IMPULSIVE STABILIZATION OF INERTIAL NEURAL NETWORKS WITH TIME-DELAY

R. Krishnasamy
Department of Mathematics, Indian Institute of Space Science and Technology, Thiruvananthapuram - 695 547, Kerala, India
Raju K. George
Department of Mathematics, Indian Institute of Space Science and Technology, Thiruvananthapuram - 695 547, Kerala, India

Abstract: Stabilization of inertial neural networks with time-delay based on impulsive control is investigated in this paper. Sufficient delay-dependent stabilization results are obtained in terms of linear matrix inequalities via Lyapunov stability theory which involves the construction of Lyapunov-Krasovskii functional. Information of time-delay is taken into account to obtain these results. Here time-delay is considered to be time-varying and the activation function is assumed to be sector bounded. Derived conditions can be validated via MATLAB. Finally, an example is provided to support the derived results.

Keywords: Inertial Neural Networks, Impulses, Lyapunov-Krasovskii Functional, Stabilization.

1. Introduction: Past decades witnessed the dynamical analysis of various types of neural networks (NNs) namely Hopfield NNs, bidirectional associative memory (BAM) NNs, cellular NNs, Cohen-Grossberg NNs and so on. In the year 1981, Babcock and Westervelt published an article on the dynamics of simple electronic NNs which initiated the research on inertial NNs (INNs). In their research work, they mentioned that inertial characteristics added to resistance and capacitance couplings can cause spontaneous oscillation, chaotic response and so on. Dynamics of INNs received much attention among researchers because of its both physical and biological significance, for details one can see [1]-[3], [8], [9] and therein.

Meanwhile, time-delays which occur in the process of information storage and transmission in NNs can cause instability, oscillation in the dynamics of NNs. Hence, there is an increasing interest to investigate the dynamics of NNs with the inclusion of time-delays, for details see [4]-[7]. On the other hand, states of NNs are often subject to abrupt change at certain moments of time due to the switching phenomenon, frequency change called impulse effects. Impulsive NNs belong to the special category which is the combination of continuous-time and discrete-time systems. Hence it is necessary to include the effects of impulses in the dynamics of NNs, see [10]-[12]. Sometimes impulsive effects can destabilize stable systems which are considered as the destructive one but on the other hand they can also be used to stabilize the de-stable systems. In this work, impulses are used to stabilize the de-stable systems.

In [10], authors established distributed delay-dependent stability criteria for impulsive INNs in which they considered both discrete and distributed time-delays. Robust stability conditions for inertial BAM NNs with time-delays and uncertainties via impulsive effect are considered in [11]. Stability analysis of Markovian jump stochastic BAM NNs with impulse control and mixed time-delays is considered in [12]. Hence in the existing literature, it can be seen that stability analysis of INNs with impulsive effects is taken in to account whereas the problem of stabilization of INNs via impulsive control has not gained attention from the researchers. This motivates us to consider the problem of stabilization of time-delay INNs via impulsive control.

Inspired by the above observations, stabilization problem of INNs based on impulsive controls is considered in this work. Here the activation function of the neural network is assumed to be sector-bounded and the time-delay is taken to be time-varying. Delay-dependent stabilization results are derived for the considered problem based on the construction of Lyapunov-Krasovskii functional (LKF) involving some quadratic and integral terms. Schur complement lemma is used in the derivation process to convert some nonlinear matrix inequalities into linear matrix inequalities (LMIs). Obtained results can be easily solved through MATLAB software. Derived theoretical results are validated through an example.

Rest of this paper is structured as follows. In Section 2, the problem description and preliminaries are given. In Section 3, sufficient delay-dependent stabilization conditions pertaining to impulsive control are presented. In Section 4, numerical example is given to illustrate the proposed results and Section 5 concludes the paper.

