EFFECTS OF THERMAL RADIATION OF AN UNSTEADY CONVECTIVE $AL_2\ O_3$ NANOFLUID FLOW PAST ON VERTICAL PLATE IN A POROUS MEDIUM

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Abstract: In this paper, we have considered the thermal radiation effects of a transient free convective aluminium oxide nanofluid flow past on a vertical plate with porous medium. Here three different nanofluids Cu, $Al_2\ O_3$ and TiO_2 are considered for study, to enhance the thermal conductivity of a fluid. Here the dimensional governing equations like momentum equations, energy equations are converted into the non-dimensional form by using the non-dimensional quantities. The solutions for velocity and temperature of a plate are derived by using analytical technique. The graphs are also plotted for different values of thermal grashof number, time, solid volume fraction, Prandtl number and different nanofluid and the discussed in a detailed manner.

Keywords: Thermal Radiation, Laplace Transform, Porous, Vertical Plate.

1. Introduction: Transport processes through porous media play important roles in diverse applications, such as in geothermal operations, petroleum industries, thermal insulation, design of solid-matrix heat exchangers, chemical catalytic reactors, and many others. The study of convective heat transfer and fluid flow in porous media has received great attention in recent years. The effect of radiation on Darcy's buoyancy induced flow of an optically dense viscous incompressible fluid along a heated inclined flat surface maintained at uniform temperature placed in a saturated porous medium with Rosseland diffusion approximation employing the implicit finite difference method together with Keller box elimination technique by Hossain and Pop[1]. Hsu et al. [2] have studied the natural convection flow over an inclined surface in porous medium, in which the normal component of the buoyancy force had been neglected in the momentum equations and obtained the similarity solutions for the flow , results of which are not valid for the small angles of inclination from the vertical that are not small. Hossain and Takhar [3] have investigated the radiation effect on mixed convection boundary layer flow of an optically dense viscous incompressible fluid along a vertical plate with uniform surface temperature.

A nonsimilar boundary layer analysis has been analysed for the free convection along a vertical plate embedded in a fluid-saturated porous medium in the presence of surface mass transfer and internal heat generation by Bakier et al [4]. Gorla and co-workers [5, 6] investigated the corresponding nonsimilar free convection boundary layer problem due to variations in surface temperature or heat flux. The effects of thermal dispersion and thermal radiation on the non-Darcy natural convection over a vertical flat plate in a fluid saturated porous medium are studied by Mohammadien and El-Amin [7].

Effects of thermal dispersion and lateral mass flux on non-Darcy natural convection over a vertical flat plate in a fluid saturated porous medium was studied by Murthy and Singh [8].

At high temperatures thermal radiation can significantly affect the heat transfer and the temperature distribution in the boundary layer flow of participating fluid. Gorla [9], and Gorla and Pop [10] has investigated the effects of radiation on mixed convection flow over vertical cylinders. Forced convection–radiation interaction heat transfer in boundary-layer over a flat plate submersed in a porous medium was analyzed by Mansour [11].

It is observed that no author has studied on a viscous flow of a natural convective flow of an Al_2O_3 nanofluid over an infinite isothermal vertical plate through a porous medium with radiation effect. Here the coupled nonlinear differential equations are converted into a radiation effect. Here the coupled non linear differential equations are converted into a non dimensional form and it is solved by Laplace transform. Three types of water based nanofluids containing nano particles of copper(cu), aluminum oxide (Al_2O_3) and titanium dioxide (TiO_2)have been considered in the present work. Effects of various parameters like thermal grashof number, time, radiation parameter and Prandtl number on the velocity of the plate and temperature of the fluid are discussed and the graphs are drawn and explained in a detailed manner.

2. Mathematical Analysis: The transient free convective viscous flow of an Al_2O_3 nanofluid through a porous infinite vertical plate with a radiation effect has-been considered. Further it is an unsteady flow of a viscous incompressible fluid which is initially at rest and surrounds an infinite vertical plate maintaining temperature T_∞. The x-axis is taken along the plate in the vertically upward direction and the y-axis is taken perpendicular to the plate. At time $t \le 0$, the plate and fluid are at the same temperature T_∞. At time $t > 0$, the plate is accelerated with a velocity $u = \frac{u_0^3}{\nu_f} t'$ its own plane against gravitational field and temperature from the plate is raised to T_w. It is also assumed that a radiative heat flux q_r is applied which is normal to the plate .The fluid is a water based nanofluid containing three types nanoparticles Cu, Al_2O_3 and TiO_2. It is further assumed that the base fluid and the suspended nanoparticles are in themal equilibrium. The thermo physical properties of the nanofluids are given in table 1.

