M/G/1 QUEUE WITH TWO STAGES IN DELAY AND RENEGING DURING DELAY PROCESS AND STANDBY SERVER DURING BERNOULLI VACATION

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Abstract: We study a M/G/1 queuing system where the arrival follows poisson process. The server provides service in a single stage. The server follows a Bernoulli schedule vacation and the system break down at random due to unavoidable circumstances. The main new assumption in this paper is the repair process is getting delayed in two stages, in which the second stage of delay is optional. Another concept of Reneging takes place during the second stage of delay process and a stand by server is provided during Bernoulli vacation. We assume that the service time, vacation time, delay time and repair time follows a general distribution. Service interruption follows exponential distribution. The steady state solution and all the other queue performance measures have been found by using supplementary variable method.

1. Queuing model: In recent years queues with server vacations have emerged as an important area of queuing theory and have been studies extensively and successfully due to their various applications in production and communication. Vacation queues have been analyzed by numerous authors . The concept of reneging, standby server and the stages in delay are the newly added concepts in this article .All the above parameter play a prominent role in real life situations where the queuing is considered to a maximum usage. Customers arrival at the system Poisson process and they are provided one by one service on a 'first come'-first served basis. $\theta_1 > 0$ is the mean arrival rate of customers. The service time follows general (arbitrary) distribution with distribution function $\bar{A}(s)$ and density function a(s). Let $\theta_2(x)\,dx$ be the conditional probability density of service completion during the interval $(x, x+dx)$,given that the elapsed time is x $so, \theta_2(x) = \frac{a(x)}{1-\bar{A}(x)}$,

$a(s) = \theta_2(s)\,e^{-\int_0^s \theta_2(x)\,dx}$.As soon as a service is completed, the server may go for a vacation with probability $m(0 \le m \le 1)$ or it may continue to serve the next customer $(1-m)$.d)The server's vacation time follows a general(arbitrary) distribution with distribution function $\bar{B}(s)$ and density function $b(s)$. Let $\theta_3(x)\,dx$ be the conditional probability of a completion of a vacation during theinterval$(x, x+dx)$,sothat$\theta_2(x) = \frac{b(x)}{1-\bar{B}(x)}$ and $b(s) = \theta_2(s)\,e^{-\int_0^s \theta_2(x)\,dx}$

A stand by server is introduced during the Bernoulli schedule server vacation. This additional concept follows exponential distribution with parameter τ >0. On returning from vacation the server instantly starts serving the customer at the head of the queue, if any. The system may breakdown at random and breakdowns are assumed to occur according to a Poisson stream with mean breakdown rate $\psi > 0$.The server's first stage of delay time follows a general(arbitrary) distribution with distribution function $C(s)$ and density function $c(s)$. Let $\gamma_1(x)\,dx$ be the conditional probability of a completion of a delay time during the interval $(x, x+dx)$, so $\gamma_1(x) = \frac{c(x)}{1-\bar{C}(x)}$ and $c(s) = \gamma_1(s)e^{-\int_0^s \gamma_1(x)dx}$.The server's second stage of delay time follows a general(arbitrary) distribution with distribution function $D(s)$ and density function $d(s)$. Let $\gamma_2(x)dx$ be the conditional probability of a completion of a delay time during the interval $(x, x+dx)$, so $\gamma_2(x) = \frac{d(x)}{1-D(x)}$ and $d(s) = \gamma_2(s)e^{-\int_0^s \gamma_2(x)dx}$.The concept of reneging takes place during the second stage of delay. Customers arriving for service may become impatient and renege after joining during second stage of delay. Reneging is assumed to follow exponential distribution with parameter ω. Hence$f(t) = \omega e^{-\omega t}dt, \omega > 0$. Thus ωdt is the probability that a customer can renege during a short interval of time $(t, t+dt)$.The server's repair time follows a general (arbitrary) distribution with distribution function $W(x)$ and density function $w(x)$. Let $\beta(x)dx$ be the conditional probability of a completion of a delay time during the interval $(x, x+dx)$, soRepair time $\beta(x) = \frac{w(x)}{1-W(x)}$ and $w(s) = \beta(s)e^{-\int_0^s \beta(x)dx}$

