APPLICATION OF VAGUE MATRIX IN MULTI-CRITERIA DECISION MAKING PROBLEM

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Abstract: The decision making is the most beneficial and fascinating pitch in the application of vague set theory. There are varieties of ideas explored in vague matrices to deal with divergent and complex aspects in different fields. This paper works towards the integrated vague preference relation matrices by the Technique for order preference by similarity to ideal solution method (TOPSIS) is extended to vague environment, which is suitable for solving decision making problem. An empirical application is given to demonstrate the proposed method.

Keywords: Vague Matrices, Vague TOPSIS Method, Vague Decision Matrix.

Introduction: Decision making is the procedure of discovering the optimal choice from the feasible alternatives. Plenty of multi criteria decision making(MCDM) approaches have been amerged and implemented in different fields. TOPSIS (Technique for the order preference by similarity to ideal solution) is one of the customary methods for multi criteria decision making which was first developed by Hwang and Yoon[3]. Current vicious competitive environment enforces many industries and cultivators to make correct decisions in location selection. A defective selection of location may lead to unessential transportation costs, scarcity of professional labor, inadequate supplies of raw materials and so on. Fuzzy set theory by Zadeh[9] has been used previously in many fields. . The concept of fuzzy matrix introduced by Thomson[7] plays a vital role in scientific development. The fuzzy matrices has been employed in many approaches to model the diagnostic and decision making process[5,6] Sanchez[4] in 1979 proposed medical diagnosis by using composite fuzzy relation. In 1993 Gau and Buehrer [2] introduced the idea of vague set which was the generalization of fuzzy set with truth membership and false membership function. The vague set theory has been investigated by many authors and has been applied in different fields. In this paper, we analyze the vague matrices in multi criteria decision making with the TOPSIS method for selecting best location for tea cultivation.

Preliminaries:

Definition 2.1: [2] A vague set A in the universe of discourse U is characterized by two membership functions given by:

(i) A true membership function $t_A : U \to [0,1]$ and

(ii) A false membership function $f_A : U \to [0,1]$

where $t_A(x)$ is a lower bound on the grade of membership of x derived from the "evidence for x", $f_A(x)$ is a lower bound on the negation of x derived from the "evidence for x", and $t_A(x) + f_A(x) \le 1$. Thus the grade of membership of u in the vague set A is bounded by a subinterval $[t_A(x), 1 - f_A(x)]$ of [0,1]. this indicates that if the actual grade of membership of x is μ(x), then, $t_A(x) \le \mu(x) \le 1 - f_A(x)$ .The vague set A is written as $A = \{\langle x, [t_A(x), 1 - f_A(x)]\rangle / u \in U\}$ where the interval $[t_A(x), 1 - f_A(x)]$ is called the vague value of x in A, denoted by $V_A(x)$.

Definition 2.4:[1] Let A and B be VSs of the form $A = \{\langle x, [t_A(x), 1 - f_A(x)]\rangle / x \in X\}$ and $B = \{\langle x, [t_B(x), 1 - f_B(x)]\rangle / x \in X\}$ Then

(i) $A \subseteq B$ if and only if $t_A(x) \le t_B(x)$ and $1 - f_A(x) \le 1 - f_B(x)$ for all x ∈ X

(ii) A=B if and only if $A \subseteq B$ and $B \subseteq A$

(iii) $A^c = \{\langle x, f_A(x), 1-t_A(x)\rangle / x \in X\}$

(iv) $A \cap B = \left\{\left\langle x, \min\left(t_A(x), t_B(x)\right), \min\left(1-f_A(x), 1-f_B(x)\right)\right\rangle / x \in X\right\}$

(v) $A \cup B = \left\{\left\langle x, \left(t_A(x) \vee t_B(x)\right), \left(1-f_A(x) \vee 1-f_B(x)\right)\right\rangle / x \in X\right\}$

For the sake of simplicity, we shall use the notation $A = \langle x, t_A, 1-f_A\rangle$ instead of $A = \{\langle x, [t_A(x), 1-f_A(x)]\rangle / x \in X\}$.