Table 1: Thermal Physical Properties of Water and Nano Particles

Physical properties	Water/base fluid	Cu(Copper)	Al_2O_3(Alumina)	TiO_2
$\rho(kg/m^3)$	997.1	8933	3970	4250
c_ρ (J/kgk)	4179	385	765	686.2
K(W/mK)	0.613	401	40	8.9538
$\beta \times 10^5 (k^{-1})$	21	1.67	0.85	0.90
0	0.0	0.05	0.15	0.2
σ(S/m)	5.5×10^{-6}	59.6×10^6	35×10^6	2.6×10^6

Then under usual Boussinessq's approximation, the unsteady viscous flow of a nano fluid is governed by the following equations:

$$\rho_{nf} \frac{\partial u}{\partial t'} = g(\rho\beta)_{nf}(T - T_\infty) + \mu_{nf} \frac{\partial^2 u}{\partial y^2} - \frac{\upsilon}{k'} u \quad (1)$$

$$(\rho c_p)_{nf} \frac{\partial T}{\partial t'} = k_{nf} \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \quad (2)$$

Where u is the velocity components along the x-direction ,T the temperature of the nanofluid ,μ_{nf} the dynamic viscosity of the nanofluid, β_{nf} the thermal expansion coefficient of the nanofluid , ρ_{nf} the density of the nano fluid , K_{nf} the thermal conductivity of the nanofluid , g the acceleration due to gravity, q_r the radiative heat flux and $(\rho c_p)_{nf}$ the heat capacitance of the nanofluid which are given by

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, \rho_{nf} = (1-\phi)\rho_f + \phi\rho_s$$
$$(\rho c_p)_{nf} = (1-\phi)(\rho c_p)_f + \phi(\rho c_p)_s$$
$$(\rho\beta)_{nf} = (1-\phi)(\rho\beta)_f + \phi(\rho\beta)_s \tag{3}$$

Where Φ is the solid volume fraction of the nano particle, ρ the density of the base fluid ,ρ_s the density of the nanoparticle, μ_f the viscosity of the base fluid, $(\rho c_p)_f$ the heat capacitance of the base fluid and $(\rho c_p)_s$ the heat capacitance of the nanoparticle. It is worth mentioning that the expressions (1) are restricted to spherical nanoparticles, where it does not account for other shapes of nano particles. The effective thermal conductivity of the nanofluid given by Hamilton and crosser model followed by kakae and Pramuanjaroenkij [12], and oztop and Abu-nada [13] is given by

$$k_{nf} = k_f \left[\frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)} \right] \tag{4}$$

Where k_f is the thermal conductivity of the base fluid and k_s the thermal conductivity of the nanoparticle. In Eqs.(1)-(4), the subscripts *nf* , *f* and *s* denote the thermophysical properties of the nano fluid ,basefluid and nano particles, respectively.
The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y} = -4a^* \sigma\left(T_\infty^4 - T^4\right) \tag{5}$$

It is assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature .This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3\, T - 3T_\infty^4 \tag{6}$$

By using equations (6) and (7) ,equation (2) reduces to

$$(\rho\, c_p)_{nf} \frac{\partial T}{\partial t'} = k_{nf} \frac{\partial^2 T}{\partial y^2} + 16a^* \sigma T_\infty^3\, (T_\infty - T) \tag{7}$$

The initial and boundary conditions of the proposed problem are given by:

$$\begin{array}{llll} u = 0, & T = T_\infty & \textit{for all} & y, t' \le 0 \\ t' > 0: \quad u = \frac{u_0^{\,3}}{\nu_f} t', & T = T_\omega & at & y = 0 \\ u \to 0 & T \to T_\infty & as & y \to \infty \end{array} \tag{8}$$