2. Steady State Equations Governing The System:

The system has the following steady state differential equations:

For service,$\frac{\partial}{\partial x}R_n(x) + (\theta_1 + \theta_2(x) + \psi)R_n(x) = \theta_1 R_{n-1}(x)$ (1)

$\frac{\partial}{\partial x}R_0(x) + (\theta_1 + \theta_2(x) + \psi)R_0(x) = 0$ (2)

For vacation ,$\frac{\partial}{\partial x}Q_n(x) + (\theta_1 + \theta_3(x) + \tau)Q_n(x) = \theta_1 Q_{n-1}(x) + \tau Q_{n+1}(x)$ (3)

$\frac{\partial}{\partial x}Q_0(x) + (\theta_1 + \theta_3(x) + \tau)Q_0(x) = \tau Q_1(x)$ (4)

For first stage of delay,$\frac{\partial}{\partial x}D_n^{(1)}(x) + (\theta_1 + \gamma_1(x))D_n^{(1)}(x) = \theta_1 D_{n-1}^{(1)}(x)$ (5)

$\frac{\partial}{\partial x}D_n^{(2)}(x) + (\theta_1 + \gamma_2(x) + \omega)D_n^{(2)}(x) = \theta_1 D_{n-1}^{(2)}(x) + \omega D_{n+1}(x)$ (6)

For second stage of delay,$\frac{\partial}{\partial x}D_0^{(2)}(x) + (\theta_1 + \gamma_2(x) + \omega)D_0^{(2)}(x) = \omega D_1(x)$ (7)

For repair process,$\frac{\partial}{\partial x}K_n(x) + (\theta_1 + \beta(x) + K_n(x)) = \theta_1 K_{n-1}(x)$ (8)

$\frac{\partial}{\partial x}K_0(x) + (\theta_1 + \beta(x) + K_0(x)) = 0$ (9)

$\theta_1 M = (1-p)\int_0^\infty R_0(x)\theta_2(x)dx + \int_0^\infty Q_0(x)\theta_3(x)dx + \int_0^\infty K_0(x)\beta(x)dx$ (10)

The following boundary conditions are used to solve the above differential equations.For $n \geq 0$

$R_n(0) = (1-p)\int_0^\infty R_{n+1}(x)\theta_2(x)dx + \int_0^\infty Q_{n+1}(x)\theta_3(x)dx + \int_0^\infty K_{n+1}(x)\beta(x)dx,$ (11)

$Q_n(0) = m\int_0^\infty R_n(x)\theta_1(x)dx, \quad n \geq 0$ (12)

$D_n^{(1)}(0) = \psi\int_0^\infty R_{n-1}(x)dx$ (13)

$D_n^{(2)}(0) = r\int_0^\infty D_n^{(1)}(x)\gamma_1(x)dx$ (14)

$K_n(0) = \int_0^\infty D_n^{(2)}(x)\gamma_2(x)dx$ (15)

$D_0^{(1)}(0) = D_0^{(2)}(0) = K_0(0) = 0$ (16)

We multiply equation (1) by z^n and sum over$n$ *from* 1 *to* ∞, add this equation to (2) we get,

$\frac{\partial}{\partial x}R_q(x,z) + (\theta_1 - \theta_1 z + \theta_2(x) + \psi)R_q(x,z) = 0$ (17)

Similarly,$\frac{\partial}{\partial x}Q_q(x,z) + \left(\theta_1 - \theta_1 z + \theta_3(x) + \tau - \frac{\tau}{z}\right)Q_q(x,z) = 0$ (18)

$\frac{\partial}{\partial x}D_q^{(1)}(x,z) + (\theta_1 - \theta_1 z + \gamma_1(x))D_q^{(1)}(x,z) = 0$ (19)

$\frac{\partial}{\partial x}D_q^{(2)}(x,z) + \left(\theta_1 - \theta_1 z + \gamma_2(x) + \omega - \frac{\omega}{z}\right)D_q^{(2)}(x,z) = 0$ (20)

$\frac{\partial}{\partial x}K_q(x,z) + (\theta_1 - \theta_1 z + \beta(x))K_q(x,z) = 0$ (21)

