DIFFERENCE CORDIAL LABELING ON FAN RELATED GRAPHS

V. Sharon Philomena
P.G. Department of Mathematics, Women's Christian College, Chennai – 600 006, Tamil Nadu, India.
M. Manjula
P.G. Department of Mathematics, Women's Christian College, Chennai – 600 006, Tamil Nadu, India.

Abstract: A *Difference cordial labeling* of a graph G is an injective function f from $V(G) \to \{1,2, \ldots |V(G)|\}$such that if each edge uv is assigned the label $|f(u) - f(v)|$, the number of the edges labeled with '1' and the number of the edges not labeled with '1' differ at most by 1. A graph with a Difference cordial labeling is called a *Difference cordial graph*. In this paper, we prove the Difference cordial labeling for Toffee graph and Uniform bow graph.

Keywords: Cordial Labeling, Difference Cordial Graph, Difference Cordial Labeling, Toffee Graph And Uniform Bow Graph.

Introduction: A graph labeling is an assignment of integers to the vertices or edges, or both, subject to certain conditions. Most graph labeling methods trace their origin to the graceful labeling introduced by Alex Rosa in 1967 [10]. Cordial labeling was introduced by Cahit in 1987 [1]. Over the years, a wide variety of labeling techniques have evolved. One such labeling technique is the difference cordial labeling. Labeled graphs serve as useful models for a broad range of applications such as astronomy, circuit design, mobile tele communication, coding theory, X-ray crystallography, radar, database management, communication network addressing and in medical field. Difference cordial labeling was introduced by Ponraj, Satish Narayanan and Kala in the year 2013 [4]. A *Difference cordial labeling* of a graph G is an injective function$f$ from $V(G) \to \{1,2, \ldots |V(G)|\}$such that if each edge uv is assigned the label$|f(u) - f(v)|$, the number of the edges labeled with '1' and the number of the edges not labeled with '1' differ at most by1.

A graph with a Difference cordial labeling is called a *Difference cordial graph* [5].The Difference cordial labeling for several graphs like path, cycle, complete graph, wheel graph, web graph and some more standard graphs have been obtained [4] - [8]. For more results on Difference cordial labeling refer to the dynamics survey by Gallian [2]. In this paper, we prove the Difference cordial labeling for Toffee graph and Uniform bow graph.

Definition 1: A *Toffee graph* is obtained from a $4-$ cycle w_1, w_2, w_3, w_4 by adjoining $w_1$with a fan graph F_n in such a way that w_1 is an apex of F_n and w_4 with a fan graph F_n in such a way that w_3 is an apex of F_n. We denote the graph by $TOG_{n,n}$ where $n \geq 5$.

Definition 2 [3]: A bow graph in which each shell has the same order l is called a *Uniform bow graph*.It is denoted by G.

Theorem 1: The Toffee graph $TOG_{n,n}$ admits Difference cordial labeling.

Proof: Let$TOG_{n,n}$ be a Toffee graph where n denotes the number of vertices in the fan graph and p and q denotes the total number of vertices and edges in $TOG_{n,n}$. The total number of vertices and edges in the Toffee graph are $p = (2n + 4)$ and $q = (6n - 2)$ respectively.

The Toffee graph is obtained from a 4 – cycle w_1, w_2, w_3, w_4 by adjoining $w_1$with a fan graph F_n in such a way that w_1 is an apex of F_n and w_4 with a fan graph F_n in such a way that w_3 is an apex of F_n.

We denote the vertices of the Toffee graph as follows
The vertices of the cycle are denoted by w_1, w_2, w_3, w_4. The vertices attached to the vertex $w_1$are denoted by $v_1, v_2, v_3, \dots v_n$ and the vertices attached to the vertex $w_4$are denoted by $u_1, u_2, u_3, \dots u_n$.

