some of them may be non-empty.

2.2 Definition: [6]: A soft set F on the universe X with the set E of parameters is defined by the set of ordered pairs
$F = \{ (e , F (e)) : e \in E , F (e) \varepsilon P (X) \}$,where F is a mapping given by $F : E \to P(X)$. The family of all soft sets over X is denoted by SS (X_E).

2.3 Definition [4]: A soft set is a parameterized family of subsets of the set X. For $e \in E$, M(e) can be considered as the set of e-approximate elements of the soft set (M, E). According to this manner, we can view a soft set (M, E, X) as a consisting of collection of approximations:
$(M, E) = \{M(e) : e \in E\}$.

2.3.1 Example: Let a soft set (M, E, X) describe the attractiveness of the skirts with respect to the parameters, which Mrs. A is going to wear. Suppose that there are four skirts in the universe
$X = \{x_1, x_2, x_3, x_4\}$ under consideration and $E = \{e_1 = \text{cheap}, e_2 = \text{expensive}, e_3 = \text{colorful}\}$ is the set of parameters. To define a soft set means to point out cheap skirts, expensive skirts and colorful skirts. Suppose that $M(e_1) = \{x_1, x_2\}$, $M(e_2) = \{x_3, x_4\}$, $M(e_3) = \{x_1, x_3, x_4\}$. Then the family $\{M(e_i) : i = 1, 2, 3\}$ of 2X is a soft set (M, E, X).

2. 4 Definition: [4]: Given two soft structures $M_1 : E \to 2\ X$, $M_2 : E \to 2\ X$ over the set X we say that M_1 weaker than M_2 if $M_1(e) \subseteq M_2(e)$ for every $e \in E$. We write in this case $M_1 \leq M_2$.

2.5 Definition: [3]: Let (X, τ, E) be a soft topological space. Then, every element of τ is called a soft open set. clearly, ϕ_E and X_E are soft open sets.

2.6 Definition: **[4]:** For two soft sets (M, E) and (N, E) we say that (M, E) is a soft subset of (N, E) and write
$(M, E) \subseteq (N, E)$ if for each $e \in E$, $M(e) \subseteq N(e)$. (M, E) is called a soft super set of (N, E) if
(N, E) is a soft subset of (M, E), and we write $(M, E) \sqsupseteq (N, E)$.

2.7 Definition: [3]: Let (X, τ, E) be a soft topological space and $(F,B) \subseteq (F,A)$. Then, (F,B) is said to be soft closed if the soft set $(F,B)^c$ is soft open.

2.8 Definition: [3]: Let (X, τ, E) be a soft topological space and $(F,B) \subseteq (F,A)$. Then, the soft interior of a soft set (F,B) is denoted by $(F,B)^\circ$ and is defined as the soft union of all soft open subsets of (F,B). Thus, $(F,B)^\circ$ is the largest soft open set contained in (F,B).

2.9 Definition: [3]: Let (X, τ, E) be a soft topological space and $(F,B) \subseteq (F,A)$ .Then, the soft closure of (F,B) denoted by $\overline{(F,B)}$ is defined as the soft intersection of all soft closed supersets of (F,B). Note that $\overline{(F,B)}$ is the smallest soft closed set containing (F,B) .

2.10 Definition [11]: A soft set (F,E) over X_E is said to be a soft element if there exist $\propto \in E$ such that
E (∝) is a singleton , say {x} and $E(\beta) = \phi_E$, $\forall \beta (\neq \propto) \in A$ such a soft element is denoted by E^x_α

2.11 Definition: **[11]:** For any pair off soft sets (F,A) and (G,B) over a common Universe X_E, (F,A) is a soft subset of (G,B), If $A \subseteq B$ and $\forall a \in A$, $F(a) \subseteq G(a)$. It is denoted by $(F,A) \subseteq (G,B)$.

2.12 Definition: **[7]:** Two soft sets (F,A) and (G,B) over a common Universe X_E is said to be soft equal if $(F,A) \subseteq (G,B)$ and $(G,B) \subseteq (F,A)$.

Result: If (F,A) = (G,B) then A = B.