Deninition 2.2:[8] Let *a*=[a^L , a^U]and *b*=[b^L ,b^U] be two interval numbers where is $0 \le a^L \le a^U \le 1$ and o$\le b^L \le b^U \le$1 then the degree of $a \ge b$ is defined as:

$$p(a \ge b) = \max\left\{1 - \max\left(\frac{b^U - a^L}{b^U - b^L + a^U - a^L}, 0\right), 0\right\} \quad \text{.....(i)}$$

that is a superior to degree of b, denoted by $a \overset{p(a \ge b)}{\succ} b$. Similarly, the degree of possibility of $b \ge a$ is defined as:

$$p(b \ge a) = \max\left\{1 - \max\left(\frac{a^U - b^L}{b^U - b^L + a^U - a^L}, 0\right), 0\right\}$$

.....(ii)

that is b superior to degree of a, denoted by $b \overset{p(b \ge a)}{\succ} a$. Let $P = p_{ij} = p(a_i \ge a_j)$ be the complementary matrix and given as follows:

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} & \cdots & p_{1n} \\ p_{2i} & p_{22} & p_{23} & \cdots & p_{2n} \\ p_{31} & p_{32} & p_{33} & \cdots & p_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ p_{m1} & p_{m2} & p_{m3} & \cdots & p_{mn} \end{bmatrix}$$

where $p_{ij} \ge 0$, $p_{ij} + p_{ji} = 1$, $p_{ii} = 0.5$ and $i, j = 1,2,3.....n$.

Summing all elements in each line of matrix P, then: $p_i = \sum_{j=1}^{n} p_{ij}.i, j = 1,2,...n$ .....(iii) Alternatives are ranked according to descending order of p_i .

Vague Sets:

Definition 3.1: A vague matrix is a matrix pairs $A = [< t_{ij}, 1-f_{ij} >]$ of non negative real numbers satisfying $0 \le t_{ij} + f_{ij} \le 1$ for all i,j, with vague hesitation degree $\pi_A(x) = 1 - (t_A(x) + f_A(x))$.

Definition 3.2: Let A and B be vague sets defined as $A = \{\langle x, [t_A(x), 1-f_A(x)]\rangle / x \in X\}$ and $B = \{\langle x, [t_B(x), 1-f_B(x)]\rangle / x \in X\}$ then Euclidean distance between the vague sets A and B is given as follows:

$$d(A,B)=\sqrt{\frac{1}{2}\sum_{i=1}\begin{bmatrix}(t_A(x)-t_B(x))^2\\+((1-f_A(x))-(1-f_B(x)))^2\\+(\pi_A(x)-\pi_B(x))^2\end{bmatrix}} \quad .....(iv)$$

Definition 3.3: Let A and B be two vague sets defined as as $A=\{\langle x,[t_A(x),1-f_A(x)]\rangle / x\in X\}$ and $B=\{\langle x,[t_B(x),1-f_B(x)]\rangle / x\in X\}$ then,

$$A\otimes B=\left\{\left\langle x, t_A(x)\cdot t_B(x), (1-f_A(x))\cdot(1-f_B(x)), \pi_{A\cdot B}(x)\right\rangle / x\in X\right\}$$

where,
$$\pi_{A\cdot B}(x)=1-f_A(x)-f_B(x)-t_A(x)\cdot t_B(x)+f_A(x)\cdot f_B(x)$$

Vague Matrices in Multi Decision Making Method: In the section, TOPSIS method is extended in vague environment which is suitable for solving decision making problems. The algorithm involving various steps of vague TOPSIS are presented as follows: Let $B=\{B_1,B_2,.....,B_n\}$ be the set of alternatives and $S=\{S_1,S_2,.....,S_n\}$ be a set of criteria. Vague TOPSIS method consists of the following steps which are given as follows:

Step 1: Construct A Vague Preference Relation Matrix:

Let $A=(a_{ij})_{n\times n}$ be a vague preference matrix of criteria as follows:

$$A=\begin{bmatrix}a_{11}&a_{12}&a_{13}&\cdots&a_{1n}\\a_{21}&a_{22}&a_{23}&\cdots&a_{2n}\\a_{31}&a_{32}&a_{33}&\cdots&a_{3n}\\\vdots&\vdots&\vdots&\ddots&\vdots\\a_{m1}&a_{m2}&a_{m3}&\cdots&a_{mn}\end{bmatrix}$$

Where $a_{ij}=(t_{ij},1-f_{ij})(i=1,2,...,m;\ j=1,2,,...,n)$ and satisfies the succeeding conditions:

$$(t_{ij})^*=\max\left\{t_{ij},\max_p\left\{\frac{t_{ip}t_{pj}}{t_{ip}t_{pj}+(1-t_{ip})(1-t_{pj})}\right\}\right\} \quad .....(1)$$

$$(1-f_{ij})^*=\max\left\{1-f_{ij},\max_p\left\{\frac{(1-f_{ip})(1-f_{pj})}{(1-f_{ip})(1-f_{pj})+(f_{ip})(f_{pj})}\right\}\right\} \quad .....(2)$$

where $(t_{ij})^*$ and $(1-f_{ij})^*$, are the components of (A^*) matrix, which are the vague membership values of the alternatives x_i over x_j, respectively and $o\le(t_{ij})^*+(f_{ij})^*\le 1$ for all i.j=1,2,....n, then we call A a multiplicative consistent vague preference relation. If $(A)^*=(a_{ij})^*_{nxn}$ does not satisfy the condition o $\le(t_{ij})^*+(f_{ij})^*\le 1$ for all $i,j=1,2,...n$, then we call A an inconsistent vague preference relation.

Step 2: Obtain the Priority Vector Of Criteria: The priority vector of criteria $w=(w_1,w_2,.....,w_n)^T$ after obtaining aggregated vague preference matrix. The priority vector can be determined with the equation given below:

$$w_j = \left[w_J^L, w_J^U\right]$$

$$= \left[\frac{1}{\sum_{j=1}^{n}\left(\frac{\left(1-t_{ij}^*\right)}{t_{ij}^*}\right)}, \frac{1}{\sum_{j=1}^{n}\left(\frac{\left(1-\left(1-f_{ij}^*\right)\right)}{\left(1-f_{ij}^*\right)}\right)}\right] \quad ...(3)$$

Step 3: Construct A Vague Decision Matrix: $R = (r_{ij})_{mxn}$ is a vague decision matrix such that:

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} & \cdots & r_{1n} \\ r_{21} & r_{22} & r_{23} & \cdots & r_{2n} \\ r_{31} & r_{32} & r_{33} & \cdots & r_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & r_{m3} & \cdots & r_{mn} \end{bmatrix}$$

where, $r_{ij} = (t_{ij}, 1-f_{ij}, \pi_{ij})(i = 1,2,..., m; j = 1,2,...., n)$ Which are contained in a vague decision matrix.

Step 4: Determine The Vague Positive Ideal Solution And The Vague Negative Ideal Solution: Let J_1 be the set of benefit criteria and J_2 be the set of cost criteria. B^* be the vague positive ideal solution and B^- be the vague negative ideal solution, then B^* and B^- can be determined respectively as follows:

$B^* = (r_1^*, r_2^*, ...r_n^*)$, $r_j^* = (t_j^*, 1-f_j^*, \pi_j^*)$, $j = 1,2,...., n$ .....(4)

$B^- = (r_1^-, r_2^-, ...r_n^-)$, $r_j^- = (t_j^-, 1-f_j^-, \pi_j^-)$, $j = 1,2,...., n$ .....(5)

Where, $t_j^* = \left\{\left(\max_i\left\{t_{ij}\right\}/\ j \in J_1\right), \left(\min_i\left\{t_{ij}\right\}/\ j \in J_2\right)\right\}$.....(6)

$1-f_j^* = \left\{\left(\max_i\left\{1-f_{ij}\right\}/\ j \in J_1\right), \left(\min_i\left\{1-f_{ij}\right\}/\ j \in J_2\right)\right\}$.....(7)

