M/M/1 TYPE QUEUEING-INVENTORY WITH SERVER VACATION, CANCELLATION AND NEGATIVE CUSTOMER

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Abstract: In this paper, We consider a continuous review queueing-inventory system with Poisson arrival process. The service facility is assumed to have a finite waiting room of capacity N. The replenishment of inventory is (s,S) policy and the lead time is assumed to follow an exponential distribution. An inventory gets added until their expiry time throughout the cancellation of purchases and the cancellation times are assumed to follow an exponential distribution. When the inventory level is zero or there is no customer in the waiting room or both cases happen, server goes to vacation which is exponentially distributed. The inventory level and the server status is obtained in the steady state. Some measures of system performance in the steady state are obtained and the results are numerically illustrated.

Keywords: Queueing-Inventory, Server Vacation, Cancellation, Negative Customer.

Introduction: Queueing-inventory has captured much attention of researchers over the last few decades. In queueing-Inventory models the availability of items are also to be considered in addition to the features in queueing theory. If the time required to serve the items to the customers is taken to be positive, then a queue is formed. Sigman and Levi [6] considered inventory models with positive service time. They assume that the processing of inventory require an arbitrarily distributed positive amount of time, thus leading to the formation of queue. Since then numerous studies on inventory models with positive service time are reported. Maike Schwarz [3] proposed the model M/M/1 Queueing systems with inventory. They derived stationary distributions of joint queue length and inventory processes in explicit product form for various M/M/1-systems with inventory under continuous review and different inventory management policies, and with lost sales. Here demand follows Poisson distribution, service times and lead times exponential distribution.

Mohammad Saffari [4] considered an M/M/1 queueing system with inventory under the (r, Q) policy and with lost sales, in which demands occur according to a Poisson process and service times are exponentially distributed. All arriving customers during stock out are lost. They derived the stationary distributions of the joint queue length and on hand inventory when lead times are random variables and can take various distributions. Krishnanmoorthy [2] has introduced cancellation of purchases and he considered two single server queueing-inventory system in which items in the inventory have random common life time. On realization of common life time, all customers in the system are flushed out. Subsequently the inventory reaches its maximum level S through a (positive lead time) replenishment for the next cycle which follows an exponential distribution. Through cancellation of purchases, inventory gets added until their expiry time and discussed cost function.

M. Rajkumar [5] considered a continue review inventory system with an orbit of infinite size and he assumed Poisson arrivals of both type of customers, exponential lead time for order placed under (s,S) ordering policy and they derived long run total expected cost rate. B. Sivakumar [7] considered a continuous review inventory system with Markovian demand. They assumed positive lead time and infinite hall waiting size for the orbit demands. The server took multiple vacation whenever the inventory level was zero and they calculated the long-run total expected cost rate. V.S. Koroliuk [1] considered queueing-inventory system with perishable inventory. The sever took vacation whenever there were no customer in the waiting room or the inventory level was zero or both cases and they developed the method of asymptotic system analysis and results.

Model Description: Consider an queueing-inventory system with maximum inventory level is S. The waiting area space is limited to accommodate maximum number of customer N including the one at the service point. We assume that the customers arrive according to a Poisson process and the demanding exactly one unit of item. Now, We make the further more assumptions:

- The customer arrive according to a Poisson process with parameter $\lambda(> 0)$.
- The negative customer arrives according to a Poisson process with rate λ_0 and the removal rule followed in this paper is RCE(Removal of a Customer at the End of the waiting line), at the time of arrival of a negative customer. Note the in this paper we assume serviced customer can't negative customer.
- When the server is vacation, the negative customer arrival rate is qλ_0 at the time of negative customer arrival and when the server is busy, the negative customer arrival rate is
- (q-1)λ_0(except at service point) at the time of negative customer arrival.
- The item is delivered to the customer after random time of service, it requires an exponentially distributed time with parameter $\alpha > 0$.
- The cancellation of purchase is followed in this paper under the condition is Return of the item without penalty, Inter cancellation time follows exponential distribution with parameter $k\beta$, when there are k items in the purchased list in the current cycle (ie there are $(S-k)$ items are in the inventory).
- An order for $Q(= S - s > s + 1)$ items is placed whenever the inventory level drops to s and the items are received only after a random time which is distributed as exponential with parameter $\mu(> 0)$.
- When the inventory level is zero or there is no customer in the waiting room or both cases happen, server goes to vacation, when server returns from the vacation, if the server find atleast one customer in the waiting area and also atleast one item in the inventory and the server immediately starts to serve the waiting customer. This follows the exponentially distributed with rate γ.