2.13 Definition: **[7]:** The complement of a soft set (F,A) denoted by (F, A) ^{c} ,It is defined by $F^c : A \to P(X_E)$ is a mapping given by $F^c(a) = U - F(a)$, ∀ a ∈ A .F^c is called the soft complement function of F . clearly $(F^c)^c$ is the same as F and $((F,A)^c)^c = (F,A)$.

2.11 Definition: **[7]:** A soft set (F, E) over X_E is said to be a null soft set , It is defined by if ∀ e ∈ E,F (e) = ϕ. It is denoted by ϕ_E

2.12 Definition:[7]: A soft set (F,E) over X is said to be absolute soft set denoted by X_E if ∀ e ∈ E , F (e)=X_E.
Clearly $X_E{}^c = \phi_E$ and $\phi_E{}^c = X_E$.

2.13 Definition:**[9]:** Let Y be a non-empty subset of X_E , then Y denotes the soft set (Y,E) over X_E for which Y (e) = Y_E, for all e ∈ E. In particular, (X,E) will denoted by X_E.

2.14 Definition:**[7]:** The union of two soft sets (F,A) and (G,B) over a common universe X_E is the soft set (H ,C) where C= A∪B and for all e ∈C , H (e) = F(e) if e ∈ A\ B , H (e) = G (e) if e ∈ B \ A and H (e) =F(e)∪G (e) if e∈A∩B . It is denoted by (F, E)∪(G, B) = (H,C).

2.15 Definition:**[7]:** The intersection of two soft sets (F,A) and (G B) over a common universe X_E is the soft set (H ,C) where C = A ∩ B and for all e ∈ C,H(e) = F (e)∩G(e). This relationship can be written as (F,A) ∩ (G,B) = (H,C).
We denote the family of these soft sets are SS $(X)_E$.

2.16 Definition: **[11]:** Let τ be the collection of a soft set over a universe X_E with a fixeed set of parameter E , then τ ⊆ SS $(X)_E$ is called a soft topology on X_E if

- X_E, $\phi_E \in \tau$.
- The union of any number of soft sets in τ belongs to τ
- The intersection of any two soft sets s in τ belongs to τ

(X, τ , E) is called a soft topological space.

2.17 Definition: **[10]:** The soft neighborhood system of a soft element E^x_α. It is denoted by $N_\alpha(E^x_\alpha)$, It is the family of all its soft neighborhoods.

2.18 Definition: **[11]:** Let (X,τ, E) be a soft topological space over X_E and (F,E) be a soft set over X_E.Then (F,E) is called soft semi open if and only if there exist an open set (G,E) such that (G,E)⊆(F,E) ⊆$\overline{(G,E)}$.

2.19 Definition: **[11]:** Let (X,τ,E) be a soft topological space over X_E. A soft set (F,E) is said to be soft semi interior if it is the union of all soft semi open sets contained in (F,E).

2.20 Definition: **[11]:** Let (X,τ, E) be a soft topological space over X_E and (O,E) be a soft set over X_E and x ∈ X_E.
Then x is said to be soft semi interior point of (G,E) if there exist a semi open set (H,E) such that x⊆(H,E)⊆ (O,E).

2.21 Definition: **[11]:** Let (X,τ,E) be a soft topological space over X_E and (N,E) be a soft set over X_E and {{x}, φ}∈ X_E. Then (N,E) is said to be soft semi neighborhood of {{x},φ} if there exist a soft semi open set (O,E) such that {{x}, φ}⊆ (O,E) ⊆ (N,E).

2.22. Definition: **[7]:** A soft subset (F,E) of a soft topological space (X,τ, E) is called $\hat{\beta}$g- soft semi closed set , if cl int cl (F,E) ⊆ (U, E) whenever (F,E)⊆(U,E) and (U, E) is soft semi open set in X_E.

Main Results

3. $\hat{\beta}$-Generalized Soft Semi Open Sets

3.1 Definition: A soft subset (F,E) in X_E is called $\hat{\beta}$g soft semi open (briefly $\hat{\beta}$g-soft semi open) in X_E if $(F, E)^c$ is $\hat{\beta}$g-soft semi closed in X_E. We denote the family of all $\hat{\beta}$g-soft semi open sets in X_E by $\hat{\beta}$ g SSO(X_E).

