REMARKS ON DECOMPOSITION OF SUPRA M- CONTINUOUS MAPPINGS

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Abstract: In this paper, supra A-set, supra t-set, supra h-set and supra C-set and some new supra topological maps are introduced. Characterizations and properties of such new notions are studied. Also investigate the relationships with other mappings like supra α^*-continuous.

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Introduction: Njastad [7] initiated the concept of nearly open sets in topological spaces. Following it many research papers were introduced by Tong[12, 13], Przemski [2] and Ganster[3] in the name of "Decompositions of Continuity" in topological spaces. In 1983, Mashhour et al. [6] introduced the supra topological spaces and studied S –continuous maps and S* - continuous maps. In 2008, Devi et al. [1] introduced and studied a class of sets called supra α-open and a class of maps called sα-continuous maps between topological spaces, respectively. Ravi et al. [9] introduced and studied a class of sets called supra β-open and a class of maps called supra β-continuous, respectively. Kamaraj et al. [5] introduced and studied the concepts of supra regular-closed sets. It is an effort based on them to bring out a paper in the name of "Decompositions of supra M-continuity" in supra topological spaces using the new sets like supra A-set, supra t-set, supra h-set and supra C-set and new mappings like supra A-continuous, supra B-continuous map, supra α^*-continuous map, supra A*-continuous map, supra B*-continuous. In this paper, we obtain some important results in supra topological spaces. In most of the occasions, our ideas are illustrated and substantiated by suitable examples.

Preliminaries: Throughout this paper (X, τ), (Y, σ) and (Z, ν) (or simply, X, Y and Z) denote topological spaces on which no separation axioms are assumed unless explicitly stated.

Definition 2.1 [6, 10]: Let X be a non-empty set. The subfamily $\mu \subseteq P(X)$ where P(X) is the power set of X is said to be a supra topology on X if $X \in \mu$ and μ is closed under arbitrary unions.
The pair (X, μ) is called a supra topological space.
The elements of μ are said to be supra open in (X, μ).
Complements of supra open sets are called supra closed sets.

Definition 2.2 [10]: Let A be a subset of (X, μ). Then
(i) the supra closure of a set A is, denoted by $cl^{\mu}(A)$, defined as
$cl^{\mu}(A) = \cap \{ B : B$ is a supra closed and $A \subseteq B\}$;
(ii) the supra interior of a set A is, denoted by $int^{\mu}(A)$, defined as
$int^{\mu}(A) = \cup\{G : G$ is a supra open and $A \supseteq G\}$.

Definition 2.3 [6]: Let (X, τ) be a topological space and μ be a supra topology on X. We call μ is a supra topology associated with τ if $\tau \subseteq \mu$.

Definition 2.4: Let (X, μ) be a supra topological space. A subset A of X is called
(i) supra semi-open set [10] if $A \subseteq cl^{\mu}(int^{\mu}(A))$;
(ii) supra α-open set [1, 10] if $A \subseteq int^{\mu}(cl^{\mu}(int^{\mu}(A)))$;
(iii) supra regular-open [9] if $A= int^{\mu}(cl^{\mu}(A))$;
(iv) supra pre-open set [11] if $A \subseteq int^{\mu}(cl^{\mu}(A))$.

The complements of the above mentioned open sets are called their respective closed sets. The family of all supra regular-closed sets of X is denoted by SRC(X).

Supra C-sets: In this section we introduce a new type of set as follows:

Definition 3.1: A subset S of X is said to be

(i) supra A-set if S = M $\cap$ N where M is supra open set and N is SRC(X);

(ii) supra t-set if $int^{\mu}(cl^{\mu}(S)) = int^{\mu}(S)$;

(iii) supra B-set if S = M$\cap$N where M is supra open and N is a supra t-set;

(iv) supra C-set if S = M $\cap$ N where M is supra open and N is a supra h-set.

(v) supra h-set if $int^{\mu}(cl^{\mu}(int^{\mu}(S))) = int^{\mu}(S)$;

Theorem 3.2: Let (X, μ) be a supra topological space. If A is a supra t-set of X and B $\subseteq$ X with A $\subseteq$ B $\subseteq$ $cl^{\mu}(A)$ then B is a supra t-set.

