THE PROPAGATION OF DILATATION WAVES IN MICRO-MORPIC MEDIUM

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Abstract: In This Paper The Propagation Of Dilatation Waves In Micro- Morphic Medium Has Been Discussed. Frequency Equations Have Been Deduced In Some Cases. It Is Found That Additional Waves Are Propagating In The Micro-Morphic Medium. The Classical Results Are Obtained As Particular Cases Of These.

I. Introduction: The Theory Of Micro-Morphic Materials Was Proposed By Erigen [1]. This Theory Was Modified By Koh [3] By Extending The Concept Of Coincidence Of Principal Directions Of Stress And Strain Of Classical Elasticity To Micro-Ealstic Solid And Assuming A Particular Form Of Isotropy. He Called This Modified Theory As Theory Of Micro-Isotropic, Micro-Elastic Materials. In This Paper The Problem Of Propagation Of Dialations And Distortional Waves Is Discussed In Medium As Developed By Koh [3].

In Classical Elasticity [2] It Was Shown That The Equations Of Motion Correspond To Two Types Of Waves Which Can Be Propagated Through An Elastic Solid. These Two Types Of Waves Are Called Dilatation And Distortional. In The Present Case, In Each Of The Situation Some Additional Waves Are Found Compared To The Classical Elasticity. These Additional Waves Are Exiting Due To Extra Degree Of Freedom Such As Micro-Rotation And Micro-Strain Of Micro-Isotropic Micro-Elastic Medium.

II. Basic Equations: The Micro-Displacement In The Micro-Elastic Continuum Is Denoted By u_k And Micro-Deformation By ϕ_{mn}. Further The Macro-Strain $e_{km} = u_{(k,m)}$, The Macro-Rotation $r_k = \frac{1}{2} \in_{kmn} u_{n,m}$, The Micro Strain $\phi_{(mn)}$ And The Micro-Rotation Vector $\phi_p = \frac{1}{2} \in_{pkm} \phi_{km}$ Where () Denotes The Symmetric Part, Comma Denotes Differentiation With Respect To The Coordinate (x_k).

The Stress Measures Are The Asymmetric Stress t_{km}, The Relative Stress σ_{km} And The Stress Moment t_{kmn}. The Couple Stress Tensor m_{kl} Is Defined By $m_{kl} = \in_{pnm} t_{kmn}$

The Constitutive Equations For The Theory Of Micro-Isotropic, Micro-Elastic Materials Are Given By Koh And Parameshwaran. They Are

$$t_{(km)} = A_1 \tau_{pp} \delta_{Km} + 2A_2 e_{Km} \qquad (1)$$

$$t_{[km]} = \sigma_{[km]} = 2A_3 \in_{pkm} (r_p + \phi_p) \qquad (2)$$

$$\sigma_{[km]} = -A_4 \phi_{pp} \delta_{km} - 2A_5 \phi_{(km)} \qquad (3)$$

$$t_{k(mn)} = B_1 \phi_{pp,k} \delta_{mn} + 2B_2 \phi_{(mn),k} \qquad (4)$$

$$m_{(kl)} = -2(B_5 \phi_{l,k} + B_4 \phi_{k,l} + B_5 \phi_{p,p} \delta_{kl}) \qquad (5)$$

Where [ ] Denotes Anti-Symmetric Part

$$A_1 = \lambda + \sigma_1, \quad B_1 = \tau_3, \; A_2 = \mu + \sigma_2, \quad 2B_2 = \tau_1 + \tau_{10}$$

$$A_3 = \sigma_5, \quad B_3 = 2\tau_4 + 2\tau_9 + \tau_7 - \tau_{10}, \; A_4 = -\sigma_1, \quad B_4 = -2\tau_4 \qquad (6)$$

$$A_5 = -\sigma_2 \quad B_5 = -2\tau_9$$

And λ, μ, σ_1, σ_2, σ_5, τ_3, τ_4, τ_7, τ_9 And τ_{10} Are Elastic Constants.
The Displacement Equations Of Motion For Micro-Elastic Body Occupying A Region R Are Given By

$$(A_1 + A_2 - A_3)u_{p,pm} + (A_2 + A_3)u_{m,pp} + 2A_3 \in_{pkm} \phi_{p,k} + \rho f_m = \rho \frac{\partial^2 u_m}{\partial t^2} \quad (7)$$

$$B_1\phi_{pp,kk}\delta_{ij} + 2B_2\phi_{(ij),kk} - A_4\phi_{pp}\delta_{ij} - 2A_5\phi_{(ij)} + \rho f_{(ij)} = \frac{1}{2}\rho j \frac{\partial^2 \phi_{(ij)}}{\partial t^2} \quad (8)$$

$$2B_3\phi_{p,mm} + 2(B_4 + B_5)\phi_{m,mp} - 4A_3(r_p + \phi_p) - \rho l_p = \rho j \frac{\partial^2 \phi_p}{\partial t^2} \quad (9)$$