Notation:
$[A]_{ij}$: The element/submatrix at $(i,j)^{th}$ position of A
0 : Zero matrix
I : Identity matrix
e : A column vector of 1^{st} appropriate dimension

Analysis: Let $X(t)$, $Y(t)$ and $I(t)$ denote the number of customers in the finite waiting room, server status and inventory level at time t. From the assumptions made on the input and output processes, it can be shown that the triplet $\{X(t), Y(t), I(t), t \geq 0\}$ is a Markov process with state space $E = E_1 \cup E_2$
Where $E_1 = \{(q, 0, i)/q = 0,1, \dots, N, i = 0,1, \dots, S\}$, $E_2 = \{(q, 1, i)/q = 1, \dots, N, i = 1, \dots, S\}$ and $Y(t) = \begin{cases} 1 & if\ server\ is\ busy \\ 0 & if\ server\ is\ vacation \end{cases}$
To determine the infinitesimal generator,
$A = (a((q,k,i),(r,l,j))), (q,k,i), (r,l,j) \in E$
can be obtained by using the following arguments:

- the arrival of a customer makes a transition with intensity λ , from $(q, 1, i)$ to $(q + 1,1, i)$, $q = 1, \dots, N - 1$, $i = 1, \dots, S$, or from $(q, 0, i)$ to $(q + 1,0, i)$, $q = 0,1, \dots, N - 1$, $i = 0,1, \dots, S$.
- the arrival of a negative customer makes a transition with intensity $(q - 1)\lambda_0$, from $(q, 1, i)$ to $(q - 1,1, i)$, $q = 2, \dots, N$, $i = 1, \dots, S$, or $q\lambda_0$, from $(q, 0, i)$ to $(q - 1,0, i)$, $q = 1, \dots, N$, $i = 0,1, \dots, S$.
- the item is delivered to the customer at service takes a transition α, from $(q, 1, i)$ to $(q - 1,1, i - 1)$, $q = 2, \dots, N$, $i = 2, \dots, S$, or from $(q, 1,1)$ to $(q - 1,0,0)$, $q = 1, \dots, N$ or from $(1,1, i)$ to $(0,0, i - 1)$, $i = 2, \dots, S$.
- when a replenishment occur takes a transition with intensity μ, from $(q, 1, i)$ to $(q, 1, i + Q)$, $q = 1, \dots, N$, $i = 1, \dots, s$, or from $(q, 0, i)$ to $(q, 0, i + Q)$, $q = 0, \dots, N$, $i = 0, \dots, s$.
- inter cancellation of the item takes a transition with intensity $(S - i)\beta$, from $(q, 1, i)$ to $(q + 1,1, i + 1)$, $q = 1, \dots, N - 1$, $i = 1, \dots, S - 1$.

- end of vacation makes a transition with γ, from $(q, 0, i)$ to $(q, 1, i)$, $q = 1, \dots, N$, $i = 1, \dots, S$.
- for other transition from, (q, k, i) to (r, l, j) except $(q, k, i) \neq (r, l, j)$, the rate is zero.

Finally, note that $a((q, k, i), (r, l, j)) = -\sum_{\substack{r\ \ l\ \ l \\ (q,k,i) \neq (r,l,j)}} \sum \sum a((q, k, i), (r, l, j))$

Hence We have, $a((q, k, i), (r, l, j))$=

$$
\begin{cases}
\lambda, & r = q+1, q = 1, \dots, N-1 \\
 & l = k, k = 1 \\
 & j = i, i = 1, \dots, S \\
 & or \\
 & r = q+1, q = 0, \dots, N-1 \\
 & l = k, k = 0 \\
 & j = i, i = 0, \dots, S \\
 & \\
q\lambda_0, & r = q-1, q = 1, \dots, N \\
 & l = k, k = 0 \\
 & j = i, i = 0, \dots, S \\
 & or \\
(q-1)\lambda_0, & r = q-1, q = 2, \dots, N \\
 & l = k, k = 1 \\
 & j = i, i = 1, \dots, S \\
 & \\
\alpha, & r = q-1, q = 2, \dots, N \\
 & l = k, k = 1 \\
 & j = i-1, i = 2, \dots, S \\
 & or \\
 & r = q-1, q = 1, \dots, N \\
 & l = k-1, k = 1 \\
 & j = i-1, i = 1 \\
 & or \\
 & r = q-1, q = 1 \\
 & l = k-1, k = 1 \\
 & j = i-1, i = 2, \dots, S \\
 & \\
\mu, & r = q, q = 1, \dots N \\
 & l = k, k = 1 \\
 & j = i+Q, i = 1, \dots, S
\end{cases}
$$