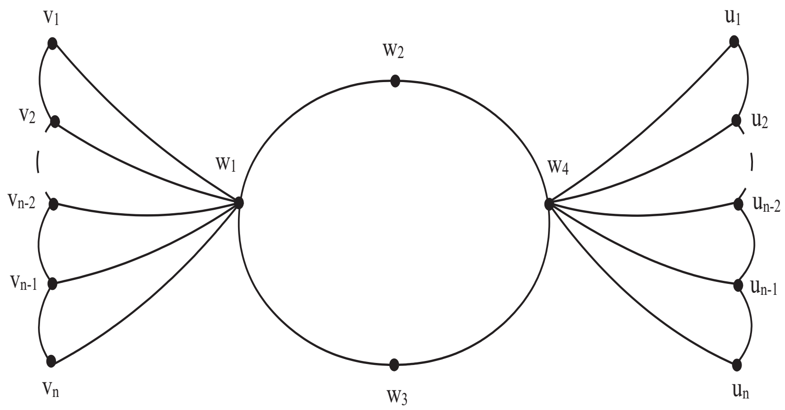

Fig. 1: Toffee graph

Define $f : V(TOG_{n,n}) \rightarrow \{1,2, \dots , n\}$
We describe the vertex labeling f of$TOG_{n,n}$ as follows:

$$f(w_i) = i, \quad for\ 1 \le i \le 4$$
$$f(u_i) = w_4 + i, \quad for\ 1 \le i \le n$$
$$f(v_i) = u_n + i, \quad for\ 1 \le i \le n$$

Using the above vertex label and the edge labels are computed.

$$e_f(0) = e_f(1) = m - 3$$

The numbers of edges with label '1' and '0' are $e_f(1) = e_f(0) = m - 3$.
Hence the condition for difference cordial labeling is
$| e_f(0) - e_f(1)| \le 1$
$| m - 3 - (m - 3)| \le 1$
Thus, $| e_f(0) - e_f(1)| \le 1$ is satisfied.
Therefore, Toffee graph admits Difference cordial labeling.
Illustration of the above theorem is given in Fig. 2

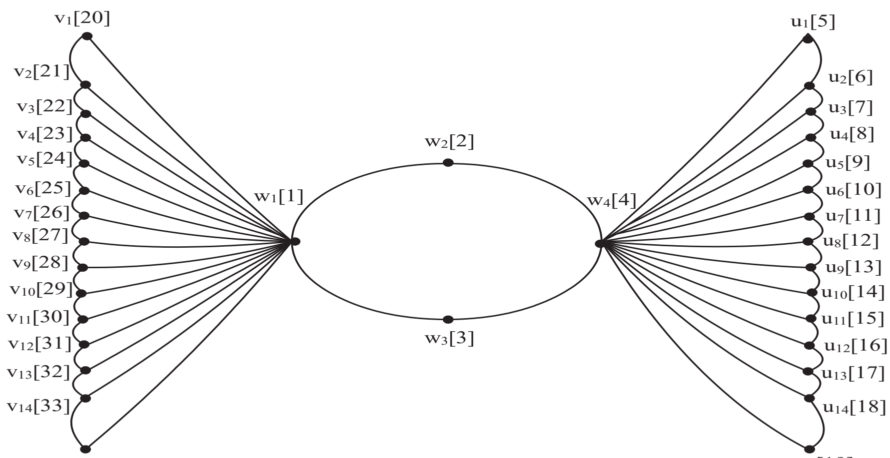

Fig. 2: Difference Cordial Labeling of$TOG_{15,15}$

Theorem 2: The Uniform bow graphadmits Difference cordial labeling.
Proof: Let$G$ be a Uniform bow graph where m denotes the number of vertices in the fan graph and p and q denotes the total number of vertices and edges in G. The total number of vertices and edges in the Uniform bow graph are $p = (2m + 1)$ and $q = (4m - 2)$ respectively.
The shell that is present to the left of the apex in the Uniform bow graph is considered as the left wing of G and the shell to the right of the apex is considered as the right wing of G.We denote the vertices of the Uniform bow graph as follows
We denote the apex of the Uniform bow graph as w. The vertices in the right wing of G as $v_1, v_2, v_3, \dots v_m$ and the vertices in the left wing of Gas $u_1, u_2, u_3, \dots u_m$.