Notations: Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n \times m$ real matrices. For symmetric matrices X and Y, the notation $X \geq Y (X > Y)$ means that $X - Y$ is positive-semidefinite (positive-definite); M^T denotes the transpose of the matrix M ; I is the identity matrix with appropriate dimension; $\|\cdot\|$ is the Euclidean norm in $\mathbb{R}^n$. $PC([-\tau_2, 0], \mathbb{R}^n)$ is the piece-wise continuous function; $\psi(s^+)$ and $\psi(s^-)$ denote the right-hand and left-hand limits of the function $\psi(s)$ respectively; $\lambda_{max}(A)$, $\lambda_{min}(A)$ denote respectively the maximum
and minimum eigenvalues of the matrix A and matrices, and matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. Problem Description and Preliminaries: Consider the following second order system of differential equation model of INNs

$$\ddot{m}_i(t) = -a_i\dot{m}_i(t) - b_i m_i(t) + \sum_{j=1}^{n} c_{ij} f_j(m_j(t)) + \sum_{j=1}^{n} d_{ij} f_j(m_j(t-\tau(t))) + I_i \quad (1)$$

for $i = 1, 2, \cdots, n$, where $\ddot{m}_i(t)$ denotes the inertial term of the i^{th} neuron at time t, $m_i(t)$ is the state of the i^{th} neuron at time t. $f_j(\cdot)$ denotes the neuron activation function of i^{th} neuron at time t and $j(0) = 0, j \in \{1, 2, \cdots, n\}$, $\tau(t)$ is the time-varying delay, I_i represents the external input on the i^{th} neuron at time t. a_i and b_i are positive constants, c_{ij} and d_{ij} are the connection weights related to the neurons without delays and with delays respectively. Initial condition of (1) is given by $m_i(s) = \varphi_i(s)$ and $\dot{m}_i(s) = \psi_i(s)$, for $-\tau_2 \leq s \leq 0$ where $\varphi_i(s)$ and $\psi_i(s)$ are bounded and continuous.

Now transform the second order differential equation model into a system of first order differential equations using the transformation $p_i(t) = \dot{m}_i(t) + m_i(t), i = 1, 2, \cdots, n$ and the resulting system is given as

$$\begin{cases} \dot{m}_i(t) = -m_i(t) + p_i(t) \\ \dot{p}_i(t) = -(a_i - 1)p_i(t) - [b_i + (1 - a_i)]m_i(t) + \sum_{j=1}^{n} c_{ij} f_j(m_i(t)) + \sum_{j=1}^{n} d_{ij} f_j(m_i(t-\tau(t))) + I_i \end{cases} \quad (2)$$

with the initial conditions given by $m_i(s) = \varphi_i(s)$ and $p_i(s) = \varphi_i(s) + \psi_i(s)$, for $-\tau_2 \leq s \leq 0$. Next, we shift the equillibrium of system (2) to the origin by using the transformation $x(t) = m(t) - m^*$ and $y(t) = p(t) - m^*$, where $m(t) = [m_1(t), m_2(t), \cdots, m_n(t)]^T$, $p(t) = [p_1(t), p_2(t), \cdots, p_n(t)]^T$. Here (m^*, p^*) is the equillibrium point of (2). Therefore, the transformed system in matrix form can be written as

$$\begin{cases} \dot{x}(t) = -x(t) + y(t) \\ \dot{y}(t) = Ay(t) - Bx(t) + Cf\left(x(t)\right) + Dg\left(x\left(t - \tau(t)\right)\right). \end{cases} \quad (3)$$

Initial conditions $x(s) = \phi(s) - m^*$ and $y(s) = \psi(s) + \phi(s) - p^*$, where $x(t) = [x_1(t), x_2(t), \cdots, x_n(t)]^T$, $y(t) = [y_1(t), y_2(t), \cdots, y_n(t)]^T$ are state vectors of (3), $g(x(t)) = [g_1(x_1(t)), g_2(x_2(t)), \cdots, g_n(x_n(t))]^T$ with $g(x(t)) = f(x(t) + m^*) - f(m^*)$, $A = diag\{(a_1 - 1), (a_2 - 1), \cdots, (a_n - 1)\}$, $B = diag\{b_1 + (1 - a_1), b_2 + (1 - a_2), \cdots, b_n + (1 - a_n)\}$, $C = (c_{ij})_{n\times n}$, $D = (d_{ij})_{n\times n}$ and $I = diag\{I_1, I_2, \cdots, I_n\}$.