$$
\begin{cases}
(S-i)\beta, & \begin{array}{l} r=q+1, q=1,\dots N-1 \\ l=k, k=1 \\ j=i+1, i=1,\dots,S-1 \end{array} \\
\gamma, & \begin{array}{l} r=q, q=1,\dots N \\ l=k+1, k=0 \\ j=i, i=1,\dots,S \end{array} \\
-(\mu+\lambda), & \begin{array}{l} r=q, q=0 \\ l=k, k=0 \\ j=i+Q, i=0,\dots,s \end{array} \\
-\lambda, & \begin{array}{l} r=q, q=0 \\ l=k, k=0 \\ j=i, i=s+1,\dots,S \end{array} \\
-(q\lambda_0+\mu+\lambda), & \begin{array}{l} r=q, q=1,\dots N-1 \\ l=k, k=0 \\ j=i, i=0 \end{array} \\
-(q\lambda_0+\mu+\gamma+\lambda), & \begin{array}{l} r=q, q=1,\dots N-1 \\ l=k, k=0 \\ j=i, i=1,\dots,s \end{array} \\
-(q\lambda_0+\gamma+\lambda), & \begin{array}{l} r=q, q=1,\dots N-1 \\ l=k, k=0 \\ j=i, i=s+1,\dots,S \end{array} \\
-(\alpha+(q-1)\lambda_0+\mu+\lambda+(S-i)\beta), & \begin{array}{l} r=q, q=1,\dots N-1 \\ l=k, k=1 \\ j=i, i=1,\dots,s \end{array} \\
-(\alpha+(q-1)\lambda_0+\lambda+(S-i)\beta), & \begin{array}{l} r=q, q=1,\dots,N-1 \\ l=k, k=1 \\ j=i, i=s+1,\dots,S \end{array}
\end{cases}
$$

$$
\begin{cases}
-(q\lambda_0+\mu), & \begin{array}{l} r=q, q=N \\ l=k, k=0 \\ j=i, i=0 \end{array} \\
-(q\lambda_0+\mu+\gamma), & \begin{array}{l} r=q, q=N \\ l=k, k=0 \\ j=i, i=1,\dots,s \end{array} \\
-(q\lambda_0+\gamma), & \begin{array}{l} r=q, q=N \\ l=k, k=0 \\ j=i, i=s+1,\dots,S \end{array} \\
-(\alpha+(q-1)\lambda_0+\mu), & \begin{array}{l} r=q, q=N \\ l=k, k=1 \\ j=i, i=1,\dots,s \end{array} \\
-(\alpha+(q-1)\lambda_0), & \begin{array}{l} r=q, q=N \\ l=k, k=1 \\ j=i, i=s+1,\dots,S \end{array} \\
0 & Otherwise
\end{cases}
$$

The More general form is:

$$A=\begin{array}{c} \\ 0\\ 1\\ 2\\ \vdots\\ N-1\\ N\end{array}\begin{array}{c}\begin{array}{ccccccc}0&1&2&3&\cdots&\text{N-1}&\text{N}\end{array}\\ \begin{pmatrix} A_0 & B_0 & 0 & 0 & \cdots & 0 & 0\\ C_1 & A_1 & B & 0 & \cdots & 0 & 0\\ 0 & C_2 & A_2 & B & \cdots & 0 & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & 0 & \cdots & A_{N-1} & B\\ 0 & 0 & 0 & 0 & \cdots & C_N & A_N \end{pmatrix}\end{array}$$