3.2 Example: The following example shows that the $\hat{\beta}$g-soft semi open sets in a soft topological space X_E.

Let X= {a_1, a_2, a_3 } E= { e_1, e_2 } τ = {Φ, X, (G_1 , E), (G_2 , E), (G_3 , E)}
{(G_4, E), (G_5, E), (G_6, E), (G_7, E)}
where (G_1, E)= {{a_1,a_2},{a_1,a_2 }}; (G_2, E) ={{a_2 }, { a_1,a_3 }}; (G_3 , E) ={{a_2,a_3},{a_1 }}
(G_4, E) = {{a_2},{a_1}}; (G_5, E) = {{a_1,a_2},X}} ; (G_6,E) = {X ,{a_1, a_2}}
(G_7, E) = {{ a_2,a_3},{ a_1,a_3}}

Solution:

Soft Set	Interior of the S.Set
{{a_1,a_2},{a_1,}} {{a_1,a_2},{a_2}} {{ a_1}{a_1,a_2 }} {{a_2},{a_1,a_2 }} {{a_1, a_2},{Φ }} {{Φ},{a_1,a_2 }} {{ a_1, {a_1 }} {{a_2}, {a_2 }} {{a_1,a_2},{a_1,a_2 }} {{a_2}, {Φ }} {{Φ},{a_1}} {{Φ},{ a_3 }} {{a_2 a_3},{Φ }} {{a_3},{a_1}} {{a_2,a_3},{ a_1}} {{a_1,a_2},{Φ }} {{a_2},{a_3 }} {{ a_1},X} {{Φ}, X} { X, {Φ }} {{a_2 a_3},{ a_3}} {{ a_3},{ a_1,a_3}} {{ a_3},{ a_3}}	❖ {{Φ },{{Φ }
{{a_2},{a_1,a_3}} {{a_2},{a_1 }} {{a_2 a_3},{a_1}} {{a_2}, X } {{X},{ a_1}} { X, {a_1,a_2 }}	❖ (G_4, E)
{{a_2}, X}	❖ (G_2, E)

Soft Set	Closure of the S. Set
$\{\{a_1,a_2\},\{a_1,\}\}$ $\{\{a_1,a_2\},\{a_2\}\}$ $\{\{a_1\}\{a_1,a_2\}\}$ $\{\{a_2\},\{a_1,a_2\}\}$ $\{\{a_1, a_2\},\{\Phi\}\}$ $\{\{\Phi\},\{a_1,a_2\}\}$ $\{\{a_1\}, \{a_1\}\}$ $\{\{a_2\}, \{a_2\}\}$ $\{\{a_1,a_2\},\{a_1,a_2\}\}$ $\{\{a_2\}, \{\Phi\}\}$ $\{\{\Phi\},\{a_1\}\}$ $\{\{\Phi\},\{a_3\}\}$ $\{\{a_2 a_3\},\{\Phi\}\}$ $\{\{a_3\},\{a_1\}\}$ $\{\{a_2,a_3\},\{a_1\}\}$ $\{\{a_1,a_2\},\{\Phi\}\}$ $\{\{a_2\},\{a_3\}\}$ $\{\{a_1\},X\}$ $\{\{\Phi\}, X\}$ $\{X, \{\Phi\}\}$ $\{\{a_2 a_3\},\{a_3\}\}$ $\{\{a_3\},\{a_1,a_3\}\}$ $\{\{a_3\},\{a_3\}\}$	❖ $\{\{X_E\},\{X_E\}\}$
$\{\{a_1\}\{a_2\}\}$ $\{\{a_1\},\{a_3\}\}$ $\{\{a_1\},\{a_2 a_3\}\}$ $\{\{\Phi\},\{a_2 a_3\}\}$ $\{\{\Phi\},\{a_3\}\}$ $\{\{a_1\},\{\Phi\}\}$	❖ $(G_3 , E)^c$
$\{\{a_3\},\{a_3\}\}$ $\{\{a_1\}\{a_2\}\}$ $\{\{a_1\},\{a_3\}\}$ $\{\{a_3\},\{a_2\}\}$ $\{\{a_3\},\{\Phi\}\}$	❖ $(G_4, E)^c$

3.3 Theorem: If (F,E) and (G,E) are $\hat{\beta}$ g- SSO(X_E) in a soft topological space X_E. Then (F,E)∩(G,E) is also $\hat{\beta}$ g SSO(X_E).