Proof: We note that $cl^{\mu}(B) \subseteq cl^{\mu}(A)$. So we have $int^{\mu}(B) \subseteq int^{\mu}(cl^{\mu}(B) \subseteq int^{\mu}(cl^{\mu}(A)) = int^{\mu}(A) \subseteq int^{\mu}(B)$. Thus $int^{\mu}(B) = int^{\mu}(cl^{\mu}(B))$ and hence B is supra t-set.

Remark 3.3:

1. The union of two supra h-set need not be a supra h-set.
2. The union of two supra t-set need not be a supra t-set.

Example 3.4: Let X = {a, b, c} with μ = {X, φ, {a, b} {a, c} {b, c}}. Here {a} and {b} are both supra t-set and supra h set but their union {a, b} is not both supra t-and supra h-sets.

Comparisons:

Theorem 4.1: Any supra open set is a supra A-set.

Proof: S = X $\cap$ S where X $\in$ SRC(X) and S is supra open. The proof is completed.

The converse of the above theorem is not true as can be seen from the following examples.

Example 4.2: Let X = {a, b, c, d} with μ = {X, φ, {a}, {a, d},{b, c, d}}. Here {d} is supra A-set but not supra open.

Theorem 4.3: Any supra closed set is a supra t-set but not converse.

Proof: Since A = $cl^{\mu}(A)$, $int^{\mu}(A) = int^{\mu}(cl^{\mu}(A))$. The proof is completed.

Example 4.4: Consider Example 4.2, {b} is supra t-set but not supra closed.

Theorem 4.5: A supra regular-open set is a supra t-set but not converse.

Proof: Since S = $int^{\mu}(cl^{\mu}(S))$, $int^{\mu}(S) = int^{\mu}(cl^{\mu}(S))$. The proof is completed.

Example 4.5: Consider Example 4.2, {b} is supra t-set but not supra regular-open.

Theorem 4.6: A supra regular-open set is supra open but not converse.

Proof: Suppose S is supra regular-open set then S = $int^{\mu}(cl^{\mu}(S))$. Then $int^{\mu}(S) = int^{\mu}(cl^{\mu}(S))$. Since S is supra regular-open, we have $int^{\mu}(S)$ = S. Thus S is supra open. The proof is completed.

Example 4.7: Consider the Example 4.2, {a, d} is supra open set but not supra regular open.

Theorem 4.8: Every supra t-set is supra B-set.

Proof: Let S be any supra t-set S = X $\cap$ S where X is supra open and S is supra t-set. The proof is completed.

The converse of the above theorem is not true as can be seen from the following example.

Example 4.9: Consider Example 4.2, {d} is supra B-set but not supra t-set.

Theorem 4.10: Any supra open set is a supra B-set.
Proof: Since S = X ∩ S where S is supra open and X is supra regular open, by Theorem 4.5, X is supra t-set. The proof is completed.
The converse of the above theorem is not true as can be seen the following example.

Example 4.11: Consider Example 4.2, {c} is supra B-set but not supra open set.

Theorem 4.12: Any supra closed is a supra B-set.
Proof: It follows from Theorem 4.3 and Theorem 4.5

Theorem 4.13: Every supra A-set is a supra B-set.
Proof: S = X ∩ S where X is supra open and S is supra regular-closed. Since S is supra closed, by Theorem 4.3, S is supra t-set. The proof is completed.
The converse of the above Theorem is not true as can be seen from the following example.

Example 4.14: Consider Example 4.2, {c} is supra B-set but not supra A-set.

Theorem 4.15: Any supra t-set is supra h-set but not converse.
Proof: Let S be supra t-set, then $int^{\mu}(S) = int^{\mu}(cl^{\mu}(S))$, $cl^{\mu}(int^{\mu}(S)) = cl^{\mu}(int^{\mu}(cl^{\mu}(S))$ implies $int^{\mu}(cl^{\mu}(int^{\mu}(S)) = int^{\mu}(cl^{\mu}(S)) = int^{\mu}(S)$. The proof is completed.
The converse of the above theorem is not true as can seen from the following example.

Example 4.16: Let X = {a, b, c} with μ = {X, φ, {a, b}, {b, c}}. Here {b} is supra h-set but not supra t-set.