$t_j^- = \left\{\left(\min_j\left\{t_{ij}\right\}/\ j \in J_1\right), \left(\max_j\left\{t_{ij}\right\}/\ j \in J_2\right)\right\}$.....(8)

$1-f_j^- = \left\{\left(\min_j\left\{1-f_{ij}\right\}/\ j \in J_1\right), \left(\max_j\left\{1-f_{ij}\right\}/\ j \in J_2\right)\right\}$.....(9)

$$\pi_j^* = \left\{\begin{matrix}\left(1-\left(\max_i\left\{t_{ij}\right\}+\max_i\left\{f_{ij}\right\}\right)/\ j \in J_1\right), \\ \left(1-\left(\min_i\left\{t_{ij}\right\}+\min_i\left\{f_{ij}\right\}\right)/\ j \in J_1\right)\end{matrix}\right\} \quad .....(10)$$

$$\pi_j^- = \left\{\begin{matrix}\left(1-\left(\min_i\left\{t_{ij}\right\}+\min_i\left\{f_{ij}\right\}\right)/\ j \in J_1\right), \\ \left(1-\left(\max_i\left\{t_{ij}\right\}+\max_i\left\{f_{ij}\right\}\right)/\ j \in J_1\right)\end{matrix}\right\} \quad .....(11)$$

Step 5: Calculate The Weighted Separation Measures: The weighted Euclidean distance is used to acquire the separation measures. The weighted lower and upper separation measures $(E_i^*)^L$, $(E_i^*)^U$ and $(E_i^-)^L$, $(E_i^-)^U$ of each alternatives from the vague positive ideal solution and the vague negative ideal solution are calculated respectively:

$$(E_i^*)^L = \sqrt{\frac{1}{2}\sum_{j=1}^{n} w_j^L \left[\begin{matrix}\left(t_{ij}-t_j^*\right)^2 + \left((1-f_{ij})-(1-f_j^*)\right)^2 \\ +\left(\pi_{ij}-\pi_j^*\right)^2\end{matrix}\right]} \quad .....(12)$$

$$(E_i^*)^U = \sqrt{\frac{1}{2}\sum_{j=1}^{n} w_j^U \left[\begin{matrix} \left(t_{ij}^U - t_j^*\right)^2 + \left((1-f_{ij}^U) - (1-f_j^*)\right)^2 \\ + \left(\pi_{ij}^U - \pi_j^*\right)^2 \end{matrix} \right]} \quad \text{.....(13)}$$

$$(E_i^-)^L = \sqrt{\frac{1}{2}\sum_{j=1}^{n} w_j^L \left[\begin{matrix} \left(t_{ij}^L - t_j^-\right)^2 + \left((1-f_{ij}^L) - (1-f_j^-)\right)^2 \\ + \left(\pi_{ij}^L - \pi_j^-\right)^2 \end{matrix} \right]} \quad \text{.....(14)}$$

$$(E_i^-)^U = \sqrt{\frac{1}{2}\sum_{j=1}^{n} w_j^U \left[\begin{matrix} \left(t_{ij}^U - t_j^-\right)^2 + \left((1-f_{ij}^U) - (1-f_j^-)\right)^2 \\ + \left(\pi_{ij}^U - \pi_j^-\right)^2 \end{matrix} \right]} \quad \text{.....(15)}$$

Step 6: Calculate The Relative Closeness Coefficient of Each Alternative to the Vague Positive And The Vague Negative Ideal Solutions: The relative closeness coefficient of an alternative B_i with respect to vague positive ideal solution B^* and the vague negative ideal solution B^- is defined as follows:

$$\left((C_i^*)^L, (C_i^*)^U\right) = \left\{\left(\frac{(E_i^-)^L}{(E_i^*)^U + (E_i^-)^U}\right), \left(\frac{(E_i^-)^U}{(E_i^*)^L + (E_i^-)^L}\right)\right\} \quad \text{.....(16)}$$

Step 7: Rank the Alternatives: Rank the alternatives according to the descending order of the relative closeness coefficients $\left((C_i^*)^L, (C_i^*)^U\right)$. The possibility degree formula is used to rank the alternatives.

