BETWEEN REGULAR OPEN SETS AND θ-OPEN SETS

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Abstract: In the year 1937 Stone introduced the concepts of regular open sets and regular closed sets to characterize Boolean Algebra. In the year 1968, Velicko studied θ-open sets for a given topology on a non-empty set X. In this paper, the notions of regular θ-open, regular θ-closed, regular θ^*- open, regular θ^*- closed, regular θ^{**}- open, regular θ^{**}- closed, θ- regular open and θ- regular closed sets are introduced and their basic properties have been discussed.

Keywords: Topology, Regular Open Sets, Θ-Open Sets.

Introduction and preliminaries: The notions of θ - open sets and regular open sets in a topological space were studied by Velicko and Stone respectively. In this paper several versions of nearly open sets have been introduced by mixing the concepts of θ- Open sets and regular open sets. Throughout this paper (X ,τ) is a topological space and A is a subset of X. The interior of A and closure of A with respect to the topology τ were denoted by int A and cl A respectively.

Definition 1.1: A is regular open [1] if A= int cl A and is regular closed if A= cl int A.

Definition 1.2: A is θ - open [2] if for all $x \in A$ there exists an open set U with $x \in U \subseteq cl\, U \subseteq A$.

Definition 1.3 : A is pre open [3] if $A \subseteq int\ cl\ A$ and is pre closed if $A \supseteq cl\ int\ A$.

Definition 1.4 : A is semi open [4] if $A \subseteq cl\ int\ A$ and is semi closed if $A \supseteq cl\ int\ A$.

θ-Regular Open Sets: In this section Θ-regular open sets and Θ-regular closed sets are introduced and studied.

Definition 2.1: A is θ-regular open if for all $x \in A$ there exists a regular open set U with $x \in U \subseteq clU \subseteq A$ and A is θ-regular closed if X– A is θ -regular open.

Proposition 2.2: Let A be a subset of a topological space (X,τ). Then
(i) A is θ -regular open⇒θ - open,
(ii) A is θ -regular closed⇒θ - closed,
(iii) the converses of (i) and (ii) are not true.

Proof: (i) and (ii) follow for Definition 1.2 and Definition 2.1. The converses are not true. For, X={a,b,c,d} τ ={ϕ, X, {a}, {a,d}, {a,b,c}}. It is easy to see that {b,c,d} is Θ- Open but not Θ - regular open.

Proposition 2.3:
(i) The intersection of two θ -regular open sets is θ -regular open.
(ii)The union of two θ -regular open sets need not be θ - regular open.
(iii) The union of two θ -regular closed sets is θ -regular closed.
(iv)The intersection of two θ -regular closed sets need not be θ - regular closed.

Proof: Let A and B be any two Θ - regular open sets in (X, τ). Let $x \in A \cup B$. Then there are regular open sets U and V with $x \in U \subseteq cl\, U \subseteq A$ and $x \in V \subseteq cl\, V \subseteq B$. That is $x \in U \cap V \subseteq cl\,(U \cap V) \subseteq Cl\, U \cap Cl\, V \subseteq A \cap B$. Since $U \cap V$ is regular open, $A \cap B$ is Θ-regular open. This proves (i). Now to prove (iii), let A and B be any two Θ- regular close sets in (X, τ). That is $X-A$ and $X-B$ are Θ-regular open in (X,τ). So, by (i), $(X-A) \cap (X-B)$ is Θ- regular open and hence $A \cup B$ is Θ-regular closed. This proves (iii). Examples can be constructed to establish (ii) and (iv).

Regular θ-Open Sets: In this section regular Θ- open sets and regular Θ- closed sets are introduced and studied.

Definition 3.1: A is regular θ-open if $A= int_\theta\, cl_\theta\, A$ and is regular θ-closed if $A= cl_\theta\, int_\theta\, A$.
The concepts of regular θ-open and θ-regular open sets are independent. Also the concepts of regular θ-closed and θ-regular closed sets are independent. The next two propositions can be proved easily and counter examples can be constructed.

Proposition 3.2:
(i) If A is θ-closed then $int_\theta\, A$ is regular θ-open.
(ii) If A and B are regular θ-open sets then $A \subseteq B \Leftrightarrow cl_\theta A \subseteq cl_\theta B$.
(iii) The intersection of two regular θ-open sets is regular θ-open .
(iv)The union of two regular θ-open sets need not regular θ-open .
(v)The complement of a regular θ-open set is regular θ-closed.

Proposition 3.3:
(i) If A is θ-open then $cl_\theta A$ is regular θ-closed.
(ii) If A and B are regular θ-closed sets then $A \subseteq B \Leftrightarrow int_\theta A \subseteq int_\theta B$.
(iii)The intersection of two regular θ-closed sets need not be regular θ-closed .
(iv)The union of two regular θ-closed sets is regular θ-closed .
(v)The complement of a regular θ-closed set is regular θ-open.

Regular θ*-open sets: In this section regular Θ^*- open sets and regular Θ^*- closed sets are introduced and studied.

Definition 4.1: A is regular θ*-open if $A= int_\theta\, cl\, A$ and is regular θ-closed if $A= cl_\theta\, int\, A$.
The concepts of regular θ-open and regular θ*- open sets are independent. Also the concepts of regular θ-closed and regular θ*- closed sets are independent. The next two propositions can be proved easily and counter examples can be constructed.

Proposition 4.2:
(i) If A is closed then $int_\theta\, A$ is regular θ*-open.
(ii) If A and B are regular θ*-open sets then $A \subseteq B \Leftrightarrow clA \subseteq clB$.
(iii)The intersection of two regular θ*-open sets need not be regular θ*-open .
(iv)The union of two regular θ*-open sets need not regular θ*-open .
(v)The complement of a regular θ*-open set is regular θ*-closed.
(vi)If A is regular θ*-open then it is pre-open.

Proposition 4.3:
(i)If A is open then $cl_\theta A$ is regular θ*-closed.
(ii) If A and B are regular θ*-closed then $A \subseteq B \Leftrightarrow intA \subseteq intB$.
(iii)The intersection of two regular θ*-closed sets need not be regular θ*-closed .
(iv)The union of two regular θ*-closed sets need not regular θ*-closed .
(v)The complement of a regular θ*-closed set is regular θ*-open.
(vi)If A is regular θ*-closed then it is pre-closed.

Regular θ-Open Sets:** In this section regular ϴ**- open sets and regular ϴ**- closed sets are introduced and studied.

Definition 5.1: A is regular θ**-open if A= int cl_θ A and is regular θ**-closed if A= cl int_θ A.
The next two propositions can be proved easily and counter examples can be constructed.

Proposition 5.2:
(i) If A is θ-closed then int A is regular θ**-open.
(ii) If A and B are regular θ**-open sets then $A \subseteq B \Leftrightarrow cl_\theta A \subseteq cl_\theta B$.
(iii)Te intersection of two regular θ**-open sets need not be regular θ**-open .
(iv)The union of two regular θ**-open sets need not regular θ**-open .
(v)The complement of a regular θ**-open set is regular θ**-closed.
(vi)If A is regular θ**-open then it is semi-closed.

Proposition 5.3:
(i) If A is θ-open then clA is regular θ**-closed.
(ii) If A and B are regular θ**-closed sets then $A \subseteq B \Leftrightarrow int_\theta A \subseteq int_\theta B$.
(iii)The intersection of two regular θ**-closed sets need not be regular θ**-closed .
(iv)The union of two regular θ**-closed sets need not regular θ**-closed.
(v)The complement of a regular θ**-closed set is regular θ**-open.
(vi)If A is regular θ**-closed then it is semi-open.

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