Multiply equation (11) by z^{n+1} and summing over$n$ *from* 1 *to* ∞, and using eq 10,wegetz$R_q(0,z) = \left[(1-x)\int_0^\infty R_q(x,z)\theta_2(x)dx + \int_0^\infty Q_q(x,z)\theta_3(x)dx + \int_0^\infty K_q(x,z)\beta(x)dx\right] - \theta_1 M$ (23)

Following the same procedure for equation (12) – (15),we get

$$Q_q(0,z) = m\int_0^\infty R_q(x,z)\,\theta_2(x)\,dx \quad (24)$$

$$D_n^{(1)}(0,z) = \psi z\int_0^\infty R_q(x,z)\,dx = \psi z R_q(z) \quad (25)$$

$$D_q^{(2)}(0,z) = r\int_0^\infty D_q^{(1)}(x,z)\gamma_1(x)\,dx \quad (26)$$

$$K_q(0,z) = \int_0^\infty D_q^{(2)}(x,z)\gamma_2(x)\,dx \quad (27)$$

Consider equation (17) and on integration,

$$R_q(x,z) = R_q(0,z)e^{-(\theta_1-\theta_1 z+\psi)x-\int_0^x \theta_2(x)\,dx} \quad (28)$$

Again integrating by parts, we get $R_q(z) = R_q(0,z)\left(\frac{1-\bar{A}(\theta_1-\theta_1 z+\psi)}{\theta_1-\theta_1 z+\psi}\right)$ (29)

Where $\bar{A}(\theta_1-\theta_1 z+\psi) = \int_0^\infty e^{-(\theta_1-\theta_1 z+\psi)x}\,dA(x)$ is the Laplace Stieltje's transform of the service time $A(x)$.

Multiply both sides of equation (28) by $\theta_2(x)$ and integrating over x, we get $\int_0^\infty R_q(x,z)\,\theta_2(x)\,dx = R_q(0,z)\,\bar{A}(\theta_1-\theta_1 z+\psi)$ (30)

Applying the same procedure for equation (18),(19),(20)and (21),we get

$$Q_q(z) = mR_q(0,z)\,\bar{A}(\theta_1-\theta_1 z+\psi)\left(\frac{1-\bar{B}\left(\theta_1-\theta_1 z+\tau-\frac{\tau}{z}\right)}{\theta_1-\theta_1 z+\tau-\frac{\tau}{z}}\right) \quad (31)$$

$$\int_0^\infty Q_q(x,z)\theta_3(x)\,dx = mR_q(0,z)\,\bar{B}\left(\theta_1-\theta_1 z+\tau-\frac{\tau}{z}\right) \quad (32)$$

$$D_q^{(1)}(z) = \psi z R_q(0,z)\left(\frac{1-\bar{A}(\theta_1-\theta_1 z+\psi)}{\theta_1-\theta_1 z+\psi}\right)\left(\frac{1-\bar{C}(\theta_1-\theta_1 z)}{(\theta_1-\theta_1 z)}\right) \quad (33)$$

$$\int_0^\infty D_q^{(1)}(x,z)\gamma_1(x)\,dx = \psi z R_q(0,z)\left(\frac{1-\bar{A}(\theta_1-\theta_1 z+\psi)}{\theta_1-\theta_1 z+\psi}\right)\bar{C}(\theta_1-\theta_1 z) \quad (34)$$

$$D_q^{(2)}(z) = r\psi z R_q(0,z)\left(\frac{1-\bar{A}(\theta_1-\theta_1 z+\psi)}{\theta_1-\theta_1 z+\psi}\right)\bar{C}(\theta_1-\theta_1 z)\left(\frac{1-\bar{D}\left(\theta_1-\theta_1 z+\omega-\frac{\omega}{z}\right)}{\theta_1-\theta_1 z+\omega-\frac{\omega}{z}}\right) \quad (35)$$

$$\int_0^\infty D_q^{(2)}(x,z)\gamma_2(x)\,dx = \frac{r\psi z R_q(0,z)\left(1-\bar{A}(\theta_1-\theta_1 z+\psi)\right)\bar{C}(\theta_1-\theta_1 z)\bar{D}\left(\theta_1-\theta_1 z+\omega-\frac{\omega}{z}\right)}{(\theta_1-\theta_1 z+\psi)} \quad (36)$$