Where ρ Is The Average Mass Density, F_m Is The Body Force Per Unit Mass, $f_{(ij)}$ Is The Symmetric Body Moment Per Unit Mass, J Is The Micro-Inertia, l_p Is The Body Couple Per Unit Mass And Do Symbol Denoted Differentiation With Respect To Time T,
Subject To The Boundary Conditions

$$t_{km}n_k = T_m \qquad t_{kmn}n_k = T_{mn} \qquad \text{On The Boundary} \quad (10)$$

Where $\vec{n}$ Is The Unit Outward Drawn Normal To The Boundary T_m And T_{mn} Are The Surface Tractions And Surface Moments Respectively.

III. Solution of The Problem:
The Equations Of Motion (7) Under The Absence Of Body Forces And Body Couples Are Given By

$$(A_1 + A_2 - A_3)\frac{\partial}{\partial x_1}\Delta + (A_2 + A_3)\nabla^2 u_1 + 2A_3\left(\frac{\partial\phi_2}{\partial x_3} - \frac{\partial\phi_3}{\partial x_2}\right) = \rho\frac{\partial^2 u_1}{\partial t^2} \quad (11)$$

$$(A_1 + A_2 - A_3)\frac{\partial}{\partial x_2}\Delta + (A_2 + A_3)\nabla^2 u_2 + 2A_3\left(\frac{\partial\phi_3}{\partial x_1} - \frac{\partial\phi_1}{\partial x_3}\right) = \rho\frac{\partial^2 u_2}{\partial t^2} \quad (12)$$

$$(A_1 + A_2 - A_3)\frac{\partial}{\partial x_3}\Delta + (A_2 + A_3)\nabla^2 u_3 + 2A_3\left(\frac{\partial\phi_1}{\partial x_2} - \frac{\partial\phi_2}{\partial x_1}\right) = \rho\frac{\partial^2 u_3}{\partial t^2} \quad (13)$$

Where
$$\Delta = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} \quad (14)$$

Similarly Under The Absence Of Body Forces And The Body Couples The Equation (9) Can Be Expressed As

$$2(B_4 + B_5)\frac{\partial}{\partial x_1}\Delta^1 + 2B_3\nabla^2\phi_1$$

$$-2A_3(u_{3,2} - u_{2,3}) - 4A_3\phi_1 = \rho j\frac{\partial^2\phi_1}{\partial t^2} \quad (15)$$

$$2(B_4 + B_5)\frac{\partial}{\partial x_2}\Delta^1 + 2B_3\nabla^2\phi_2$$

$$-2A_3(u_{1,3} - u_{3,1}) - 4A_3\phi_2 = \rho j\frac{\partial^2\phi_2}{\partial t^2} \quad (16)$$

$$2(B_4 + B_5)\frac{\partial}{\partial x_3}\Delta^1 + 2B_3\nabla^2\phi_3$$

$$-2A_3\left(u_{2,1}-u_{1,2}\right)-4A_3\phi=\rho j\frac{\partial^2\phi_3}{\partial t^2} \tag{17}$$

Where $\Delta^1=\frac{\partial\phi_1}{\partial x_1}+\frac{\partial\phi_2}{\partial x_2}+\frac{\partial\phi_3}{\partial x_3}$ (18)

Differentiating (11) With Respect To x_1 We Get,

$$\left(A_1+A_2-A_3\right)\frac{\partial^2}{\partial x_1^2}\Delta+\left(A_2+A_3\right)\nabla^2\frac{\partial u_1}{\partial x_1}$$

$$+2A_3\left(\phi_{2,31}-\phi_{3,12}\right)=\rho\frac{\partial^2}{\partial t^2}\left(\frac{\partial u_1}{\partial x_1}\right) \tag{19}$$

Similarly, Differentiating (12) With Respect To x_2 We Get,

$$\left(A_1+A_2-A_3\right)\frac{\partial^2}{\partial x_2^2}\Delta+\left(A_2+A_3\right)\nabla^2\frac{\partial u_2}{\partial x_2}$$

$$+2A_3\left(\phi_{3,12}-\phi_{1,32}\right)=\rho\frac{\partial^2}{\partial t^2}\left(\frac{\partial u_2}{\partial x_2}\right) \tag{20}$$

Differentiating (13) With Respect To x_3 We Get

$$\left(A_1+A_2-A_3\right)\frac{\partial^2}{\partial x_3^2}\Delta+\left(A_2+A_3\right)\nabla^2\frac{\partial u_3}{\partial x_3}$$

$$+2A_3\left(\phi_{1,23}-\phi_{2,13}\right)=\rho\frac{\partial^2}{\partial t^2}\left(\frac{\partial u_3}{\partial x_3}\right) \tag{21}$$