Illustrative Example: A tea manufacturing industries is planning to locate the best tea cultivation area. Here five tea cultivation areas such as B_1 , B_2 , B_3 , B_4 and B_5 are chosen for evaluation. In order to evaluate the location for tea cultivation the following evaluation factors are considered: Soil fertility and climatic factors (S_1), expansion possibility (S_2), storage problem (S_3), transportation facility (S_4) and labour demand and cost (S_5).

Step 1: Construct a vague preference relation matrix: Let $A = (a_{ij})_{5\times5}$ be a vague preference matrix of criteria

$$A = \begin{bmatrix} [0.2,0.7] & [0.6,0.8] & [0.3,0.4] & [0.6,0.7] & [0.1,0.8] \\ [0.3,0.6] & [0.5,0.7] & [0.2,0.8] & [0.2,0.3] & [0.4,0.6] \\ [0.1,0.5] & [0.3,0.4] & [0.3,0.9] & [0.5,0.8] & [0.2,0.8] \\ [0.2,0.6] & [0.2,0.3] & [0.1,0.6] & [0.3,0.6] & [0.1,0.8] \\ [0.4,0.5] & [0.1,0.3] & [0.4,0.7] & [0.4,0.5] & [0.1,0.3] \end{bmatrix}$$

where $A = (a_{ij})_{5\times5}$ has been the consistent vague preference matrix since it is satisfying the condition (1) and (2) the following matrices exist as follows:

$$A^* = \begin{bmatrix} [0.2,0.8] & [0.6,0.8] & [0.3,0.9] & [0.6,0.8] & [0.1,0.8] \\ [0.31,0.6] & [0.5,0.7] & [0.31,0.8] & [0.31,0.6] & [0.4,0.6] \\ [0.14,0.8] & [0.3,0.63] & [0.3,0.9] & [0.5,0.8] & [0.2,0.9] \\ [0.2,0.8] & [0.2,0.63] & [0.1,0.9] & [0.3,0.8] & [0.1,0.8] \\ [0.4,0.5] & [0.1,0.3] & [0.4,0.7] & [0.4,0.5] & [0.1,0.3] \end{bmatrix}$$

Step 2: Obtain the priority vector of criteria: the priority vector of criteria has been established by utilizing equation (3) are as follows:

$w_1 = [0.05999, 0.90090]$, $w_2 = [0.10881, 0.37319]$, $w_3 = [0.06329, 0.76494]$,

$w_4 = [0.03530, 0.69191]$ and $w_5 = [0.0444, 0.14107]$.

Tea Cultivators	Criteria				
	S_1	S_2	S_3	S_4	S_5
B_1	**[0.012, 0.63,0.6]**	**[0.065, 0.30,0.23]**	**[0.019, 0.31,0.26]**	**[0.021, 0.41,0.39]**	**[0.004, 0.1,0.11]**
B_2	**[0.018, 0.54,0.5]**	**[0.054, 0.26,0.21]**	**[0.013, 0.612,0.6]**	**[0.007, 0.21,0.20]**	**[0.02, 0.09,0.1]**
B_3	**[0.006, 0.45,0.4]**	**[0.033, 0.15,0.12]**	**[0.019, 0.69,0.67]**	**[0.071, 0.35,0.33]**	**[0.009, 0.1,0.10]**
B_4	**[0.01, 0.45,0.4]**	**[0.02, 0.11,0.09]**	**[0.006, 0.46,0.45]**	**[0.011, 0.42,0.38]**	**[0.004, 0.1,0.11]**
B_5	**[0.02, 0.45,0.4]**	**[0.011, 0.112,0.1]**	**[0.025, 0.54,0.51]**	**[0.014, 0.35,0.33]**	**[0.004, 0.04,0.03]**

Step 3: Construct a vague decision matrix: The vague decision matrix has been constructed in the table 1 given below : Table 1: the vague decision matrix