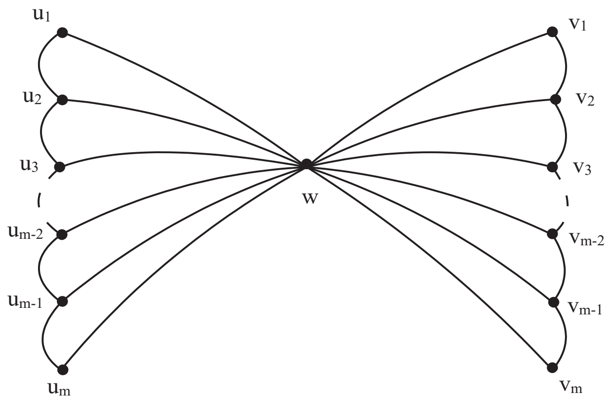

Fig. 3: Uniform bow graph

Define $f : V(G) \to \{1,2, \dots, n\}$
We describe the vertex labeling f of G as follows:

$f(w) = 1$
$f(v_i) = w + i, \quad for \ 1 \le i \le m$
$f(u_i) = v_m + i, \quad for \ 1 \le i \le m$

Using the above vertex label and the edge labels are computed.

$$e_f(0) = e_f(1) = n - 2$$

The numbers of edges with label '1' and '0'are $e_f(1) = e_f(0) = n - 2$.
Hence the condition for difference cordial labeling is
$| e_f(0) - e_f(1)| \le 1$
$| n - 2 - (n - 2)| \le 1$
Thus, $| e_f(0) - e_f(1)| \le 1$ is satisfied.
Therefore, Uniform bow graph admits Difference cordial labeling.
Illustration of the above theorem is given in Fig.4

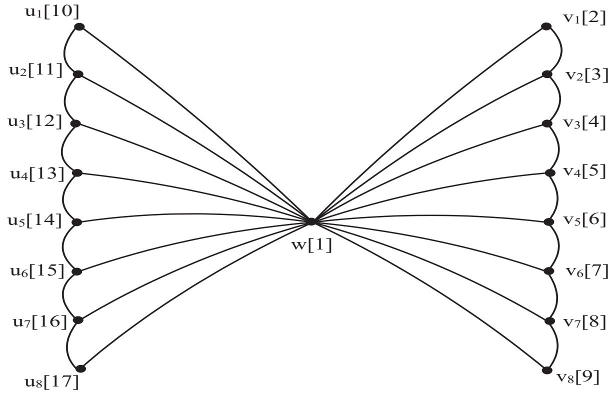

Fig. 4: Difference Cordial Labeling of$G$

Conclusion: In this paper, we have proved the Difference cordial labeling on Toffee graph and Uniform bow graph. We further intend to prove Difference cordial labeling for many new graphs.

References:

1. I. Cahit, Cordial graphs: A weaker version of graceful andharmonious graphs, ArsCombinatorial, Vol. 23, (1987), pp. 201-207.
2. J. A. Gallian, A Dynamic Survey of Graph Labeling, The Electronic journal ofCombinatorics (2016).
3. J. JebaJesintha, K. Ezhilarasi Hilda,"All Uniform bow graph are graceful", Math.Comput. Sciences, Vol. 5, (2015), pp.185-191.
4. R. Ponraj, S. Satish Narayanan and R.Kala, "Difference cordial labeling of corona graphs J. Math. Comput.Sciences: Theory and Practical, Vol. 5,(2013), pp.185-196.
5. R. Ponraj, S. Satish Narayanan and R.Kala, "Difference cordial labeling of Graphs", Global J. Math. Sciences:Theory and Practical, Vol. 5, (2013) pp.160-184.
6. R. Ponraj and S. Satish Narayanan, "Difference cordial labeling of some derived graphs", International Journalofmathematics, Vol. 2, (2014), pp. 40-45.
7. R. Ponraj, S. Satish Narayanan and R. Kala, "A note on Difference cordial graphs", Palestine Journal of mathematics, Vol. 5, (2015), pp. 105-124.
8. R. Ponraj, S. Satish Narayanan and R. Kala,"Some results and examples on Difference cordial of graphs", Tbilisi Journal ofmathematics, Vol. 40, (2016), pp. 417-427.
9. R. Ponraj, S. Satish Narayanan andR.Kala,"Difference cordial labeling ofsome special graphs", Palestine Journal ofmathematics, Vol. 6, (2017), pp. 135-140.
10. Rosa A "On certain valuation of the vertices of a graph theory of graphs" Int. Symposium Rome Gordon and Breach, NY Dound, Paris, (1967), pp 349-355.

***