Proof: Let (F,E) and (G,E) are $\hat{\beta}$g- SSO(X_E) in a soft topological space X_E. Then $(F, E)^C$ and $(G, E)^C$ are $\hat{\beta}$g-soft semi closed sets in X_E. we have $(F, E)^C \cup (G, E)^C$ is also $\hat{\beta}$g-Soft semi closed set in X_E. That is $(F, E)^C \cup (G, E)^C = ((F, E) \cap (G, E))^C$ is a $\hat{\beta}$g-Soft semi closed set in X_E.

Therefore (F,E)∩(G,E) is $\hat{\beta}$g- SSO(X_E).

3.4 Remark: The $\hat{\beta}$g- soft semi nbhd (N,E) of $\{\{a_2\}, \varphi\}$need not be a $\hat{\beta}$g- SSO(X_E)

Example: Let X={ a_1, a_2, a_3 }, $\tau = \{\{X,\varphi,(G_1,E),(G_2,E),(G_3,E)\ldots\ldots (G_7, E)\}\}$.

Then $\hat{\beta}$g SSO(X)={ $X_E,\varphi,\{\{a_3\},\{\Phi\}\},\{\{a_1\},\{a_2a_3\}\}\{\{a_1\}\{a_2\}\}\{\{a_3\},\{a_2\}\}$.......}.

Now $\{\{\Phi\},\{a_3\}\}$ is a $\hat{\beta}$g-soft.semi nbhd of $\{\{\Phi\},\{\Phi\}\}$ but it is not a $\hat{\beta}$g- SSO(X_E).

Hence $\hat{\beta}$g- soft semi nbhd (N,E) of $\{\{\Phi\},\{a_3\}\} \in X_E$ need not be $\hat{\beta}$g- SSO(X_E).

3.5 Theorem: If a soft subset (N,E) of a space X_E is $\hat{\beta}$ g- SSO(X_E).(N,E) is a $\hat{\beta}$g-soft semi nbhd of each of its points.

Proof: Suppose (N,E) is $\widehat{\beta}$ g- SSO(X_E). Let {{a_1}, φ } ϵ (N, E) is $\hat{\beta}$g-soft semi nbhd of X_E. For a soft set (N,E) is a $\widehat{\beta}$ g- SSO(X_E) such that {{a_1}, φ}∈(N,E) contained (N,E) .
Since {{a_1},φ} is an arbitrary point of (N,E) it follows that (N,E) is a $\hat{\beta}$g-Soft semi nbhd of each of its points.

3.6 Remark: The converse of the above theorem need not be true as seen from the following
Example: Let X={ a_1, a_2, a_3 }, τ ={X,φ,(G_1,E),(G_2,E),(G_3,E)...... (G_7, E)} .
Then $\hat{\beta}$g S.S.O(X) = { G_1, E), (G_2, E) .... (G_7, E) }.
Now the soft set = {X ,{a_2}} is a $\widehat{\beta}$g-soft semi nbhd of {{ φ },{a_2}},{{ a_1}{ a_2 }}.
Since the $\hat{\beta}$g.soft open set (G_6,E)={{X ,{a_1, a_2}}is such that {{ φ },{a_2,a_3}}⊂{X ,{a_2}}.
But the soft semi open set {X,{a_2}}} is not $\widehat{\beta}$ g- SSO(X_E) .

3.7 Theorem: Let (X,τ, E) be a topological space. If (F,E) is a $\hat{\beta}$g-soft semi closed subset of X_E and {{a}, φ}∈ $(F,E)^C$. Prove that there exists a $\hat{\beta}$g-soft semi nbhd (N,E) of {{a}, φ}such that (N,E) ∩(F,E) = {φ, φ}.
Proof: Let (F,E) is a $\hat{\beta}$g- soft semi closed subset of X_E and {{a}, φ }∈ $(F,E)^c$.
Then $(F,E)^c$ is $\widehat{\beta}$ g- SSO(X_E). So by theorem 3.5 $(F,E)^c$contains a $\hat{\beta}$g- Soft semi nbhd of each of its points. Hence there exists a $\hat{\hat{\beta}}$g- Soft semi nbhd (N,E) of {{a}, φ } such that (N,E) ⊂ $(F,E)^c$.
That is (N,E) ∩(F,E) ={φ, φ}.

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