Step 4: Determine the vague positive ideal solution and the vague negative ideal solution:

Considering the soil fertility and climatic factors, expansion possibility, storage problem are considered as the benefit criteria, $J_1 = \{S_1, S_2, S_3\}$ and the transportation facility, labour demand and cost are the cost criteria $J_2 = \{S_4, S_5\}$. Then the vague positive and the vague negative ideal solution have been obtained by employing equations (4) to (11) as follows:

$$B^* = \begin{Bmatrix} [0.024, 0.63, 0.606],\ [0.065, 0.30, 0.235],\ [0.025, 0.69, 0.67] \\ [0.007, 0.21, 0.203],\ [0.004, 0.042, 0.038] \end{Bmatrix} \text{ and}$$

$$B^- = \begin{Bmatrix} [0.0059, 0.45, 0.4],\ [0.011, 0.112, 0.10],\ [0.006, 0.31, 0.30] \\ [0.021, 0.415, 0.394],\ [0.0178, 0.113, 0.0952] \end{Bmatrix}$$

Step 5: Calculate The Weighted Separation Measures: Vague negative and vague positive separation measures based on the weighted lower and upper Euclidian distance for each cultivation area have been calculated by using the equation (12) to (15).

Tea cultivators	$(E_i^*)^L$	$(E_i^*)^U$	$(E_i^-)^L$	$(E_i^-)^U$
B_1	0.1064	0.3805	0.0708	0.2007
B_2	0.0480	0.1093	0.0966	0.3288
B_3	0.0688	0.2172	0.1028	0.3318
B_4	0.0909	0.2852	0.0256	0.1557
B_5	0.0849	0.2663	0.0572	0.1973

Step 6: Calculate the relative closeness coefficient of each alternative to the Vague negative and vague positive ideal solution: The relative closeness coefficient of an each cultivation field to the Vague negative and vague positive ideal solution have been calculated by using equation (16) as follow:

$\left(\left(C_1^*\right)^L,\left(C_1^*\right)^U\right)=\{[0.1218,1.1326]\}$ $\left(\left(C_2^*\right)^L,\left(C_2^*\right)^U\right)=\{[0.2205,2.2739]\}$

$\left(\left(C_3^*\right)^L,\left(C_3^*\right)^U\right)=\{[0.1872,1.9336]\}$ $\left(\left(C_4^*\right)^L,\left(C_4^*\right)^U\right)=\{[0.0581,1.3365]\}$

$\left(\left(C_5^*\right)^L,\left(C_5^*\right)^U\right)=\{[0.1243,1.3885]\}$

Step 7: Rank the location of the tea cultivation according to the descending order of the relative closeness coefficients. Five locations have been ranked according to the descending order of the relative closeness coefficients. The location has been ranked by using the possibility degree formula and the following matrix is constructed below:

$$P=\begin{bmatrix}0.5 & 0.7 & 0.7 & 0.5 & 0.6\\ 0.3 & 0.5 & 0.5 & 0.3 & 0.4\\ 0.3 & 0.5 & 0.5 & 0.4 & 0.4\\ 0.5 & 0.7 & 0.6 & 0.5 & 0.5\\ 0.4 & 0.6 & 0.6 & 0.5 & 0.5\end{bmatrix}$$

Summing all the elements in each row of the matrix P, then: $p_1=3.0, p_2=2.0, p_3=2.1\ p_4=2.8,$ $p_5=2.6$. The location for tea cultivation is ranked as B_1, B_4, B_5, B_3 and B_2 according to descending order of $p_i(i=1,2,3,4,5)$. Thus B_1 has been selected as the most desirable facility location among all the location

Conclusion: The success of the best yield in tea cultivation depends on their capability on making equitable strategic decisions. The location for best tea cultivation selection is a multi criteria decision making problem which includes both quantitative and qualitative approach. In this paper a vague TOPSIS method is used to find the best and suitable place for tea cultivation. In this case study only five criteria is used for our analysis more evaluation criteria can be used to improve the effectiveness of the decision making process.

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