Adding (19) To (21) We Obtain

$$\left(A_1+A_2-A_3\right)\nabla^2\Delta+\left(A_2+A_3\right)\nabla^2\Delta=\rho\frac{\partial^2\Delta}{\partial t^2}$$

I.E., $\left(A_1+2A_2\right)\nabla^2\Delta=\rho\frac{\partial^2\Delta}{\partial t^2}$

$$\therefore\nabla^2\Delta=\frac{1}{C_1^2}\frac{\partial^2\Delta}{\partial t^2} \tag{22}$$

Where $C_1^2=\frac{A_1+2A_2}{\rho}$ (23)

The Equation (22) Shows That The Volume Dilation Or Compression Is Transmitted In The Form Of Waves Through The Micro-Morphic Medium With A Velocity $C_1=\sqrt{\frac{A_1+2A_2}{\rho}}$.

This Wave Is Called A Wave Of Dilatation. The Equation (22) Corresponds To The Classical Elasticity. In View Of (6) The Equation (23) Reduces To

$$C_1^2=\frac{A_1+2A_2}{\rho}=\frac{\lambda+2\mu+\sigma_1+2\sigma_2}{\rho}$$

And It Reduces To $\frac{\lambda+2\mu}{\rho}$ When $\sigma_1=0$, $\sigma_2=0$, Which Is The Classical Result.[2]

Now, Differentiating (15), (16), (17) With Respect To x_1, x_2 And x_3 Respectively, We Have

$$2(B_4+B_5)\frac{\partial^2}{\partial x_1^2}\Delta^1+2B_3\nabla^2\frac{\partial\phi_1}{\partial x_1}-2A_3(u_{3,21}-u_{2,31})$$

$$-4A_3\frac{\partial\phi_1}{\partial x_1}=\rho j\frac{\partial^2}{\partial t^2}\left(\frac{\partial\phi_1}{\partial x_1}\right) \quad (24)$$

$$2(B_4+B_5)\frac{\partial^2}{\partial x_2^2}\Delta^1+2B_3\nabla^2\frac{\partial\phi_2}{\partial x_2}-2A_3(u_{1,32}-u_{3,12})$$

$$-4A_3\frac{\partial\phi_2}{\partial x_2}=\rho j\frac{\partial^2}{\partial t^2}\left(\frac{\partial\phi_2}{\partial x_2}\right) \quad (25)$$

$$2(B_4+B_5)\frac{\partial^2}{\partial x_3^2}\Delta^1+2B_3\nabla^2\frac{\partial\phi_3}{\partial x_3}-2A_3(u_{2,13}-u_{1,23})$$

$$-4A_3\frac{\partial\phi_3}{\partial x_3}=\rho j\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2\phi_3}{\partial x_3}\right) \quad (26)$$

Adding (24), (25) And (26) We Get

$$2(B_4+B_5)\nabla^2\Delta^1+2B_3\nabla^2\Delta^1-4A_3\Delta^1=\rho j\frac{\partial^2\Delta^1}{\partial t^2}$$

I.E. $$\left[2(B_4+B_5)+2B_3\right]\nabla^2\Delta^1-4A_3\Delta^1=\rho j\frac{\partial^2\Delta^1}{\partial t^2} \quad (27)$$

Plane Waves Advancing In The Positive Direction Of The Unit Vector $\vec{n}$ May Be Expressed As

$$\Delta^1=A\exp\{ik(\vec{n}.\vec{r}-vt)\} \quad (28)$$

Where A Is A Constant, K Is The Wave Number, $\vec{r}$ Is The Position Vector, V Is The Velocity Of The Wave.

Substituting (28) In (27) We Get

$$2(B_3+B_4+B_5)k^2+4A_3=\rho j^2k^2v^2$$

$$2(B_3+B_4+B_5)+\frac{4A_3}{k^2}=\rho jv^2 \quad (29)$$

If We Introduce The Angular Frequency $\omega=kv$ (30)

The Equation (29) Reduces To

$$C_2^2=\frac{2(B_3+B_4+B_5)}{\rho j\left(1-\frac{\omega_1^2}{\omega^2}\right)} \quad (31)$$

Where $\omega_1^2=\frac{4A_3}{\rho j}$ (32)

The Speed Of The Waves Depends On The Frequency. Hence, They Are Dispersive. Further These Waves Depend On Purely Micromorphic Elastic Constants Which Are Not Encountered In Classical Elasticity.