System (3) with continuous and impulsive control becomes

$$\begin{cases} \dot{x}(t) = -x(t) + y(t) + E_1 u_1(t), \ t \neq t_k, \\ \Delta x(t) = (F_1 - I)x(t^-) + E_2 u_2(t), \ t = t_k, \\ \dot{y}(t) = Ay(t) - Bx(t) + Cf\left(x(t)\right) + Dg\left(x\left(t - \tau(t)\right)\right) + E_3 u_3(t), \ t \neq t_k, \\ \Delta y(t) = (F_2 - I)y(t^-) + E_4 u_4(t), \ t = t_k, \end{cases} \quad (4)$$

where $u_1, u_3 \in \mathbb{R}^{n_1}$ are continuous control inputs and $u_2, u_4 \in \mathbb{R}^{n_2}$are impulse control inputs. E_1, E_2, E_3, E_4, F_1 and F_2 are known constant matrices. $\Delta x(t)|_{t=t_k} = x(t_k^+) - x(t_k^-)$ and $\Delta y(t)|_{t=t_k} = y(t_k^+) - y(t_k^-)$. $x(t_k)$ and $y(t_k)$ denote the impulse at the moment t_k. Here the discrete time sequence t_k satisfies $0 = t_0 < t_1 < t_2 < \cdots < t_k < \cdots$ and $lim_{k\to\infty} t_k = \infty$. Both $x(t_k)$ and $y(t_k)$ are assumed to be right continuous, i.e., $x(t_k) = x(t_k^+)$ and $y(t_k) = y(t_k^+)$. Initial conditions $\phi(t), \psi(t) \in PC([-\tau_2, 0], \mathbb{R}^n)$ are piece-wise continuous functions at finite number of points.
Next design the controllers u_1, u_2, u_3 and u_4 as follows

$$u_1(t) = K_1 x(t), u_2(t^-) = K_2 x(t^-), u_3(t) = K_3 y(t), u_4(t) = K_4 y(t^-), \quad (5)$$

where K_1, K_2, K_3 and K_4 are control gain matrices to be designed. Also use the fact that $\Delta x(t) = x(t_k) - x(t_k^-)$ and $\Delta y(t) = y(t_k) - y(t_k^-)$ then system (4) becomes

$$\begin{cases} \dot{x}(t) = -x(t) + y(t) + E_1 K_1 x(t), \ t \neq t_k, \\ \Delta x(t) = (F_1 - I)x(t^-) + E_2 K_2 x(t^-), \ t = t_k, \\ \dot{y}(t) = Ay(t) - Bx(t) + Cf\left(x(t)\right) + Dg\left(x\left(t - \tau(t)\right)\right) + E_3 K_3 y(t), \ t \neq t_k, \\ \Delta y(t) = (F_2 - I)y(t^-) + E_4 K_4 y(t^-), \ t = t_k. \end{cases} \quad (6)$$

The following assumptions, definition and lemma are required to prove the main results.

Assumption 2.1: The neuron activation function $g_i(\cdot)$ in (6) is globally Lipschitz in $x(t)$, $g(0) = 0$ and satisfies the following condition

$$\left(g\left(x(t)\right) - G_1 x(t)\right)^T \left(g\left(t, x(t)\right) - G_2 x(t)\right) \leq 0, \qquad \forall \ x(t) \in \mathbb{R}^n,$$

where G_1 and G_2 are known real constant matrices of appropriate dimensions.

Assumption 2.2: Time-varying delay $\tau(t)$ satisfies

$$0 \leq \tau(t) \leq \tau_2, \qquad \dot{\tau}(t) \leq \mu < 1,$$

where τ_2 and μ are constants.