Where, $A_q=\begin{bmatrix} A_{00}^{(q)} & A_{01}^{(q)}\\ 0 & A_{11}^{(q)}\end{bmatrix}, q=1,\dots,N,\qquad A_0=[A_{00}^{(0)}]$

$$A_{00}^{(q)}=\begin{array}{c} \\ 0\\ 1\\ 2\\ \vdots\\ s\\ s+1\\ \vdots\\ S\end{array}\begin{array}{c}\begin{array}{ccccccccc}0&1&2&3&\cdots&Q&Q+1&\cdots&S\end{array}\\ \begin{pmatrix} f_0^{(q)} & 0 & 0 & 0 & \cdots & \mu & 0 & \cdots & 0\\ 0 & f_0^{(q)} & 0 & 0 & \cdots & 0 & \mu & \cdots & 0\\ 0 & 0 & f_0^{(q)} & 0 & \cdots & 0 & 0 & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & \mu\\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & f_0^{(q)} \end{pmatrix}\end{array}, \qquad q=0,1,\dots,N$$

$$f_i^{(0)}=\begin{cases}-(\mu+\lambda), & i=0,\dots,s\\ -(\lambda), & i=s+1,\dots,S\end{cases}$$

$$f_i^{(q)}=\begin{cases}-(q\lambda_0+\mu+\lambda), & i=0\\ -(q\lambda_0+\mu+\gamma+\lambda), & i=1,\dots,s\\ -(q\lambda_0+\gamma+\lambda), & i=s+1,\dots,S\end{cases}, \quad q=1,\dots,\mathrm{N}-1$$

$$f_i^{(N)}=\begin{cases}-(q\lambda_0+\mu), & i=0\\ -(q\lambda_0+\mu+\gamma), & i=1,\dots,s\\ -(q\lambda_0+\gamma), & i=s+1,\dots,S\end{cases}$$

$$A_{01}^{(q)}=\begin{array}{c} \\ 0\\ 1\\ 2\\ \vdots\\ \mathrm{N}\end{array}\begin{array}{c}\begin{array}{cccccc}0&1&2&3&\cdots&\mathrm{N}\end{array}\\ \begin{pmatrix} 0 & 0 & 0 & 0 & \cdots & 0\\ \gamma & 0 & 0 & 0 & \cdots & 0\\ 0 & \gamma & 0 & 0 & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots & & \vdots\\ 0 & 0 & 0 & 0 & \cdots & \gamma \end{pmatrix}\end{array}, \qquad q=1,\dots,N$$

$$A_{11}^{(q)}=\begin{array}{c} \\ 0\\ 1\\ 2\\ \vdots\\ s\\ s+1\\ \vdots\\ S\end{array}\begin{array}{c}\begin{array}{ccccccccc}0&1&2&3&\cdots&Q&Q+1&\cdots&S\end{array}\\ \begin{pmatrix} g_0^{(q)} & 0 & 0 & 0 & \cdots & \mu & 0 & \cdots & 0\\ 0 & g_0^{(q)} & 0 & 0 & \cdots & 0 & \mu & \cdots & 0\\ 0 & 0 & g_0^{(q)} & 0 & \cdots & 0 & 0 & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & \mu\\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & g_0^{(q)} \end{pmatrix}\end{array}, \qquad q=1,\dots,N$$

Where,

$$g_i^{(q)} = \begin{cases} -(\alpha + (q-1)\lambda_0 + \mu + \lambda + (S-i)\beta), & i = 1,\dots,s \\ -(\alpha + (q-1)\lambda_0 + \lambda + (S-i)\beta), & i = s+1,\dots,S \end{cases}, \quad q = 1,\dots,\mathrm{N}-1$$

$$g_i^{(N)} = \begin{cases} -(\alpha + (q-1)\lambda_0 + \mu), & i = 1,\dots,s \\ -(\alpha + (q-1)\lambda_0), & i = s+1,\dots,S \end{cases}$$

$$C_q = \begin{bmatrix} C_{00}^{(q)} & 0 \\ C_{10}^{(\mathrm{q})} & C_{11}^{(\mathrm{q})} \end{bmatrix}, q = 2,\dots,N \ , \quad C_0 = \begin{bmatrix} C_{00}^{(1)} \\ C_{10}^{(1)} \end{bmatrix}$$