Now The Equations Of Motion Under The Absence Of Forcen And Couples, The Equation (8) Involving Micro-Strains Can Be Expressed As

$$B_1\phi_{pp,kk} + 2B_2\phi_{11,kk} - A_4\phi_{pp} - 2A_5\phi_{11} = \frac{1}{2}\rho j\frac{\partial^2\phi_{11}}{\partial t^2} \quad (33)$$

$$B_1\phi_{pp,kk} + 2B_2\phi_{22,kk} - A_4\phi_{pp} - 2A_5\phi_{22} = \frac{1}{2}\rho j\frac{\partial^2\phi_{22}}{\partial t^2} \quad (34)$$

$$B_1\phi_{pp,kk} + 2B_2\phi_{33,kk} - A_4\phi_{pp} - 2A_5\phi_{33} = \frac{1}{2}\rho j\frac{\partial^2\phi_{33}}{\partial t^2} \quad (35)$$

$$2B_2\phi_{(12),kk} - 2A_5\phi_{(12)} = \frac{\rho j}{2}\frac{\partial^2\phi_{(12)}}{\partial t^2} \quad (36)$$

$$2B_2\phi_{(13),kk} - 2A_5\phi_{(13)} = \frac{\rho j}{2}\frac{\partial^2\phi_{(13)}}{\partial t^2} \quad (37)$$

$$2B_2\phi_{(23),kk} - 2A_5\phi_{(23)} = \frac{\rho j}{2}\frac{\partial^2\phi_{(23)}}{\partial t^2} \quad (38)$$

We Rewrite These Equations For The Convenience. Adding (33) To (35) We Get

$$(2B_1 + 2B_2)\phi_{pp,kk} - (3A_4 + 2A_5)\phi_{pp} = \frac{\rho j}{2}\frac{\partial^2\phi_{pp}}{\partial t^2} \quad (39)$$

Subtracting (34) From (33) It Yields

$$2B_2(\phi_{11} - \phi_{22})_{,kk} - 2A_5(\phi_{11} - \phi_{22})_{,kk} = \frac{1}{2}\rho j\frac{\partial^2}{\partial t^2}(\phi_{11} - \phi_{22})_{,kk} \quad (40)$$

And Subtracting (35) From (33) We Get

$$2B_2(\phi_{11} - \phi_{33})_{,kk} - 2A_5(\phi_{11} - \phi_{33}) = \frac{1}{2}\rho j\frac{\partial^2}{\partial t^2}(\phi_{11} - \phi_{33})_{,kk} \quad (41)$$

Now We Discuss The Waves Corresponding To (36) To (41). Plane Waves Advancing In The Positive Direction Of Unit Vector $\vec{n}$ May Be Expressed As

$$\phi_{(ij)} = D_{(ij)} \exp\{ik(\vec{n}.\vec{r} - vt)\} \quad (42)$$

Where D_{ij} Are Constants And

$$D_{ij} = D_{(ji)} \quad (43)$$

Substituting (42) In The Equation (39) And Using (30), We Get

$$[(3B_1 + 2B_2)k^2 + (3A_4 + 2A_5)]\phi_{pp} = \frac{\rho j}{2}\omega^2$$

We Obtain An Expression For The Phase Velocity V_3 As

$$V_3^2 = \frac{\omega^2}{k^2} = \frac{2(3B_1 + 2B_2)}{\rho j(\omega^2 - \omega_2^2)} \quad (44)$$

Where $\omega_2^2 = 2\frac{(3A_4 + 2A_5)}{\rho j}$ (45)

It Is Longitudinal Micro-Dilation Wave. V_3 Is Real Not Finite Or Imaginary According As $\omega > \omega_2$, $\omega = \omega_2$ And $\omega < \omega_2$ Respectively.

Now Substituting (42) In The Equation (40) We Obtain The Phase Velocity V_4 And It Is Given By

$$V_4^2 = \frac{\omega^2}{k^2} = \frac{4B_2\omega^2}{\rho j\left(\omega^2 - \omega_3^2\right)} \quad (46)$$

Where $\omega_3^2 = \frac{4A_5}{\rho j}$ (47)

We Call This Wave As Transverse Micro-Extensional Wave.

IV. Conclusion: We Can Observe That The Equation (26) To (28) And (41) Are Identical To The Equation (40). Thus, We Have Another Four Waves Whose Phase Velocities Are v_5, v_6, v_7, v_8 Each Of Which Is Equal To v_4 .The Wave Corresponds To The Equation (41), We Call It As Transverse Micro-Extensional Wave. The Waves Given By (36) To (38) And Called Micro-Shear Waves. All These Waves Are Dispersive And Having Some Cut-Off Frequencies. Thus It Is Possible To Have Eight Waves Propagating In Micromorphic Medium With Four District Velocities.

The Equations Of Motion Involving Micro-Strains Give Another Six Waves Propagating With Two District Velocities And All Are Dispersive.

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