Definition 2.3: The equilibrium point of INNs with impulses and time-delay (6) is said to be exponentially stable under impulsive control with convergence rate $\alpha > 0$ if there exists $\lambda > 0$ such that

$$\| x(t) \|^2 + \| y(t) \|^2 \leq \lambda \, e^{-2\alpha(t-t_0)} Sup_{-\tau_2 \leq s \leq 0}[\| \phi(s) \|^2 + \| \psi(s) \|^2], \qquad \forall \ t \geq t_0.$$

Definition 2.4: The function $V: [0, \infty) \times \mathbb{R}^n \times \mathbb{R}^n \to [0, \infty)$ belong to class Ω_0 if

1. the function V is continuous on each of the sets $[t_{k-1}, t_k) \times \mathbb{R}^n \times \mathbb{R}^n, \forall \ t \geq 0$ and $V(t, 0, 0) = 0$,
2. $V(t, x, y)$ is locally Lipschitzian,
3. for each $k \in \mathbb{N}$, there exist finite limits $lim_{(t,\bar{x},\bar{y})\to(t_k^-,x,y)} V(t_k^-, x, y)$ and $lim_{(t,\bar{x},\bar{y})\to(t_k^+,x,y)} V(t_k^+, x, y)$ with $V(t_k, x, y) = V(t_k^+, x, y)$.

3. Stabilization Results for Impulsive INNs: In this section, we derive the stabilization conditions for the time-delay INNs based on impulsive control. Here, control gain matrices introduced in (5) is taken as

$$K_1 = L_1 X^{-1}, \ K_2 = L_2 X^{-1}, \ K_3 = L_3 Y^{-1}, \ K_4 = L_4 Y^{-1}, \quad (7)$$

where L_1, L_2, L_3 and L_4 are unknown matrices to be determined.

Theorem: System (6) is exponentially stable under the Assumptions 2.1 and 2.2 if there exist scalars α, λ, symmetric positive-definite matrices $P_1, P_2, \hat{Q}$ and R, matrices L_1, L_2, L_3 and L_4 such that the following symmetric LMIs hold

$$\Omega_1 < 0, \Omega_2 < 0, \Omega_3 < 0, \quad (8)$$

where $\Omega_1 = \begin{bmatrix} \widehat{\Omega} & S_1 & S_2 \\ * & -I & 0 \\ * & * & -I \end{bmatrix}$, $\Omega_2 = \begin{bmatrix} -X & XF_1^T + L_2^T E_2^T \\ * & -X \end{bmatrix}$, $\Omega_3 = \begin{bmatrix} -Y & YF_2^T + L_4^T E_4^T \\ * & -Y \end{bmatrix}$

with the stabilizing gains given by (7). Here $\widehat{\Omega} = (\widehat{\Omega}_{p,q})_{5\times5}$ with $\widehat{\Omega}_{1,1} = -2X + \hat{Q} + 2\alpha X + E_1 L_1$, $\widehat{\Omega}_{1,3} = Y - XB^T$, $\widehat{\Omega}_{1,4} = X(G_1^T + G_2^T)$, $\widehat{\Omega}_{\{2,2\}} = -e^{-2\alpha\tau_2}\hat{Q}$, $\widehat{\Omega}_{3,3} = -2AY + 2\alpha Y + E_3 L_3$, $\widehat{\Omega}_{3,4} = C$, $\widehat{\Omega}_{3,5} = D$, $\widehat{\Omega}_{4,4} = R - 2I$, $\widehat{\Omega}_{5,5} = -(1-\mu)R$, $S_1 = [G_1 X \ \ 0 \ \ 0 \ \ 0 \ \ 0]^T$, $S_2 = [G_2 X \ \ 0 \ \ 0 \ \ 0 \ \ 0]^T$ and remaining entries of symmetric block are zero.