$$K_q(z) = \frac{r\psi z R_q(0,z)\left(1-\bar{A}(\theta_1-\theta_1 z+\psi)\right)\bar{C}(\theta_1-\theta_1 z)\bar{D}\left(\theta_1-\theta_1 z+\omega-\frac{\omega}{z}\right)\left(\theta_1-\theta_1 z+\omega-\frac{\omega}{z}\right)\left(1-\bar{W}(\theta_1-\theta_1 z)\right)}{(\theta_1-\theta_1 z+\psi)(\theta_1-\theta_1 z)} \quad (37)$$

$$\int_0^\infty K_q(x,z)\,\beta(x)\,dx = \frac{r\psi z R_q(0,z)\left(1-\bar{A}(\theta_1-\theta_1 z+\psi)\right)\bar{C}(\theta_1-\theta_1 z)\,\bar{D}\left(\theta_1-\theta_1 z+\omega-\frac{\omega}{z}\right)\bar{W}(\theta_1-\theta_1 z)}{(\theta_1-\theta_1 z+\psi)} \quad (38)$$

Substituting(30),(32)and(38)in(23),weget$\boldsymbol{R_q(0,z)} = \frac{-(\theta_1 M)(\theta_1-\theta_1 z+\psi)}{\mathrm{d}(z)}$ (39)

Substituting (39) in 29, 31, 33, 35, 37, we get$\boldsymbol{R_q(z)} = \frac{-\theta_1 M\left(1-\bar{A}(\theta_1-\theta_1 z+\psi)\right)}{\mathrm{d}(z)}$

$$\boldsymbol{Q_q(z)} = \frac{-\theta_1 Mm\bar{A}(\theta_1-\theta_1 z+\psi)\left(1-\bar{B}\left(\theta_1-\theta_1 z+\tau-\frac{\tau}{z}\right)\right)}{\mathrm{d}(z)}$$

$$\boldsymbol{D_q^{(1)}(z)} = \frac{-\theta_1 M\psi z\left(1-\bar{A}(\theta_1-\theta_1 z+\psi)\right)\left(1-\bar{C}(\theta_1-\theta_1 z)\right)}{\mathrm{d}(z)}$$

$$\boldsymbol{D_q^{(2)}(z)} = \frac{-\theta_1 Mr\psi z\left(1-\bar{A}(\theta_1-\theta_1 z+\psi)\right)\bar{C}(\theta_1-\theta_1 z)\{1-\bar{D}\left(\theta_1-\theta_1 z+\omega-\frac{\omega}{z}\right)\}}{\mathrm{d}(z)}$$

$$K_q(z) = \frac{-\theta_1 Mr\psi\left(1-\bar{A}(\theta_1-\theta_1 z+\psi)\right)\bar{C}(\theta_1-\theta_1 z)\bar{D}\left(\theta_1-\theta_1 z+\omega-\frac{\omega}{z}\right)\left(1-\bar{W}(\theta_1-\theta_1 z)\right)}{\mathrm{d}(z)}$$

Where

$$d(z) = (\theta_1-\theta_1 z+\psi)\left(z-\bar{A}(\theta_1-\theta_1 z+\psi)-m\bar{A}(\theta_1-\theta_1 z+\psi)\bar{B}\left(\theta_1-\theta_1 z+\tau-\frac{\tau}{z}\right)\right)-r\psi z\left(1-\bar{A}(\theta_1-\theta_1 z+\psi)\right)\bar{C}(\theta_1-\theta_1 z)\bar{D}\left(\theta_1-\theta_1 z+\omega-\frac{\omega}{z}\right)\bar{W}(\theta_1-\theta_1 z)$$

