TRIPLE CONNECTED COMPLEMENTARY TREE EQUITABLE DOMINATION

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Abstract. A dominating set $S \subseteq V(G)$ is said to be Triple connected complementary tree equitable dominating set if S is a triple connected equitable dominating set and $\langle V-S \rangle$ is a tree. The minimum cardinality taken over all triple connected complementary tree equitable dominating sets is called the triple connected complementary tree equitable domination number of G and is denoted by $\gamma_{tc-cteq}$. In this paper, we introduce the concept of triple connected complementary tree equitable dominating set and we found this number for some standard graphs. Also some results on triple connected complementary tree equitable dominating sets are established.

Keywords: Triple Connected Dominating Set, Equitable Dominating Set, Triple Connected Complementary Equitable Dominating Set.

1. Introduction: A subset S of V is called a dominating set of G if every vertex in V -S is adjacent to at least one vertex in S. The domination number $\gamma(G)$ is the minimum cardinality taken over all dominating sets in G. A dominating set S of a connected graph G is said to be a connected dominating set of G if the induced sub graph $\langle S \rangle$ is connected. The minimum cardinality taken over all connected dominating sets is the connected domination number and is denoted by $\gamma_c(G)$. Many authors have introduced different types of domination parameters by imposing conditions on the dominating set [5]. Recently, the concept of triple connected graphs has been introduced by Paulraj Joseph *et.al* [4] by considering the existence of a path containing any three vertices of G. They have studied the properties of triple connected graphs and established many results on them. A graph G is said to be triple connected if any three vertices of G lie on a path in G. All paths, cycles, complete graphs and wheels are some standard examples of triple connected graphs. G. Mahadevan etal., introduced triple connected domination number of a graph. A subset S of V of a nontrivial connected graph G is said to be triple connected dominating set, if S is a dominating set and the induced sub graph $\langle S \rangle$ is triple connected. The minimum cardinality taken over all triple connected dominating sets is called the triple connected domination number of G and is denoted by $\gamma_{tc}(G)$. A dominating set $S \subseteq V(G)$ is said to be equitable dominating set if for every $v \in V-S$, there exists a vertex $u \in S$ such that $uv \in E(G)$ and $|\deg(u)-\deg(v)| \leq 1$. A dominating set $S \subseteq V(G)$ is said to be triple connected equitable dominating set if S is a equitable dominating set and $\langle S \rangle$ is triple connected In this paper, we introduce the concept of triple connected complementary tree equitable dominating set and we found this number for some standard graphs. Also some results on triple connected complementary tree equitable dominating sets are established.

Theorem1.1[4] A connected graph G is not triple connected if and only if there exists a H-cut with $\omega(G-H) \geq 3$, such that $|V(H) \cap N(C_i)| = 1$ for at least three components C_1, C_2 and C_3 of G-H.

2. Triple Connected Complementary Tree Equitable Domination:

Definition 2.1: A dominating set $S \subseteq V(G)$ is said to be Triple connected complementary tree equitable dominating set if S is a triple connected equitable dominating set and $\langle V - S \rangle$ is a tree. The minimum cardinality taken over all triple connected complementary tree equitable dominating sets is called the triple connected complementary tree equitable domination number of G and is denoted by $\gamma_{tc-cteq}$.

Example 2.2: For the graph G in Figure 2.1, $S = \{v_2, v_3, v_5\}$ forms a $\gamma_{tceq-ct}$ set of G. Hence $\gamma_{tc-cteq}(G) = 3$.

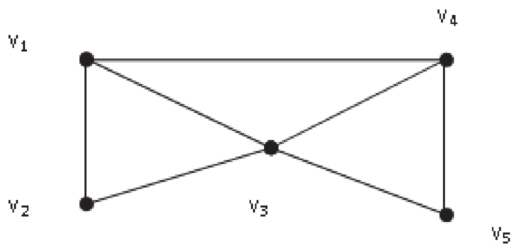

Figure 2.1

Observation 2.3: Triple connected complementary tree equitable dominating set does not for all graph and if exists then $\gamma_{tc-cteq}(G) \geq 3$.

Example 2.4: For K_2, there does not exist a triple connected complementary tree equitable dominating set.

Throughout, this paper we consider only connected graphs for which Triple connected complementary tree equitable dominating set exists.

Observation 2.5: The complement of the triple connected complementary tree equitable dominating set need not be a triple connected complementary tree equitable dominating set.

Example 2.6: For the graph G in Figure 2.2, S = $\{v_2, v_3, v_4\}$ forms a triple connected complementary tree equitable dominating set of G. But the complement V-S = $\{v_1, v_5\}$ is not a triple connected complementary tree equitable dominating set.

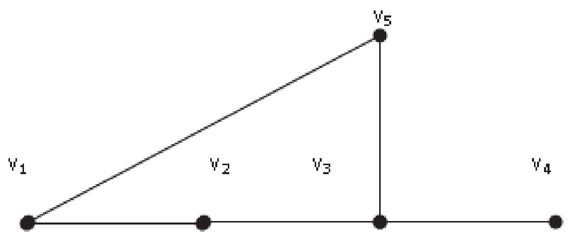

Figure 2.2

Observation 2.8: Every triple connected complementary tree equitable dominating set is a dominating set but not conversely.

Example 2.9: For the graph G in Figure 2.3, S={v_3,v_4,v_5} forms the triple connected complementary tree equitable dominating set as well as the dominating set of G. But S = {v_1,v_2} is a dominating set but not a triple connected complementary tree equitable domination set.

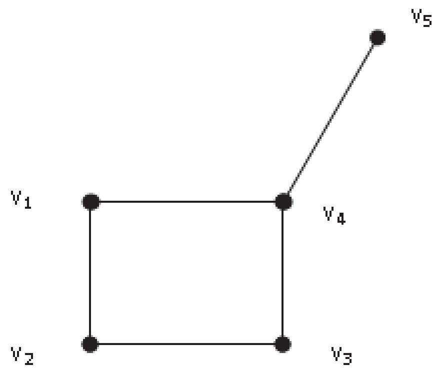

Figure 2.3

Theorem 2.10: If the induced sub graph of each connected dominating set of G has more than two pendent vertices, then G does not contain a triple connected complementary tree equitable dominating set.
Proof: The proof follows from theorem 1.1

Exact Value for Some Standard Graphs:

1. For any cycle of order p ≥ 3,

$$\gamma_{tc-cteq}(C_p) = p \text{ if } p < 5$$
$$= p - 2 \text{ if } p \geq 5$$

2. For any path of order p ≥ 3,

$$\gamma_{tc-cteq}(P_p) = p \text{ if } p = 3$$
$$= p - 1 \text{ if } p = 4$$
$$= p - 2 \text{ if } p \geq 5$$

3. For any complete graph of order p ≥ 3,

$$\gamma_{tc-cteq}(K_p) = p$$

4. For any complete bipartite graph of order p ≥ 4,

$$\gamma_{tc-cteq}(K_{m,n}) = p \text{ where } m + n = p$$

5. For any wheel graph of order p ≥ 3,

$$\gamma_{tc-cteq}(W_p) = p$$

Theorem 2.11: For any connected graph G with p ≥ 3, we have $3 \leq \gamma_{tc-cteq}(G) \leq n$ and the bounds are sharp.
Proof: The lower and upper bound follows from definition 2.1.

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