On introducing the following non dimensional quantities are:

$$U = \frac{u}{u_0}\ ,\ t = \frac{t' u_0^{\,2}}{\nu_f},\ Y = \frac{y u_0}{\nu_f}, \tag{9}$$

$$\theta=\frac{T-T_\infty}{T_w-T_\infty},\ Gr=\frac{g\,v_f\,\beta(T_w-T_\infty)}{u_0^{\,3}},\ R=\frac{16\,a^*\,\sigma T_\infty^{\,3}}{k_f}\left(\frac{v_f^{\,2}}{u_0^{\,2}}\right),\Pr=\frac{\mu c_p}{k_f}$$

By using equations (3), (4) and (9),equations (1) and (7) leads to,

$$L_1\frac{\partial U}{\partial t}=L_3\frac{\partial^2 U}{\partial Y^2}+L_2 Gr\theta$$

$$L_5\frac{\partial\theta}{\partial t}=L_6\frac{1}{\Pr}\frac{\partial^2\theta}{\partial Y^2}-\frac{R}{\Pr}\theta \tag{10}$$

Where $L_1=(1-\phi)+\phi\left(\frac{\rho_s}{\rho_f}\right),L_2=(1-\phi)+\phi\left(\frac{(\rho\beta)_s}{(\rho\beta)_f}\right),L_3=\frac{1}{(1-\phi)^{2.5}}$

$$L_5=(1-\phi)+\phi\left(\frac{(\rho c_p)_s}{(\rho c_p)_f}\right),L_6=\left[\frac{k_s+2k_f-2\phi(k_f-k_s)}{k_s+2k_f+\phi(k_f-k_s)}\right] \tag{11}$$

Where R is the radiation parameter, pr is the Prandtl number, Gr is the thermal grashof number, Gr approximates the ratio of the buoyancy force to the viscous force acting, Large R signifies a large radiation effect while R→o corresponds to zero radiation effect.

The corresponding initial and boundary conditions are,

$$\begin{array}{lllll} & U=0, & \theta=0, & for\ all & Y,t\le 0\\ t>0: & U=gt, & \theta=1, & at & Y=0\\ & U\to 0, & \theta\to 0, & as & Y\to\infty\end{array} \tag{12}$$

Where $g=\frac{L_1}{L_3}$

Methods of solutions

Equation (10) is solved subject to the initial and boundary conditions with the help of laplace transform analytically. The exact solutions are expressed in terms of exponential function and complementary error function.

$$\theta=\frac{1}{2}\left[\exp(2\eta\sqrt{abt})\ erfc\left(\eta\sqrt{a}+\sqrt{bt}\right)+\exp(-2\eta\sqrt{abt})\ erfc\left(\eta\sqrt{a}-\sqrt{bt}\right)\right] \tag{13}$$

$$U = \frac{gt}{2}\begin{bmatrix} \exp(2\eta\sqrt{ht})\ erfc\left(\eta\sqrt{g}+\sqrt{ht}\right) \\ +\exp(-2\eta\sqrt{ht})\ erfc\left(\eta\sqrt{g}-\sqrt{ht}\right) \end{bmatrix}$$

$$-\frac{\eta\sqrt{g}\sqrt{t}}{2\sqrt{h}}\begin{bmatrix} \exp(-2\eta\sqrt{ht})\ erfc\left(\eta\sqrt{g}-\sqrt{ht}\right) \\ +\exp(2\eta\sqrt{ht})\ erfc\left(\eta\sqrt{g}+\sqrt{ht}\right) \end{bmatrix}$$

$$+\frac{c}{2d}\begin{bmatrix} \exp(2\eta\sqrt{ght})\ erfc\left(\eta\sqrt{g}+\sqrt{ht}\right) \\ +\exp(-2\eta\sqrt{ght})\ erfc\left(\eta\sqrt{g}-\sqrt{ht}\right) \end{bmatrix}$$

$$-\frac{c\exp(dt)}{2d}\begin{bmatrix} \exp(2\eta\sqrt{g(h+d)t})\ erfc\left(\eta\sqrt{g}+\sqrt{(h+d)t}\right) \\ +\exp(-2\eta\sqrt{g(h+d)t}\ erfc\left(\eta\sqrt{g}-\sqrt{(h+d)t}\right) \end{bmatrix}$$