3. The Distribution of Queue Length at Any Point of Time: The Probability generating function of the queue size is

$L_q(z) = R_q(z) + Q_q(z) + D_q^{(1)}(z) + D_q^{(2)}(z) + K_q(z)$

That is adding all the above we get the probability generating function of the queue size as

$$M_q(Z) = \frac{N(Z)}{D(Z)} \quad (40)$$

$\boldsymbol{D(z)} = \{(\theta_1 - \theta_1 z + \psi)\left(z - \bar{A}(\theta_1 - \theta_1 z + \psi) - m\bar{A}(\theta_1 - \theta_1 z + \psi)\bar{B}\left(\theta_1 - \theta_1 z + \tau - \frac{\tau}{z}\right)\right) - r\psi z(1 - \bar{A}(\theta_1 - \theta_1 z + \psi))\bar{C}(\theta_1 - \theta_1 z)\bar{D}\left(\theta_1 - \theta_1 z + \omega - \frac{\omega}{z}\right)\bar{W}(\theta_1 - \theta_1 z)\}(\theta_1 - \theta_1 z)\left(\theta_1 - \theta_1 z + \omega - \frac{\omega}{z}\right)\left(\theta_1 - \theta_1 z + \tau - \frac{\tau}{z}\right)$

$\boldsymbol{N(z)} = (-\theta_1 M)\left\{(1 - \bar{A}(\theta_1 - \theta_1 z + \psi))\left(\theta_1 - \theta_1 z + \tau - \frac{\tau}{z}\right)(\theta_1 - \theta_1 z)\left(\theta_1 - \theta_1 z + \omega - \frac{\omega}{z}\right)\right\} + (-\theta_1 Mm)\left\{\bar{A}(\theta_1 - \theta_1 z + \psi)\left(1 - \bar{B}\left(\theta_1 - \theta_1 z + \tau - \frac{\tau}{z}\right)\right)(\theta_1 - \theta_1 z + \psi)(\theta_1 - \theta_1 z)\left(\theta_1 - \theta_1 z + \omega - \frac{\omega}{z}\right)\right\} + (-\theta_1 m\psi z)\left\{(1 - \bar{A}(\theta_1 - \theta_1 z + \psi))(1 - \bar{C}(\theta_1 - \theta_1 z))\left(\theta_1 - \theta_1 z + \tau - \frac{\tau}{z}\right)\left(\theta_1 - \theta_1 z + \omega - \frac{\omega}{z}\right)\right\} + (-\theta_1 mr\psi z)\{(1 - \bar{A}(\theta_1 - \theta_1 z + \psi))\bar{C}(\theta_1 - \theta_1 z)\left(1 - \bar{D}\left(\theta_1 - \theta_1 z + \omega - \frac{\omega}{z}\right)\right)(\theta_1 - \theta_1 z)\left(\theta_1 - \theta_1 z + \tau - \frac{\tau}{z}\right)\} + (-\theta_1 Mr\psi)\{(1 - \bar{A}(\theta_1 - \theta_1 z + \psi))\bar{C}(\theta_1 - \theta_1 z)\bar{D}\left(\theta_1 - \theta_1 z + \omega - \frac{\omega}{z}\right)(1 - \bar{W}(\theta_1 - \theta_1 z))\left(\theta_1 - \theta_1 z + \tau - \frac{\tau}{z}\right)\left(\theta_1 - \theta_1 z + \omega - \frac{\omega}{z}\right)\}$

Since Equation (39) is indeterminate of the form $\frac{0}{0}$ at $z = 1$, we use L'Hopital's Rule on Equation (39) to obtain