Theorem 4.17: Let (X, μ) be the supra topological Space.
(a) Any supra A-set is supra C-set.
(b) Any supra open set is supra C-set
Proof:
(a) S = X ∩ S where X is supra open and S is supra h-set. The proof is completed.
(b) S = X ∩ S where X is supra h-set and S is supra open set. The proof is completed.
The converse of the above Theorem is not true as can be seen from the following example.

Example 4.18: Let X = {a, b, c} with μ = {X, φ, {a, b},{b, c}}. Here {b, c} is supra C-set but not supra h-set. Also {a} is supra C-set but not supra open set.

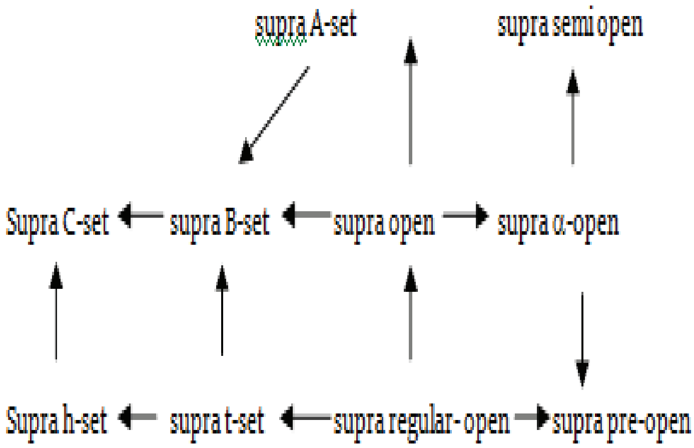
Theorem 4.19: Every supra B-set is supra supra C-set.
Proof: S = X ∩ S where X is supra open and S is supra t-set. By Theorem 4.15, S is supra h-set. The proof is completed.
The converse of the above theorem is not true as can be seen from following Example.

Example 4.20: Consider Example 4.18, {b} is supra C-set but not supra B-set.

Remark 4.21: Supra A-set and supra semi open-sets are independent.
Consider Example 4.2. Here {d} is supra A-set but not supra semi-open set. Also {a, b, d} is supra semi open set but not supra A-set.

Remark 4.22: From the above discussions we have the following diagram of implications

Decomposition of Supra M-Continuity:
Definition 5.1: Let (X, τ) and (Y, σ) be two topological spaces with τ ⊆ μ and σ ⊆ λ. A map f: (X, μ) → (Y, λ) is said to be
(i) supra M-continuous (supra -irresolute [9]) if $f^{-1}(V)$ is supra open in X for every supra open V of Y;
(ii) supra α-continuous [1] if $f^{-1}(V)$ is supra α-open in X for every open V of Y.
We introduce a new class of mappings as follows.



None of the Above Implications is Reversible

Definition 5.2: Let (X, τ) and (Y, σ) be two topological spaces with $\tau \subseteq \mu$ and $\sigma \subseteq \lambda$. A map f: (X, μ)→(Y, λ) is said to be

(i) supra α*-continuous if $f^{-1}(V)$ is supra α-open in X for every supra open set V of Y;
(ii) supra A-continuous if $f^{-1}(V)$ is supra A set in X for every open set V of Y;
(iii) supra A*-continuous if $f^{-1}(V)$ is supra A set in X for every supra open set V of Y;
(iv) supra B-continuous if $f^{-1}(V)$ is supra B-set in X for every open set V of Y;
(v) supra B*-continuous if $f^{-1}(V)$ is supra B-set in X for every supra open set V of Y;
(vi) supra C-continuous if $f^{-1}(V)$ is supra C-set in X for every open set V of Y;
(vii) supra C*-continuous if $f^{-1}(V)$ is supra C-set in X for every supra open set V of Y.

Theorem 5.3: A set S of X is supra regular-open if and only if S is supra pre-open and supra t-set.
Proof: Let S be supra regular-open. By theorem 4.5, S is supra t-set. Also By Theorem 4.6, S is supra open. Thus S is supra pre-open.
Conversely, Let S be supra pre-open and supra t-set. Since $int^{\mu}(S) \subseteq S \subseteq int^{\mu}(cl^{\mu}(S)) = int^{\mu}(S)$, $S = int^{\mu}(cl^{\mu}(S))$. Hence, S is supra regular open.