Proof: Stabilization results for system (6) can be obtained from the construction of LKF as follows

$$V\big(t,x(t),y(t)\big) = x^T(t)P_1x(t) + y^T(t)P_2y(t) + \int_{t-\tau_2}^{t} e^{2\alpha(s-t)}\, x^T(s)Qx(s)ds$$
$$+\int_{t-\tau(t)}^{t} e^{2\alpha(s-t)}\, g^T\big(x(s)\big)Rg\big(x(s)\big)ds, \qquad (9)$$

where P_1, P_2, Q and R are unknown symmetric positive definite matrices of appropriate dimensions. From Definition 2.4, one can see that (9) is locally Lipschitz for each $t \in [t_{k-1}, t_k)$ and $V(t,0,0) = 0$. Moreover, all the conditions of Definition 2.4 hold and hence $V\big(t,x(t),y(t)\big) \in \Omega_0$ for each $t \in [t_{k-1}, t_k)$. Dini's upper right hand derivative of (9) along trajectory of (6) can be determined as follows:

$$D^+V\big(t,x(t),y(t)\big) \le 2x^T(t)P_1\dot{x}(t) + 2y^T(t)P_2\dot{y}(t) + x^T(t)Qx(t) - e^{-2\alpha\tau_2}x^T(t-\tau_2)Qx(t-\tau_2)$$
$$+g^T(x(t))Rg(x(t)) - (1-\mu)g^T x(t-\tau(t))Rg(x(t-\tau(t)))$$
$$-2\alpha\Big(\int_{t-\tau_2}^{t} e^{2\alpha(s-t)}\, x^T(s)Qx(s)ds + \int_{t-\tau(t)}^{t} e^{2\alpha(s-t)}\, g^T\big(x(s)\big)Rg\big(x(s)\big)ds. \qquad (10)$$

According to Assumption 2.1, we have

$$\begin{bmatrix} x(t) \\ g\big(x(t)\big)\end{bmatrix}^T \begin{bmatrix} -(G_1^TG_2 + G_2^TG_1) & G_1^T + G_2^T \\ * & -2I \end{bmatrix}\begin{bmatrix} x(t) \\ g\big(x(t)\big)\end{bmatrix} \ge 0. \qquad (11)$$

Combining inequalities (10) and (11), we get

$$D^+V\big(t,x(t),y(t)\big) + 2\alpha\, V\big(t,x(t),y(t)\big) \le \xi^T(t)\Omega\xi(t) < 0 \qquad (12)$$

which implies that

$$\Omega < 0, \qquad (13)$$

$\forall t \in [t_{k-1}, t_k)$ where $\xi^T(t) = [x^T(t) \quad x^T(t-\tau_2) \quad y^T(t) \quad g^T(x(t)) \quad g^T(x(t-\tau(t)))\,]$ and $\Omega = \big(\Omega_{l,m}\big)_{5\times 5}$ with

$\Omega_{1,1} = -2P_1 + Q + 2\,\alpha\, P_1 + P_1E_1K_1 - (G_1^TG_2 + G_2^TG_1)$, $\Omega_{1,3} = P_1 - B^TP_2$, $\Omega_{1,4} = G_1^T + G_2^T$, $\Omega_{2,2} = -e^{-2\alpha\tau_2}Q$, $\Omega_{3,3} = -2P_2A + 2\alpha\, P_2 + P_2E_3K_3$, $\Omega_{3,4} = P_2C$, $\Omega_{3,5} = P_2D$, $\Omega_{4,4} = R - 2I$, $\Omega_{5,5} = -(1-\mu)R$.

From (9), at $t = t_k$, we have

$$V(t_k,x(t_k),y(t_k)) - V(t_k^-,x(t_k^-),y(t_k^-)) = x^T(t_k)P_1x(t_k) + y^T(t_k)P_2y(t_k) - x^T(t_k^-)P_1x(t_k^-)$$
$$-y^T(t_k^-)P_2\, y(t_k^-)$$
$$= x^T(t_k^-)\{-P_1 + (K_2^TE_2^T + F_1^T)P_1(F_1 + E_2K_2)\}x(t_k^-)$$
$$+y^T(t_k^-)\{-P_2 + (K_4^TE_4^T + F_2^T)P_2(F_2 + E_4K_4)\}y(t_k^-)$$

which implies that

$$-P_1 + (K_2^TE_2^T + F_1^T)P_1(F_1 + E_2K_2) < 0 \qquad (14)$$
$$-P_2 + (K_4^TE_4^T + F_2^T)P_2(F_2 + E_4K_4) < 0. \qquad (15)$$