$\mathrm{Mq}(1) = 6\{(\tau - \theta_1)\theta_1(\omega - \theta_1)(1 - \bar{A}(\psi))(E(C) + E(D) + E(W) - 1) + E(B)(\tau - \theta_1)\psi\theta_1(\omega - \theta_1)\bar{A}(\psi)\}\{6(-\theta_1)(\omega - \theta_1)(\tau - \theta_1)\}[[8((-\theta_1)(1 - E(A)(-\theta_1) - mE(A)(-\theta_1)\bar{B} + \bar{A}(\psi)E(B)(-\theta_1 + \tau)) + 3((\psi)(-E(A^2)\theta_1^2 - m[E(A^2)\theta_1^2 + \theta_1 E(A)(\tau - \theta_1)E(B) + E(A)\theta_1 E(B)(\tau - \theta_1) + E(B^2)(\tau - \theta_1)^2) - 6((-\psi)r[2(1 - \bar{A}(\psi))(-\theta_1)E(C) + 2(1 - \bar{A}(\psi))(\omega - \theta_1)E(D) + (1 - \bar{A}(\psi))(-\theta_1)E(W) + (-\theta_1)^2(-E(A^2) + 2\theta_1 E(A)(\omega - \theta_1)E(D) - \theta_1^2 E(A)E(W) + (1 - \bar{A}(\psi))\theta_1^2 E(C^2) + (1 - \bar{A}(\psi))\theta_1^2 E(C)E(W) + 2(1 - \bar{A}(\psi))(-\theta_1)E(C)E(D)(\omega - \theta_1) + (1 - \bar{A}(\psi))E(D^2)(\omega - \theta_1)^2 + (1 - \bar{A}(\psi))E(D)(-\theta_1)(\omega - \theta_1) - 2\theta_1 E(A)]) + \psi(1 - E(A)(-\theta_1) - mE(A)(-\theta_1)\bar{B} + \bar{A}(\psi)E(A)(-\theta_1 + \tau)) + (1 - \bar{A}(\psi) - m[E(B)(\omega - \theta_1)\bar{A}(\psi)](-\theta_1)]^{-1}$

In order to determine the probability of idle time Q, we use the normalizing condition $M_q(1) + Q = 1$ and Utilization factor ρ can be found

4. Performance Measures of The Model Defined: Let L_q denote the mean queue size under the steady state

Then

$$L_q = \frac{d}{dz}M_q(z)_{z=1} = \frac{D^{iv}(1)N^{v}(1) - D^{v}(1)N^{iv}(1)}{2\left(D^{iv}(1)\right)^2} \quad (41)$$

Where primes in the above condition indicate fourth and fifth primes at z = 1 individually. Completing the subordina at z =1, we have

$\boldsymbol{N^{iv}(1)} = 6\{(\tau - \theta_1)\theta_1(\omega - \theta_1)(1 - \bar{A}(\psi))(E(C) + E(D) + E(W) - 1) + E(B)(\tau - \theta_1)\psi\theta_1(\omega - \theta_1)\bar{A}(\psi)\}$

$$N^{v}(1) = 6\bar{A}'(\psi)\theta_1^2(\omega-\theta_1)(\tau-\theta_1)[3\psi+3E(C)+3E(D)+3E(W)-4]+6(1-\bar{A}(\psi))\theta_1[4\tau(\omega-\theta_1)-3E(B)(\omega-\theta_1)(\tau-\theta_1)\theta_1-2E(C^2)(\omega-\theta_1)(\tau-\theta_1)\theta_1-4E(C)\omega(\tau-\theta_1)-4E(C)\tau(\omega-\theta_1)+3E(C)(\omega-\theta_1)(\tau-\theta_1)+2E(D^2)(\tau-\theta_1)(\omega-\theta_1)^2-4E(D)\omega(\tau-\theta_1)-4E(D)\tau(\omega-\theta_1)+3E(D)(\omega-\theta_1)(\tau-\theta_1)+3E(C)E(D)\theta_1(\omega-\theta_1)(\tau-\theta_1)+2E(W^2)\theta_1(\omega-\theta_1)(\tau-\theta_1)-4E(W)\tau(\omega-\theta_1)-4E(W)\omega(\tau-\theta_1)-3E(D)E(W)(\tau-\theta_1)(\omega-\theta_1)^2+3E(C)E(W)(\omega-\theta_1)(\tau-\theta_1)+2[E(B^2)(\omega-\theta_1)(\tau-\theta_1)\psi-4E(B)\psi\tau(\omega-\theta_1)+2E(B)\psi(\omega-\theta_1)(\tau-\theta_1)]-12\psi\theta_1(\omega-\theta_1)[E(B^2)(\tau-\theta_1)^2-2E(B)\tau+E(B)(\tau-\theta_1)]$$