$$-\frac{c}{2d}\begin{bmatrix} \exp(2\eta\sqrt{abt})\ erfc\left(\eta\sqrt{a}+\sqrt{bt}\right) \\ +\exp(-2\eta\sqrt{abt})\ erfc\left(\eta\sqrt{a}-\sqrt{bt}\right) \end{bmatrix}$$

$$+\frac{c\exp(dt)}{2d}\begin{bmatrix} \exp(2\eta\sqrt{a(b+d)t})\ erfc\left(\eta\sqrt{a}+\sqrt{(b+d)t}\right) \\ +\exp(-2\eta\sqrt{a(b+d)t})\ erfc\left(\eta\sqrt{a}-\sqrt{(b+d)t}\right) \end{bmatrix} \quad (14)$$

Where $a=\frac{L_5\Pr}{L_6}, b=\frac{R}{L_5\Pr}, c=\frac{L_2 Gr}{L_1-L_3 a}, d=\frac{L_3 ab}{L_1-L_3 a}, g=\frac{L_1}{L_3}, h=\frac{1}{KL_1}$

3. Results and Discussion: In order to get physical insight in to the problem, the computed values of different parameters like thermal Grashof number, Prandtl number, time, volume solid fraction and radiation parameter on the temperature of the fluid and velocity of the plate are considered and it is depicted in graph. The effects are explained in a detailed manner. By changing the values of various parameters for the above expression, the numerical values are computed until they converge to free stream boundary conditions. We have considered three different types of nanofluids containing Copper (Cu), Aluminum oxide (Al_2O_3) and Titanium oxide (TiO_2) with water as a base fluid. The nanoparticle volume fraction is considered in the range of $0 \le \Phi \le 0.2$. In this study, we have considered spherical nanoparticles with thermal conductivity and dynamic viscosity shown in model I in table 2.

Table 2: Thermal Conductivity and Dynamic Viscosity for Various Shapes of Nanoparticles

Model	Shape Of Nanoparticles	Thermal Conductivity	Dynamic Viscosity
I	Spherical	$\frac{k_{nf}}{k_f}=\frac{k_s+2k_f-2\phi(k_f-k_s)}{k_s+2k_f+\phi(k_f-k_s)}$	$\mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}}$
II	Spherical (nanotubes)	$\frac{k_{nf}}{k_f}=\frac{k_s+2k_f-2\phi(k_f-k_s)}{k_s+2k_f+\phi(k_f-k_s)}$	$\mu_{nf}=\mu_f(1+7.3\phi+12.3\phi^2)$
III	Cylindrical	$\frac{k_{nf}}{k_f}=\frac{k_s+\frac{1}{2}k_f-\frac{1}{2}\phi(k_f-k_s)}{k_s+\frac{1}{2}k_f+\phi(k_f-k_s)}$	$\mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}}$

IV	Cylindrical(nanotubes)	$\frac{k_{nf}}{k_f}=\frac{k_s+\frac{1}{2}k_f-\frac{1}{2}\phi(k_f-k_s)}{k_s+\frac{1}{2}k_f+\phi(k_f-k_s)}$	$\mu_{nf}=\mu_f(1+7.3\phi+12.3\phi^2)$

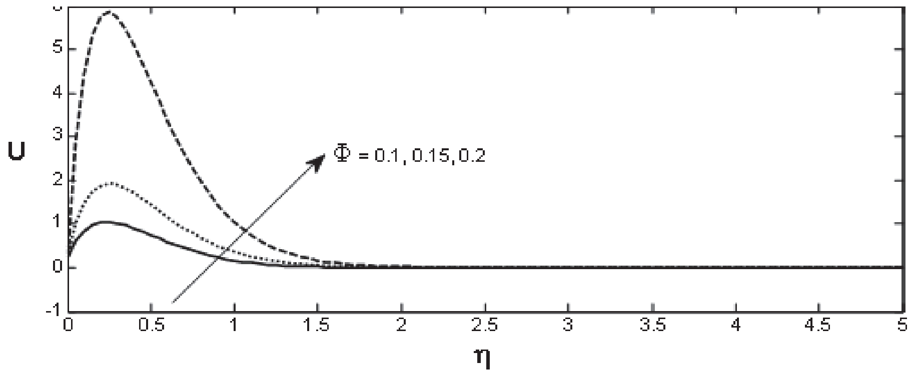

Fig.3.1 Velocity profiles for different values of Φ

Fig.3.1 explains the effect of volume of solid fraction (Φ = 0.1, 0.15, 0.2) parameter of Al_2O_3-water nanoparticles on the velocity profile and it can be easily seen that the velocity profile increases with increasing values of volume solid fraction. It is also revealed that the increase in the value of Φ results in the increase of velocity of the plate.