$$C_{00}^{(q)} = \begin{array}{cc} & \begin{array}{ccccc} 0 & 1 & 2 & \cdots & \mathrm{S} \end{array} \\ \begin{array}{c} 0\\1\\2\\\vdots\\\mathrm{S} \end{array} & \begin{pmatrix} \mathrm{q}\lambda_0 & 0 & 0 & \cdots & 0 \\ 0 & \mathrm{q}\lambda_0 & 0 & \cdots & 0 \\ 0 & 0 & \mathrm{q}\lambda_0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & \mathrm{q}\lambda_0 \end{pmatrix} \end{array}, \ q = 1,\dots,N \ , \quad C_{10}^{(1)} = \begin{array}{cc} & \begin{array}{ccccc} 0 & 1 & 2 & \cdots & S \end{array} \\ \begin{array}{c} 0\\1\\2\\\vdots\\\mathrm{S} \end{array} & \begin{pmatrix} \alpha & 0 & 0 & \cdots & 0 \\ 0 & \alpha & 0 & \cdots & 0 \\ 0 & 0 & \alpha & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & \alpha \end{pmatrix} \end{array}$$

$$C_{10}^{(\mathrm{q})} = \begin{array}{cc} & \begin{array}{ccccc} 0 & 1 & 2 & \cdots & S \end{array} \\ \begin{array}{c} 0\\1\\2\\\vdots\\\mathrm{S} \end{array} & \begin{pmatrix} \alpha & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix} \end{array}, \quad C_{11}^{(\mathrm{q})} = \begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & \cdots & \mathrm{S\text{-}1} & \mathrm{S} \end{array} \\ \begin{array}{c} 1\\2\\3\\\vdots\\\mathrm{S} \end{array} & \begin{pmatrix} (\mathrm{q}-1)\lambda_0 & 0 & \cdots & 0 & 0 \\ \alpha & (\mathrm{q}-1)\lambda_0 & \cdots & 0 & 0 \\ 0 & \alpha & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \alpha & (\mathrm{q}-1)\lambda_0 \end{pmatrix} \end{array},$$

$$q = 2,\dots,N \qquad\qquad q = 2,\dots,N$$

$$B = \begin{bmatrix} B_{00} & 0 \\ 0 & B_{11} \end{bmatrix}, \quad B_0 = \begin{bmatrix} B_{00} & 0 \end{bmatrix}, \quad B_{00} = \begin{array}{cc} & \begin{array}{ccccc} 0 & 1 & 2 & \cdots & S \end{array} \\ \begin{array}{c} 0\\1\\2\\\vdots\\\mathrm{S} \end{array} & \begin{pmatrix} \lambda & 0 & 0 & \cdots & 0 \\ 0 & \lambda & 0 & \cdots & 0 \\ 0 & 0 & \lambda & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & \lambda \end{pmatrix} \end{array}$$

$$\mathrm{B}_{11} = \begin{array}{cc} & \begin{array}{cccccc} 1 & 2 & 3 & \cdots & \mathrm{S\text{-}1} & \mathrm{S} \end{array} \\ \begin{array}{c} 1\\2\\\vdots\\\mathrm{S}-1\\\mathrm{S} \end{array} & \begin{pmatrix} \lambda_0 & (\mathrm{S}-1)\beta & 0 & \cdots & 0 & 0 \\ 0 & \lambda & (\mathrm{S}-2)\beta & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \lambda & (\mathrm{S}-(\mathrm{S}-1)\beta \\ 0 & 0 & 0 & \cdots & 0 & \lambda \end{pmatrix} \end{array}$$

It may be noted that the matrices A_q, C_q, B are square matrices of order 2 and B_{11},$C_{11}^{(q)}$,B_{00},$C_{00}^{(q)}$,$A_{00}^{(q)}$ square matrices of order $S+1$,$A_{11}^{(q)}$ square matrices of order S. $A_{01}^{(q)}$, $C_(10)^{(q)}$ rectangular matrices of order $S \times (S+1)$, C_0 rectangular matrices of order 2×1 and B_0 rectangular matrices of order 1×2, $A_{00}^{(0)}$ square matrices of order 1×1

Steady State Analysis: Let Φ be the steady-state probability vector of A. That is Φ satisfies $\Phi A = 0$, $\Phi e = 1$.
The vector Φ can be represented by
$\Phi = (\Phi^{(0)}, \Phi^{(1)}, \dots, \Phi^{(N)})$, where $\Phi^{(q)} = (\Phi^{(q,0)}, \Phi^{(r,1)})$, $q = 0,1,\dots,N$ Which is partitioned as follows,
$\Phi^{(q,0)} = (\Phi^{(q,0,0)}, \Phi^{(q,0,1)}, \dots, \Phi^{(q,0,N)})$, $\quad q = 0,1,\dots,N$
$\Phi^{(r,1)} = (\Phi^{(q,1,1)}, \Phi^{(q,1,2)}, \dots, \Phi^{(q,1,N)})$, $\quad r = 1,\dots,N$.