$$D^{(iv)}(1) = \{6(-\theta_1)(\omega-\theta_1)(\tau-\theta_1)\}[[8((-\theta_1)(1-E(A)(-\theta_1)-mE(A)(-\theta_1)\bar{B}+\bar{A}(\psi)E(B)(-\theta_1+\tau))+3((\psi)(-E(A^2)\theta_1^2-m[E(A^2)\theta_1^2+\theta_1E(A)(\tau-\theta_1)E(B)+E(A)\theta_1E(B)(\tau-\theta_1)+E(B^2)(\tau-\theta_1)^2)-6((-\psi)r[2(1-\bar{A}(\psi))(-\theta_1)E(C)+2(1-\bar{A}(\psi))(\omega-\theta_1)E(D)+(1-\bar{A}(\psi))(-\theta_1)E(W)+(-\theta_1)^2(-E(A^2)+2\theta_1E(A)(\omega-\theta_1)E(D)-\theta_1^2E(A)E(W)+(1-\bar{A}(\psi))\theta_1^2E(C^2)+(1-\bar{A}(\psi))\theta_1^2E(C)E(W)+2(1-\bar{A}(\psi))(-\theta_1)E(C)E(D)(\omega-\theta_1)+(1-\bar{A}(\psi))E(D^2)(\omega-\theta_1)^2+(1-\bar{A}(\psi))E(D)(-\theta_1)(\omega-\theta_1)-2\theta_1E(A)])+\psi(1-E(A)(-\theta_1)-mE(A)(-\theta_1)\bar{B}+\bar{A}(\psi)E(A)(-\theta_1+\tau))+(1-\bar{A}(\psi)-m[E(B)(\omega-\theta_1)\bar{A}(\psi)](-\theta_1)]$$

$$D^{v}(1) = \{6(-\theta_1)(\omega-\theta_1)(\tau-\theta_1)[10(-\theta_1)(1-E(A)(-\theta_1)-m\bar{A}'(-\theta_1)\bar{B}+\bar{A}(\psi)E(B)(-\theta_1+\tau))+2((\psi)(-E(A^2)\theta_1^2-m[E(A^2)\theta_1^2+\theta_1E(A)(\tau-\theta_1)E(B)+E(A)\theta_1E(B)(\tau-\theta_1)+E(B^2)(\tau-\theta_1)^2)-4((-\psi)r[2(1-\bar{A}(\psi))(-\theta_1)E(C)+2(1-\bar{A}(\psi))(\omega-\theta_1)E(D)+(1-\bar{A}(\psi))(-\theta_1)E(W)+(-\theta_1)^2(-E(A^2)+2\theta_1E(A)(\omega-\theta_1)E(D)-\theta_1^2E(A)E(W)+(1-\bar{A}(\psi))\theta_1^2E(C^2)+(1-\bar{A}(\psi))\theta_1^2E(C)E(W)+2(1-\bar{A}(\psi))(-\theta_1)E(C)E(D)(\omega-\theta_1)+(1-\bar{A}(\psi))E(D^2)(\omega-\theta_1)^2+(1-\bar{A}(\psi))E(D)(-\theta_1)(\omega-\theta_1)-2\theta_1E(A)]+(-\theta_1)\{(-E(A^2)\theta_1^2-m[E(A^2)\theta_1^2+\theta_1E(A)(\tau-\theta_1)E(B)+E(A)\theta_1E(B)(\tau-\theta_1)+E(B^2)(\tau-\theta_1)^2\}\}+\{11[(-\theta_1)(-2\omega)(\omega-\theta_1)]+12[(-\theta_1)(\omega-\theta_1)(-2\tau)]\}[\psi(1-\bar{A}'(-\theta_1)-m\bar{A}'(-\theta_1)\bar{B}+\bar{A}\,\mathrm{E(B)}(-\theta_1+\tau))+\psi(1-E(A)(-\theta_1)-m\bar{A}'(-\theta_1)+\bar{A}(\psi)E(B)(-\theta_1+\tau))-r\psi[1-\bar{A}(\psi)+E(A)(-\theta_1)+(1-\bar{A}(\psi))\theta_1E(C)+(1-\bar{A}(\psi))\theta_1E(W)]$$

Other Performance measures L, W, W_q can be derived using Little's formula L = Lq + ρ, W q = Lq/λ , W = L/λ

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