One can notice that inequalities (13)-(15) are not LMIs and hence cannot be solved directly by using LMI solvers. So apply Schur complement lemma to inequalities (14) and (15) gives

$$\begin{bmatrix} -P_1 & (F_1^T + K_2^TE_2^T)P_1 \\ * & -P_1 \end{bmatrix} < 0,\ \begin{bmatrix} -P_2 & (F_2^T + K_4^TE_4^T)P_2 \\ * & -P_2 \end{bmatrix} < 0. \qquad (16)$$

Now pre- and post-multiply inequalities (16) by $diag\{X,X\}$ on both sides and using the relation $X = P_1^{-1}$, we get Ω_2 and Ω_3. Inorder to get Ω_1, pre- and post-multiply both sides of (13) by $diag\{X,I,Y,I,I\}$, where $Y = P_2^{-1}$ and also use the relation $-X(G_1^TG_2)X \le XG_1^TG_1X + XG_2^TG_2X$. After some algebraic manipulations, one can get Ω_1.

From inequality (12), we can obtain $V\big(t,x(t),y(t)\big) \le e^{-2\alpha\,(t-t_{k-1})}V(t_{k-1},x(t_{k-1}),y(t_{k-1}))$ for each $t \in [t_{k-1},t_k)$ and $V\big(t,x(t),y(t)\big) \le e^{-2\alpha\,(t-t_0)}V(t_0,x(t_0),y(t_0))$.

From LKF, we get $V\big(t,x(t),y(t)\big) \ge \lambda_1 \parallel x(t) \parallel^2$ and $V\big(t_0,x(t_0),y(t_0)\big) \le \lambda_2 Sup_{-\tau_2 \le s \le 0}[\parallel \phi(s) \parallel^2 + \parallel \psi(s) \parallel^2]$, where $\lambda_1 = \lambda_{min}(P_1) + \lambda_{min}(P_2)$, $\lambda_2 = \lambda_{max}(P_1) + \lambda_{max}(P_2) + \tau_2\lambda_{max}(Q) + \tau_2\lambda_{max}(R)$ and hence $\parallel x(t) \parallel^2 + \parallel y(t) \parallel^2 \le \lambda\, e^{-2\alpha(t-t_0)}Sup_{-\tau_2 \le s \le 0}[\parallel \phi(s) \parallel^2 + \parallel \psi(s) \parallel^2]$, $\lambda = \frac{\lambda_2}{\lambda_1}$. Hence by definition of exponential stability, system (6) is exponentially stable with stabilizing gains (7). This completes the proof of the theorem.

Remark: This research work investigates the stabilization analysis of INNs with time-delays under the influence of impulsive controls. Here information of both time-varying delay and its derivative is considered. Even though the stabilization problem of INNs with both discrete and distributed delay under impulsive control is investigated in [10], information on the bound of discrete delay is not considered in [10]. This work is focused to derive stabilization results of INNs which include the information on time-delay and its derivative.

4. Numerical Examples: In this section, a numerical example is presented to demonstrate the validity of the derived results.

Example: Consider system (6) with the following parameters

$A = diag\{4, 4\}, B = diag\{12, 11\}, C = \begin{bmatrix} 0.3 & 0 \\ -0.6 & -0.4 \end{bmatrix}, D = \begin{bmatrix} 0.2 & 0.3 \\ 0.4 & 0.7 \end{bmatrix}, E_1 = 0.3I, E_2 = E_3 = 0.2I,$

$E_4 = 0.2I, F_1 = F_2 = 0.1I, G_1 = 0.2I, G_2 = 0.1I, \tau_2 = 0.6, \mu = 0.2, \alpha = 0.1,$