D.P.Gaver discussed, Whenever the Markov process in a Markovian environment. The A is have same structure, Hence using the same arguments, We can calculate the limiting probability vectors. We provide the algorithm here.

Algorithm: 1. Determine recursively the matrices
$D_0 = A_0$
$D_n = A_n + C_n(-D_{n-1}^{-1})B_{n-1}$, for $n = 1$
$D_n = A_n + C_n(-D_{n-1}^{-1})B$, for $n = 2, \ldots, N$
2. Compute recursively the vectors $\Phi^{(n)}$ using
$\Phi^{(n)} = \Phi^{(n+1)}C_{n+1}(-D_n^{-1})$, for $n = 0,1, \ldots, N-1$
3. solve the system of equations
$\Phi^N D_N = 0$
$\sum_{n=0}^{N} \Phi^n e = 1.$
From the system of equation $\Phi^N D_N = 0$, Vector $\Phi^{(n)}$ could be determined uniquely, upto a multiplicative constant. This constant is decided by
$\Phi^{(n)} = \Phi^{(n+1)}C_{n+1}(-D_n^{-1})$, for $n = 0, \ldots, N-1$ and $\sum_{n=0}^{N} \Phi^n e = 1.$

System Performance Measures: In this section, we derive some system performance measures with respected to the steady state analysis.

Expected Inventory Level: Let η_I denote the mean inventory level in the steady state. Since $\Phi^{(q,k,i)}$ denote the steady state probability when the Number of customers in the waiting area q, the server states is k and the inventory level is i, the mean inventory level is given by
$\eta_I = \sum_{q=0}^{N} \sum_{i=1}^{S} i\Phi^{(q,0,i)} + \sum_{q=1}^{N} \sum_{i=1}^{S} i\Phi^{(q,1,i)}$

Expected Reorder Rate: Let η_R denote the expected reorder rate in the steady state. We note that a reorder is triggered when the inventory level drops from s+1 to s, we also note that reordering will occur when the server is on vacation and working condition.
$\eta_R = \sum_{q=1}^{N} \mu\Phi^{(q,1,s+1)}$

Expected Cancellation Rate: Let η_c denote the expected cancellation rate in the steady state. $\eta_c = \sum_{q=1}^{N} \sum_{i=1}^{S-1} \beta\Phi^{(q,1,i)}$

Expected Number of Customers Waiting Server is on Vacation: Let η_{wv} denote the mean number of customers waiting while server is on vacation in the steady state.
$\eta_{wv} = \sum_{q=1}^{N} \sum_{i=0}^{S} q\Phi^{(q,0,i)}$

Expected Loss of Customers: Let η_L denote the expected loss of customers cost due to negative customers per unit time. $\eta_L = \lambda_0\left[\sum_{q=1}^{N} \sum_{i=1}^{S} \Phi^{(q,1,i)} + \sum_{q=1}^{N} \sum_{i=0}^{S} \Phi^{(q,0,i)}\right]$

Cost Analysis: Let TC(s,S) denote the long-run expected cost rate under the following cost structure:
c_s: Setup cost per order.
c_n: The cost due to the arrival of negative customers per unit time.
c_w: The waiting cost of the customers per unit time.
c_h: The inventory carrying cost per unit time.
c_c: the system due to per unit cancellation of inventory purchased.
Then
$TC(s,S) = c_c\eta_c + c_s\eta_R + c_n\eta_L + c_h\eta_I + c_w\eta_{wv}$.

Conclusion : In this paper, we have considered an M/M/1 queueing inventory system with server vacation, cancellation and negative customers, we have developed the steady state distribution of the system using matrix and several system performance measures also been calculated. The long-run total expected cost rate is derived and also we have derived numerical illustrations.

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