Theorem 5.4: A subset S of X is supra open if and only if it is both supra α-open and supra A-set.
Proof: Let S be supra open. Then S is supra α-open and by Theorem 4.1, S is supra A-set. Conversely, Let S be supra α-open and supra A-set. Since S is supra A-set, S = X ∩ S where X is supra open and S ∈ SRC(X). Since S is supra α-open,

$X \cap S \subseteq int^{\mu}(cl^{\mu}(int^{\mu}(X \cap S))$
$= int^{\mu}(cl^{\mu}(int^{\mu}(X) \cap int^{\mu}(S)))$
$= int^{\mu}(cl^{\mu}(X \cap int^{\mu}(S)))$ (as X is supra open)
$\subseteq int^{\mu}(cl^{\mu}(X) \cap cl^{\mu}(int^{\mu}(S)))$
$= int^{\mu}(cl^{\mu}(X) \cap S)$ as $S \in SRC(X)$
$\subseteq int^{\mu}(cl^{\mu}(X) \cap int^{\mu}(S)$ --------------------- (1)

Now since $X \subseteq int^{\mu}(cl^{\mu}(X)$, by(1)

S $= X \cap S = (X \cap S) \cap X$
$\subseteq (int^{\mu}(cl^{\mu}(X) \cap int^{\mu}(S)) \cap X$
$\subseteq X \cap int^{\mu}((S)) \cap X$
$= X \cap int^{\mu}(S)$
$= int^{\mu}(S)$

Therefore $S \subseteq int^{\mu}(S)$ But $int^{\mu}(S) \subseteq S$. Hence S is supra-open.

Theorem 5.5: A subset S of X is supra open if and only if supra α-open and supra B-set.
Proof: Let S be a supra open set. Then S is supra α-open. Also, by Theorem 4.10, S is supra B-set. Conversely let S be supra α-open and supra B-set. Since S is supra B-set, S = X $\cap$ S where X is supra open and S is supra t-set. Then S = X $\cap$ S $\subseteq$ X $\cap$ $int^{\mu}(cl^{\mu}(S))$ (as S is supra pre-open) = X $\cap$ $int^{\mu}(S)$ (as S is supra t-set). We have S $\subseteq$ X $\cap$ $int^{\mu}(S)$ implies S $\subseteq$ $int^{\mu}(S)$. But always $int^{\mu}(S) \subseteq S$. Thus S = $int^{\mu}(S)$ and S is supra open.

Theorem 5.6: A subset S is supra open in X if and only if S is supra α-open set and supra C-set.
Proof: Let S be supra open in X. Then S is supra α-open set and by Theorem 4.17, S is supra C-set. Conversely, let S be a supra α-open set and supra C-set. Since S is supra C-set, S = X $\cap$ S where X is supra open and S is supra h-set. Since S is supra α-open and S is supra h-set. Since S is supra α-open set, S $\subseteq$ $int^{\mu}(cl^{\mu}(int^{\mu}(S)))$ = $int^{\mu}(cl^{\mu}(int^{\mu}(X \cap S)))$ $\subseteq$ $int^{\mu}(cl^{\mu}(int^{\mu}(X)))$ $\cap int^{\mu}(cl^{\mu}(int^{\mu}(S)))$ = $int^{\mu}(cl^{\mu}(X))$ $\cap$ $int^{\mu}(S)$ (as X is supra open and S is supra h-set). Now S = X $\cap$ S = X $\cap$ (X $\cap$ S) = X $\cap$ S $\subseteq$ X $\cap$ $(int^{\mu}(cl^{\mu}(X)) \cap int^{\mu}(S))$ $\subseteq$ X $\cap$ $int^{\mu}(S)$ (as X $\subseteq$ $int^{\mu}(cl^{\mu}(X))$. S $\subseteq$ $int^{\mu}(S)$. but $int^{\mu}(S) \subseteq S$. Thus S = $int^{\mu}(S)$ and S is supra open.

Theorem 5.7: Let (X, τ) and (Y, σ) be two topological space with $\tau \subseteq \mu$, $\sigma \subseteq \lambda$. Let f: (X, μ)$\rightarrow$(Y, σ) be a mapping. Then f is supra M-continuity if and only if

(i) f is supra α^*-continuous and supra A-continuous.
(ii) f is supra α^*-continuous and supra B-continuous.
(iii) f is supra α^*-continuous and supra C-continuous.

Proof: It is the decompositions of supra M-continuity from Theorem 5.4, 5.5, 5.6.

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