For the above parameters, delay-dependent stabilization condition obtained in Theorem 3.1 is solved through MATLAB LMI solvers which show that the considered system (6) is exponentially stabilizable. Feasible matrices are given below

$$X = \begin{bmatrix} 10.39 & -0.31 \\ -0.31 & 11.26 \end{bmatrix}, Y = \begin{bmatrix} 68.92 & -1.68 \\ -1.68 & 68.16 \end{bmatrix}, \ Q = \begin{bmatrix} 37.76 & 0 \\ 0 & 37.76 \end{bmatrix}, R = \begin{bmatrix} 4.61 & 2.17 \\ 2.17 & 7.97 \end{bmatrix}.$$

Stabilizing gains are given by

$$K_1 = \begin{bmatrix} -15.32 & -0.38 \\ 0.15 & -12.72 \end{bmatrix}, K_2 = K_4 = \begin{bmatrix} -0.5 & 0 \\ 0 & -0.5 \end{bmatrix}, K_3 = \begin{bmatrix} 16.65 & 0.02 \\ -0.04 & 16.12 \end{bmatrix},$$

$\lambda_1 = 0.10, \ \lambda_2 = 27.23, \lambda = 266.36$. Hence we get

$$\| x(t) \|^2 + \| y(t) \|^2 \leq 266.36 \, e^{-0.2t} Sup_{-0.6 \leq s \leq 0} [\| \phi(s) \|^2 + \| \psi(s) \|^2]$$

which shows that system (6) is exponentially stabilizable.

5. Conclusion: In this work, stabilizability problem of INNs with time-delay under impulsive control is investigated. Both time-varying and constant type of time-delay is taken into account and the corresponding results are presented. LKF involving exponential terms are utilized in the process of obtaining stabilization results. Those conditions are checked using MATLAB through LMI solvers. The problem considered in this work can be extended with the incorporation of multiple time-delays in the place of single delay and also with parameter uncertainties.

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References:

1. J.F. Ashmore, D. Attwell, Models for electrical tuning in hair cells, Proceedings of the Royal Society of London. Series B, Biological Sciences, 226 (1985) 325-344.
2. K.L. Babcock, R.M. Westervelt, Stability and dynamics of simple electronic neural networks with added inertia, Physica D, 23 (1986) 464-469.
3. K.L. Babcock, R.M. Westervelt, Dynamics of simple electronic neural networks, Physica D, 28 (1987) 305-316.
4. K. Gu, L. Kharitonov, J. Chen, Stability of Time Delay Systems, Birkhauser, Boston, 2003.
5. S. Lakshmanan, C.P. Lim, M. Prakash, S. Nahavandi, P. Balasubramaniam, Neutral-type of delayed inertial neural networks and their stability analysis using the LMI approach, Neurocomputing, 230 (2017) 243-250.
6. Q. Liu, X. Liao, Y. Liu, S. Zhou, S. Guo, Dynamics of an inertial two-neuron system with time delay, Nonlinear Dynamics, 58 (2009) 573-609.
7. Y. Liu, W. Liu, L. M.A. Obaid, I.A. Abbas, Exponential stability of Markovian jumping Cohen-Grossberg neural networks with mixed mode-dependent time-delays, Neurocomputing, 177 (2016) 409-415.

8. M. Prakash, P. Balasubramaniam, S. Lakshmanan, Synchronization of Markovian jumping inertial neural networks and its applications in image encryption, Neural Networks, 83 (2016) 86-93.
9. R. Rakkiyappan, S. Premalatha, A. Chandrasekar, J. Cao, Stability and synchronization analysis of inertial memristive neural networks with time delays, Cognitive Neurodynamics, 10 (2016) 437-451.
10. T. Yu, H. Wang, M. Su, D. Cao, Distributed-delay-dependent exponential stability of impulsive neural networks with inertial term, Neurocomputing, doi:10.1016/j.neucom.2018.06.033.
11. W. Zhang, T. Huang, C. Li, J. Yang, Robust stability of inertial BAM neural networks with time delays and uncertainties via impulsive effect, Neural Processing Letters, doi:10.1007/s11063-017-9713-4.
12. Q. Zhu, J. Cao, Stability analysis of Markovian jump stochastic BAM neural networks with impulse control and mixed time delays, IEEE transactions on neural networks and learning systems, 23 (2012) 467-479.

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