Fig.3.2 shows that the velocity profile of Al_2O_3-water nanoparticles for various values of Prandtl number (Pr = 0.71, 6.2, 7.1) with coordinate η. The values of different parameters t= 0.2, Gr = 5, ∅ = 0.1 is considered. It is noticed that velocity of the plate increases for the increasing values of Prandtl number.

The effect of radiation parameter (R=0.5, 1, 2) on the velocity profile is considered when Pr=0.71 which correspond to air ,thermal grashof number Gr=5 and time t=0.5. It is noticed that the velocity profile enhances as the value of radiation parameter increases is noticed in Fig.3.3.

The value profile increases sharply and it remains respective maxima, the curve's settle down to the corresponding asymptotic value, is observed in Fig.3.4 and it shows the influence of Al_2O_3-water nanoparticles on the velocity profile for the different time (t=0.1,0.2,0.3). It is also observed that whenever the time increases, the velocity of the plate also increases.

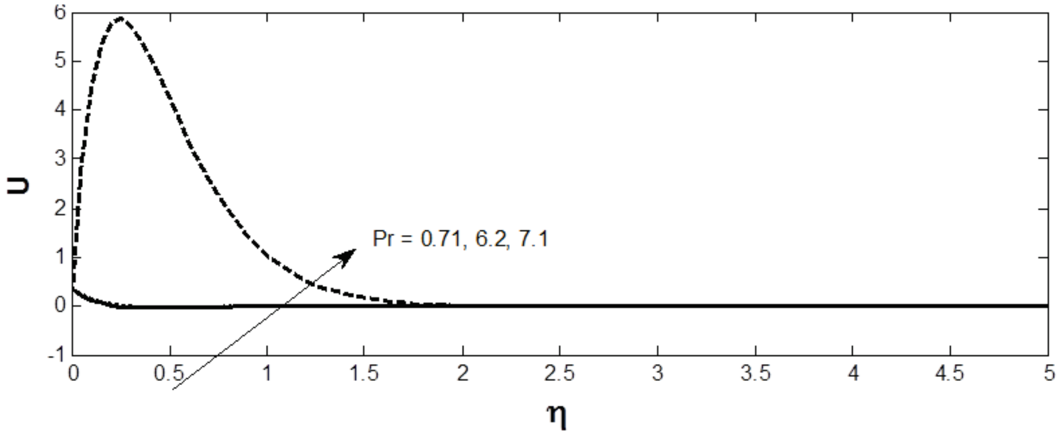

Fig.3.2 Velocity profiles for different values of Pr

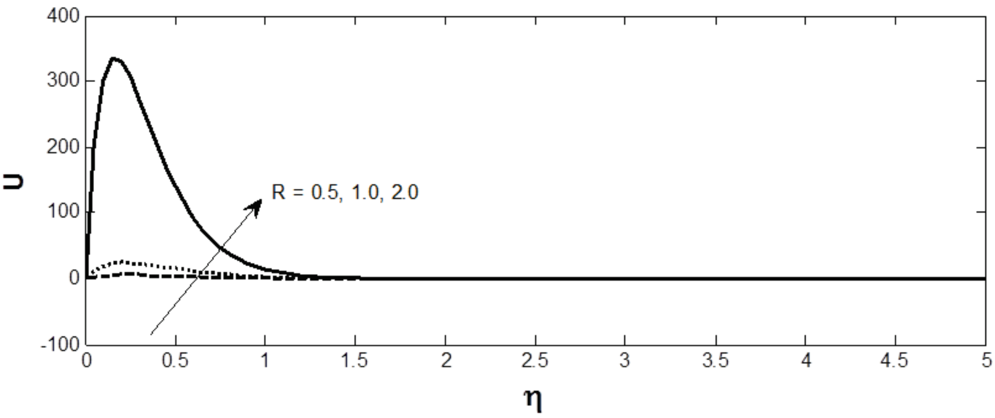

Fig.3.3 Velocity profiles for different values of R

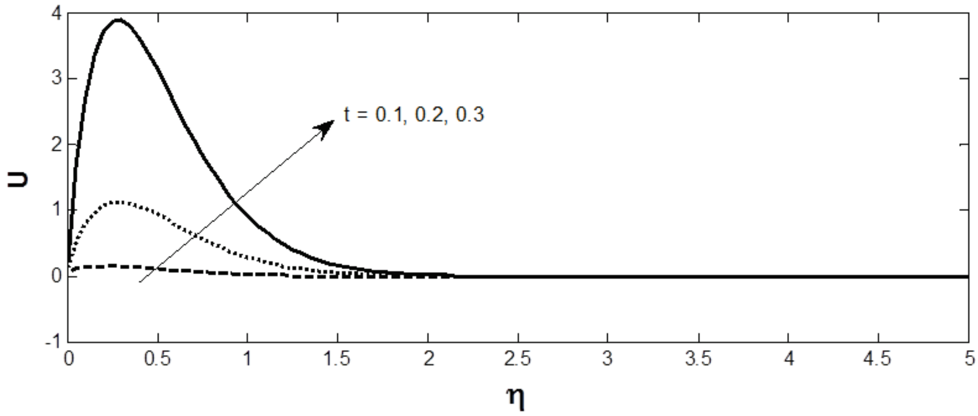

Fig.3.4 Velocity profiles for different values of t

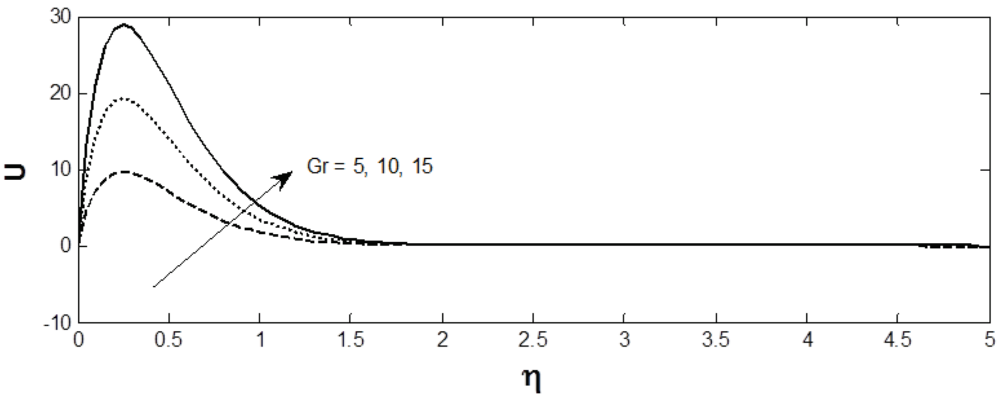

Fig.3.5 Velocity profiles for different values of Gr

The velocity profiles of Al_2O_3-water nanofluid with coordinate η for different values of thermal grashof number, radiation parameter ,time and volume solid fraction when Pr=0.71 are presented in, the effect of thermal grashof number on the velocity profile is shown in Fig.3.5 Thermal grashof number is the ratio of the buoyancy force due to spatial variation in the fluid density to the viscous force and grashof number (Gr = 5, 10, 15) represents the cooling effect near the plate. It is noticed that the velocity increases with increasing values of thermal grashof number.

