GLOBAL TRIPLE CONNECTED COMPLEMENTARY ACYCLIC DOMINATION

N. Saradha
Assistant Professor, SCSVMV, Enathur, Kanchipuram.
V. Swaminathan
Ramanujan Research Center in Mathematics,Saraswathi Narayanan College,Madurai, India
Arumugam
Research Scholar, SCSVMV, Enathur, Kanchipuram

Abstract. A dominating set $S \subseteq V(G)$ is said to be global Triple connected dominating set if S is a global dominating set and $\langle S \rangle$ is a triple connected. The minimum cardinality taken over all global triple connected dominating sets is called the global triple connected domination number of G and is denoted by γ_{gtc}. A dominating set $S \subseteq V(G)$ is said to be global Triple connected complementary acyclic dominating set if S is a global triple connected dominating set and $\langle V-S \rangle$ is acyclic. The minimum cardinality taken over all global triple connected complementary acyclic dominating sets is called the global triple connected complementary acyclic domination number of G and is denoted by γ_{gtc-ca}. In this paper, we introduce the concept of global triple connected complementary acyclic dominating set and we found this number for some standard graphs. Also some results on global triple connected complementary acyclic dominating sets are established.

Keywords: Triple Connected Dominating Set, Global Dominating Set, Global Triple Connected Dominating Set, Global Triple Connected Complementary Acyclic Dominating Set.

1. Introduction: A subset S of V is called a dominating set of G if every vertex in V -S is adjacent to at least one vertex in S. The domination number $\gamma(G)$ is the minimum cardinality taken over all dominating sets in G. A dominating set S of a connected graph G is said to be a connected dominating set of G if the induced sub graph $\langle S \rangle$ is connected. The minimum cardinality taken over all connected dominating sets is the connected domination number and is denoted by $\gamma_c(G)$. Many authors have introduced different types of domination parameters by imposing conditions on the dominating set [4]. Recently, the concept of triple connected graphs has been introduced by Paulraj Joseph *et.al* [3] by considering the existence of a path containing any three vertices of G. They have studied the properties of triple connected graphs and established many results on them. A graph G is said to be triple connected if any three vertices of G lie on a path in G. All paths, cycles, complete graphs and wheels are some standard examples of triple connected graphs. G. Mahadevan etal., introduced triple connected domination number of a graph. A subset S of V of a nontrivial connected graph G is said to be triple connected dominating set, if S is a dominating set and the induced sub graph $\langle S \rangle$ is triple connected. The minimum cardinality taken over all triple connected dominating sets is called the triple connected domination number of G and is denoted by $\gamma_{tc}(G)$. A dominating set $S \subseteq V(G)$ is said to be Global dominating set if for every $v \in V-S$, there exists a vertex $u \in S$ such that v is not adjacent to u. A dominating set $S \subseteq V(G)$ is said to be global triple connected dominating set if S is a global dominating set and $\langle S \rangle$ is triple connected In this paper, we introduce the concept of global triple connected complementary acyclic dominating set and we found this number for some standard graphs. Also some results on global triple connected complementary acyclic dominating sets are established.

Theorem1.1[4] A connected graph G is not triple connected if and only if there exists a H-cut with $\omega(G-H)\geq 3$, such that $|V(H)\cap N(C_i)|=1$ for at least three components C_1, C_2 and C_3 of G-H.

2. *Global Triple Connected Complementary Acyclic Domination:*

Definition 2.1: A subset S of V of a non trivial graph G is said to be a Global Triple Connected Complementary acyclic dominating set if S is a Global triple connected dominating set and the induced sub graph $\langle V-S\rangle$ is acyclic. The minimum cardinality taken over all global triple connected complementary acyclic dominating sets is called the global triple connected complementary acyclic domination number of G and is denoted by $\gamma_{gtc-ca}(G)$.

Example 2.2: For the graph K4, $S=\{v_1,v_2,v_3,v_4\}$ forms a global triple connected complementary acyclic dominating set.

Example 2.3: Global triple connected complementary acyclic dominating set does not exist for all graphs p

Observation 2.4: Every global triple connected complementary acyclic dominating set is a triple connected dominating set. But every triple connected dominating set is not a global triple connected complementary acyclic dominating set.

Observation 2.5: For any connected graph G with p vertices,
$\gamma(G)\leq\gamma_c(G)\leq\gamma_{gc}(G)\leq\gamma_{gtc}(G)\leq\gamma_{gtc-ca}(G)\leq p$ and this inequalities are strict.

Example 2.6: $\gamma(K_n)=1,\gamma_c(K_n)=2,\gamma_{gc}(K_n)=n,\gamma_{gtc}(K_n)=n,\gamma_{gtc-ca}(K_n)=n$

Theorem 2.7: If the induced sub graph of all connected dominating set of G has more than two pendent vertices then G does not contains a global triple connected complementary acyclic dominating set.

Example 2.8; Global triple connected complementary acyclic domination number for some standard graphs.

1. For any complete graph G with p vertices $\gamma_{gtc-ca}(G\circ K_1)=p$
2. For any path of order $p\geq 3$, $\gamma_{gtc-ca}(P_p)=3$ if $p<5$
   $=p-2$ if $p\geq 5$
3. For any cycle of length p for $p\geq 3$, $\gamma_{gtc-ca}(C_p)=3$ if $p<5$
   $=p-2$ if $p\geq 5$
4. For the complete bipartite graph $K_{m,n}(m,n\geq 2)$, $\gamma_{gtc-ca}(K_{m,n})=m+n-3$
5. For any star $K_{1,p-1}(p\geq 3)$, $\gamma_{gtc-ca}(K_{1,p-1})=3$
6. For any complete graph Kp $p\geq 3$, $\gamma_{gtc-ca}(K_p)=p$
7. For any wheel Wn , $\gamma_{gtc-ca}(W_p)=3$

Theorem 2.9: For any connected graph G with $p\geq 4$ we have $3\leq\gamma_{gtc-ca}\leq p$ and the bound is sharp.

References:

1. F.Harary, Graph Theory, Addison –Wessley, Reading Mass. (1969).
2. T.W. Haynes, S.T. Hedetniemi and P. J. Slater, Fundamentals of domination in graphs, Marcel Dekker Inc. New Yory(1998).
3. Paulraj Joseph J and Arumugam S, Dominatinon and connectivity in graphs, International Journal of Management and Systems, Vol. 8 No. 3, 233-236.
4. Teresa W. Haynes, Stephen T. Hedetniemi, Peter J. Slater, Fundamentals of domination in graphs, Marcel Decker, Inc., New York 1998.
5. J.A. Bondy and U.S.R. Murty, Graph Theory with Applications, The Macmillan Press Ltd, 1976.
6. G. Mahadevan, A. Selvam, J. Paulraj Joseph and T. Subramanian, Triple connected domination number of a graph, International Journal of Mathematical combinatorics, Vol.3(2012), 93-104.
7. E.Sampathkumar, Global domination number of a graph, Jour. Math. Phy. Sci., Vol. 23, Vol. 5, October 1989.

***