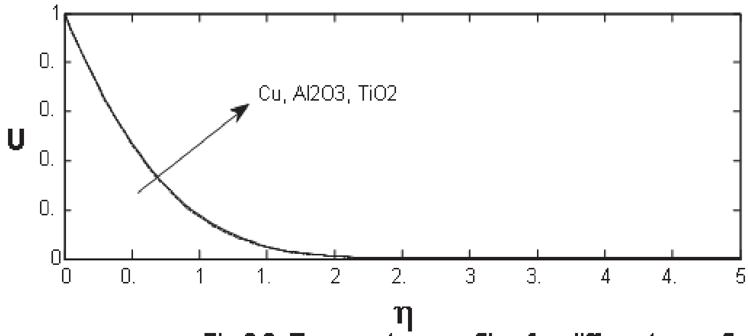

Fig.3.6. Temperature profiles for different nanofluids

Fig.3.6 reveals that the fluid temperature variations for the three types of water based nanofluids Al_2O_3 – water and TiO_2 -water when Pr=0.71 and Gr=5 at t=0.5.Due to higher thermal conductivity of Cu-water nanofluids, the temperature of Cu-water nanofluid is found to be higher than Al_2O_3 –water and TiO_2 – water nanofluids .It is also seen that the thermal boundary layer thickness is more for Al_2O_3 -water than Cu – water and TiO_2 – water nanofluids.

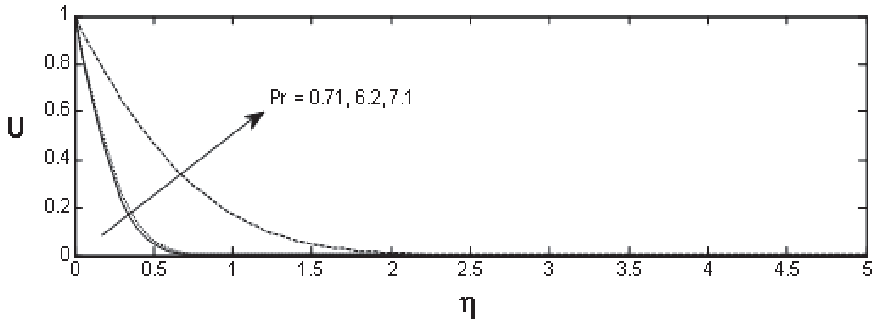

Fig.3.7. Temperature profiles for different values of Pr

In Fig. 3.7. temperature profile for different values of Prandtl number (Pr = 0.71 ,3, 7.1) is presented when R=3,Gr=5 at t=0.5. The size of the thermal boundary layer increases with increasing Prandtl number because the Boussinesq's approximation in the momentum equation consists of assuming that density of the fluid varies with temperature linearly.

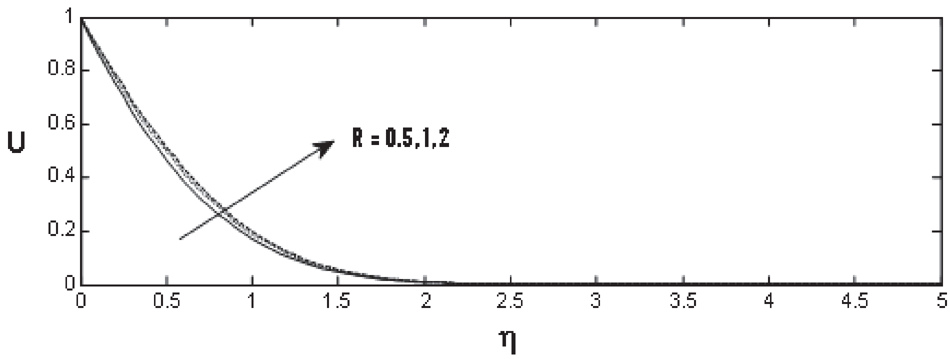

Fig.3.8. Temperature profiles for different values of

Here temperature profile gradually reduces to reach the free stream temperature which is shown in Fig.3.8. It can be seen that the effect of radiation parameter (R=0.5,1,2) on the temperature profile when Pr=0.71,Gr=5 at t=0.5.The increase in radiation parameter means the exhausting of heat energy from the flow region and so the temperature of the fluid increases.

Conclusion: The transient free convective Al_2O_3 nanofluid flow of a isothermal vertical plate in the presence of thermal radiation with porous medium was considered. The coupled governing equations are solved by using laplace transform technique. The effect of various parameters like Thermal grashof number, time Prandtl number, Radiation parameter on the velocity of the plate and temperature of the fluid are discussed in a detailed way and graphs are also depicted and the conclusions are as follows:

1. An increase in the radiation parameter as well as Prandtl number leads to the decrease in the temperature of the fluid.
2. An increase in the volume solid fraction, time, radiation parameter, Prandtl number and thermal grashof number leads to an increase in the velocity of the plate.

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